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A FLUID-PARTICLE MODEL WITH ELECTRIC FIELDS NEAR A LOCAL MAXWELLIAN WITH RAREFACTION WAVE*†

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Dedicated to the 90th Birthday of Xiaqi Ding

Abstract

The paper is concerned with time-asymptotic behavior of solution near a local Maxwellian with rarefaction wave to a fluid-particle model described by the Vlasov-Fokker-Planck equation coupled with the compressible and inviscid fluid by Euler-Poisson equations through the relaxation drag frictions, Vlasov forces between the macroscopic and microscopic momentums and the electrostatic potential forces. Precisely, based on a new micro-macro decomposition around the local Maxwellian to the kinetic part of the fluid-particle coupled system, which was first developed in [16], we show the time-asymptotically nonlinear stability of rarefaction wave to the one-dimensional compressible inviscid Euler equations coupled with both the Vlasov-Fokker-Planck equation and Poisson equation.

Keywords fluid-particle model; rarefaction wave; time-asymptotic stability
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1 Introduction and Main Result

This paper is devoted to the time-asymptotic behavior of solutions to a system modeling the evolution of dispersed particles in a compressible charged fluid under the effect of electrostatic potential forces and at the microscopic scale, the cloud of

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particles is described by a Vlasov-Fokker-Planck equation, and the charged fluid is modeled by Euler-Poisson equations (denoted by EP-VFP in abbreviation) with electric fields. The fluid-particle interactions are driven by a relaxation drag frictions force exerted by the macroscopic fluid onto the particles. Such a model was first introduced by Williams in the context of combustion theory [30], and can also be found in [3], which reads

$$\begin{cases} f_t + v \cdot \nabla_x f - \nabla_x \Phi \cdot \nabla_v f = \mathrm{div}_v[(v-u)f + \nabla_v f], \\ \rho_t + \mathrm{div}_x(\rho u) = 0, \\ (\rho u)_t + \mathrm{div}_x(\rho u \otimes u) + \nabla_x p(\rho) = \displaystyle\int_{\mathbb{R}^3} (v-u) f \mathrm{d}v + \rho \nabla_x \Phi, \\ -\Delta_x \Phi = \displaystyle\int f \mathrm{d}v - \rho, \end{cases} \tag{1.1}$$

where the time variable $t \in \mathbb{R}^+$, spatial variables $x = (x_1, x_2, x_3) \in \mathbb{R}^3$ and the particle velocity $v = (v_1, v_2, v_3) \in \mathbb{R}^3$. In system (1.3), $f = f(t, x, v)$ is the distribution function of particles at time t and position x and with microscopic velocity v and $\rho = \rho(t, x) > 0$ and $u = u(t, x) = (u_1, u_2, u_3)(t, x)$ are the fluid density and velocity, respectively. Here the fluid pressure $p = p(\rho)$ is given by the usual γ-laws

$$p(\rho) = A\rho^\gamma, \quad \gamma \geq 1, \tag{1.2}$$

with the fluid constant $A > 0$ and will be normalized to be 1 in the present paper.

Fluid-particle model arises in a lot of industrial applications. One example is the analysis of sedimentation phenomenon, with applications in medicine, chemical engineering or waste water treatment. Such systems are also used in the modeling of aerosols and sprays with applications, for instance, in the study of Diesel engines (see [5, 30]). First, when the self-consistent electric field is ignored, some mathematical process has been made to the fluid-particle model (1.3). For example, some stability properties and a formal asymptotic analysis to this coupled system were first investigated by Carrillo and Goudon in [5]. Then Mellet and Vasseur [26] used the relative entropy method to give a rigorously mathematical proof for the formal asymptotic analysis to the NS-VFP system in [5]. Furthermore, Mellet and Vasseur [25] established the global existence of weak solutions to a system of a kinetic equation coupled with compressible Navier-Stokes equations, the fluid is assumed to be barotropic with γ-pressure law ($\gamma > 3/2$). In [6], Chae, Kang and Lee showed the existence of the global classical solutions close to an equilibrium, and further proved that the solutions converged to the equilibrium exponentially for the NS-VFP system on three-dimensional torus. Then Li, Mu and Wang [12] established the global well-posedness of a strong solution to the 3D NS-VFP system in whole space with the small initial perturbation of some given equilibrium. Moreover, the algebraic rate of convergence of a solution toward the equilibrium state was obtained. For the