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Existence and Nonlinear Stability of Stationary Solutions to the Viscous Two-Phase Flow Model in a Half Line

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Abstract. The outflow problem for the viscous two-phase flow model in a half line is investigated in the present paper. The existence and uniqueness of the stationary solution is shown for both supersonic state and sonic state at spatial far field, and the nonlinear time stability of the stationary solution is also established in the weighted Sobolev space with either the exponential time decay rate for supersonic flow or the algebraic time decay rate for sonic flow.

AMS subject classifications: 35B35, 35B40, 76T10

Key words: Two-phase flow, outflow problem, stationary solution, nonlinear stability.

1 Introduction

The two-phase flow model plays a practically important part in nuclear, engineering, oil-and-gas, the analysis of fluidization and the study of sedimentation phenomena which is used in medicine, chemical engineering or waste water treatment [5, 8, 15]. For strongly coupled motions of two phases, it is appropriate to simplify the two-phase flow model as the drift-flux model which is based on

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the mixture momentum equation with the constitutive equation [8]. Due to the simplicity, accurateness and practically broad applications, the drift-flux model is significantly important for two-phase flow models and was mainly derived by Zuber and Findlay [31], Ishii and Hibiki [8]. In this paper, we consider the drift-flux model which was formally obtained from a Vlasov-Fokker-Planck equation coupled with compressible Navier-Stokes equations by some asymptotic limits in [1,15].

We are concerned with the initial boundary value problem for the drift-flux model as follows:

$$\begin{cases} n_t+(nu)_x=0, \\ \rho_t+(\rho u)_x=0, \\ \big[(\rho+n)u\big]_t+\big[(\rho+n)u^2+p(n,\rho)\big]_x=\mu u_{xx}, \quad x>0, \quad t>0, \end{cases} \tag{1.1}$$

where u is the mixed velocity of the fluid and particle, $\rho>0$ and $n>0$ stand for the densities of two fluids. The constant μ is the viscosity coefficient. The pressure satisfies

$$p(n,\rho)=A_1\rho^\gamma+A_2n^\alpha \tag{1.2}$$

with four constants $A_1>0$, $A_2>0$, $\gamma\geq 1$ and $\alpha\geq 1$. The initial data satisfy

$$(n,\rho,u)(0,x)=(n_0,\rho_0,u_0)(x), \quad \inf_{x\in\mathbb{R}_+} n_0(x)>0, \quad \inf_{x\in\mathbb{R}_+}\rho_0(x)>0, \tag{1.3}$$

and the outflow boundary condition is imposed

$$u(t,0)=u_-<0, \tag{1.4}$$

where ρ_+, n_+, u_+ and u_- are constants.

There have been important progress about incompressible inviscid limit, global existence, time decay rate of solutions made recently on the drift-flux model. For instance, the incompressible inviscid limit of the solution to Cauchy problem for (1.1)-(1.2) in 3D have been shown in [13]. The global existence of weak solution to Dirichlet's problem in 3D for (1.1)-(1.2) have been proved in [22]. For (1.1)-(1.2) with magnetic field, Wen and Zhu in [23] obtained the well-posedness and time decay estimates of strong solution to Cauchy problem in 3D. For (1.1)-(1.2) with magnetic field, it should be emphasized that Yin and Zhu in [29] got the nonlinear stability with the exponential or the algebraic time decay rate to outflow problem for the supersonic case. The existence of global weak solutions to Cauchy problem for (1.1) was studied in [3,25]. The existence of global weak solutions to free boundary value problem for (1.1) in 1D can be found in [2,4,26,27]. The global well-posedness of strong solution to Cauchy problem in Besov space