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LONG-TERM DYNAMIC ANALYSIS OF ENDANGERED SPECIES WITH STAGE-STRUCTURE AND MIGRATIONS IN POLLUTED ENVIRONMENTS*†

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Abstract

We propose a stochastic stage-structured single-species model with migrations and hunting within a polluted environment, where the species is separated into two groups: the immature and the mature, which migrates from one patch to another with different migration rates. By constructing a Lyapunov function, together with stochastic analysis approach, the stochastic single-species model admits a unique global positive solution. We then utilize the comparison theorem of stochastic differential equations to investigate the extinction and persistence of solution to stochastic single-species model. The main results indicate that the species densities all depend on the intensities of random perturbations within both patches. As a consequence, we further provide several strategies for protecting endangered species within protected and unprotected patches.

Keywords protection zones; stage-structure; random perturbations; migration; extinction and persistence

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1 Introduction

The establishment of the protection zones within some countries or areas around the world has been widely accepted and recognized as an efficient strategy to avoid the endangered species from extinction when faced the excessive activities of human beings. The researchers proposed and formulated the population models between unprotected and protected patches to investigate how the dynamic mechanics of the

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species undertook in the long run. For instance, Zou and Wang [2] studied the dynamic behaviors of a deterministic single-species model with diffusion between two patches

$$\begin{cases} \dot{x}(t) = rx(t)\Big(1 - \dfrac{x(t)}{K}\Big) - \dfrac{d}{H}(x(t) - y(t)) - Ex(t), \\ \dot{y}(t) = ry(t)\Big(1 - \dfrac{y(t)}{K}\Big) + \dfrac{d}{h}(x(t) - y(t)), \end{cases} \tag{1}$$

where $x(t)$ and $y(t)$ respectively represent population densities of the species in unprotected and protected patches at time t, r means the intrinsic growth rate, K refers to the carrying capacity of environments, d is the diffusion coefficient, E denotes the hunting rate in unprotected patch. By assuming that the size of the unprotected patches is H and the size of the protected patches is h; the diffusion term is proportional to the differences of densities between two patches, by considering a size-dependent single-species model with migration and hunting, they derived that reducing the diffusion coefficient and increasing the size of the protection zone are both propitious for the enhancement of population levels within protected patch. Other latest results regarding the protection zones could be found in the literatures [4,13,21,31-34].

Pollution of environments often seriously threatens spaces for survival for most species on the globe. The fact is that, some species become extinction, and more species are endangered when they simultaneously face environmental and human pollution. Nowadays, it therefore is of especial importance to study the persistence and extinction for endangered species within polluted environments. Recently, many scholars have conducted extensive researches on endangered species regarding polluted environment and toxins distribution, and have obtained some good related results. For example, Srinivsu [26] studied a single-population model in which the input toxin is a constant, and obtained sufficient conditions for the consistence, persistence and extinction. Later, Yan *et al.* [27] found the criteria for survival and extinction by comparison theorem of ordinary differential equations to study the dynamic behaviors of a single-population model, when the population growth rate in the polluted environment is nonlinearly related to the toxin concentration in species. Almost at the same year, Yang *et al.* [28] investigated the persistence of a single-population model in polluted environment by Dulac function method, and obtained sufficient conditions for the global stability of positive equilibrium state to the model, and further explained its corresponding biological meaning.

We in this paper propose a single-species model in which the species moves between patches: unprotected patch and protected patch. We denote the density of individuals for endangered species in unprotected patch at time t by $x(t)$, and the