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ANALYSIS OF AN EPIDEMIC MODEL WITH CROWLEY-MARTIN INCIDENCE RATE AND HOLLING TYPE II TREATMENT*†

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Abstract

We propose an epidemic model consisting five compartments within a total population with Crowley-Martin incidence rate and Holling type II treatment, where total population is separated by the susceptible, the vaccinated, the exposed, the infected and the removed in this paper. We firstly prove that the epidemic model admits a unique global positive solution by contradiction. We then find out that diseases tend to extinction provided that the basic reproduction number is less than one. Moreover, the sufficient conditions of persistence for infectious diseases are obtained by constructing suitable Lyapunov functions.

Keywords epidemic model; Crowley-Martin incidence rate; extinction; persistence

2000 Mathematics Subject Classification 60H10

1 Introduction

Spread of infectious diseases normally have showed great impacts upon individuals health and society economy, even upon social panic and extinction for some species. Then in applications, how to prevent and control infectious diseases is a challenge for human being. And in the view of mathematicians, prevention and control of infectious diseases will be a main topic when human beings encounter modified epidemic models, which usually consider the susceptible (S, for short), the infected (I, for short) and the recovered (R, for short) to be basic compartments of a total population. For instance, some epidemic models [1–5] described spread mech-

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anisms and diffusion approaches for some specific topics. Recent contributions have discussed epidemic models in multiple views. We here mentioned that the interactions between the susceptible and the infected have been paid much attention for several decades. For example, some researchers considered that interaction between susceptible individuals might be restricted by their own density, hence a saturated incidence rate $\frac{\alpha SI}{1+\beta S}$ [6,7] was introduced into epidemic models instead of bilinear incidence rate αSI [8], where α means the infection rate and β denotes the saturation constant which describes the epidemiological spread of infectious diseases. Here, Zhang *et al.* analyzed an SIR epidemic model with incubation time and saturated incidence rate $\frac{\alpha SI}{1+\beta S}$ in [6], in which they showed that if the basic reproduction number satisfies $R_0 < 1$, the disease-free equilibrium is globally asymptotically stable and the disease dies out. While for the case $R_0 > 1$, the epidemic model admits an endemic equilibrium. Similarly, saturated incidence rate $\frac{\alpha SI}{1+\beta I}$ was also extensively considered by authors, among which the inhibitory effect of infected individuals was more focused, see [9–11] and references therein. And also, several types of incidence rates have been raised, for example, [12, 13] involved Beddington-DeAngelis type incidence rate, and Crowley-Martin type with form of $\frac{\alpha SI}{(1+\beta S)(1+\gamma I)}$ was considered in [14–17].

Epidemic models with saturated treatment rate were extended by authors in [21–23]. Especially, Upadhyay *et al.* [23] proposed the following SEIR model with Crowley-Martin incidence rate and treatment function $h(I)$:

$$\begin{cases} \dfrac{\mathrm{d}S(t)}{\mathrm{d}t} = \Lambda - \dfrac{\alpha SI}{(1+\beta S)(1+\gamma I)} - \delta_0 S, \\ \dfrac{\mathrm{d}E(t)}{\mathrm{d}t} = \dfrac{\alpha SI}{(1+\beta S)(1+\gamma I)} - (\delta_0+\delta_1)E, \\ \dfrac{\mathrm{d}I(t)}{\mathrm{d}t} = \delta_1 E - \dfrac{\delta_2 uI}{1+\alpha_1 uI} - h(I) - (\delta_0+\delta_3)I, \\ \dfrac{\mathrm{d}R(t)}{\mathrm{d}t} = \dfrac{\delta_2 uI}{1+\alpha_1 uI} + h(I) - \delta_0 R, \end{cases} \tag{1}$$

where $E(t)$ represents the density of exposed individuals at time t, and δ_0 is the natural death rate, δ_1 means the transmission rate from E to I, and δ_3 indicates the death rate of disease, u is the treatment control constant, δ_2 and α_1 are non-negative. The treatment function $h(I)$ here is displayed by

$$h(I) = \frac{\beta_1 I}{1+\alpha_2 I}, \quad \text{Holling type II} \tag{2}$$

or

$$h(I) = \frac{\beta_1 I^2}{1+\alpha_2 I^2}, \quad \text{Holling type III,} \tag{3}$$