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# THERMAL CREEP FLOW IN THE RAREFIED GAS*

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**Dedicated to the 90th Birthday of Xiaqi Ding**

**Abstract**

The usual heat flow moves along the direction from high temperature place to the low one, as often observed in the daily life. However, when the gas is very rarefied, the gas may move along a different way, that is, the so-called thermal creep flow moves along the direction from the low temperature place to the high one. In this note, we will survey our recent mathematical works on this topic, mainly based on [27] and [25].

**Keywords** thermal creep flow; rarefied gas; Boltzmann equation; low Mach limit; Compressible Navier-Stokes equations

**2000 Mathematics Subject Classification** 76P05

## 1 Introduction

The usual heat flow moves along the direction from high temperature place to the low one, as often observed in the daily life. However, when the gas is very rarefied, an interesting phenomenon may happen. A so-called thermal creep flow moves along a different direction, that is, from the low temperature area to the high one, see the pictures below,

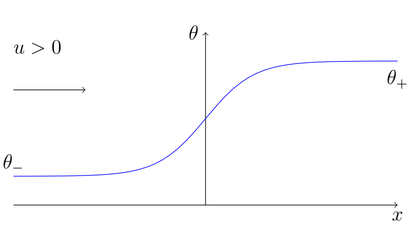


Figure 1: $\theta_- < \theta_+$

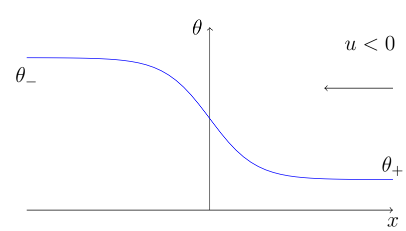


Figure 2: $\theta_- > \theta_+$

where $\theta$ denotes the temperature and $u$ denotes the velocity of the flow. The thermal creep flow in a rarefied gas was known since the time of Maxwell. Since the funda-

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mental equation in statistical physics for rarefied gas is the Boltzmann equation, it is naturally conjectured that the Boltzmann equation provides more information in the microscopic level and can model the thermal creep flow. Although there have been a large of numerical computations on the basis of kinetic equation since 1960s, see [46–48] and the references therein, rigorous mathematical analysis is few. The first mathematial analysis was given in [10], later by [27]. Recently the same phenomenon was also observed in the low Mach limit of compressible Navier-Stokes equations with non-trivial profile, see [25]. In the following two sections, we will survey the works on the thermal creep flow in both the Boltzmann equation [27] and the low Mach limit of the compressible Navier-Stokes equations [25].

## 2 Boltzmann Equation

The non-dimensional Boltzmann equation takes the form

$$Sh\partial_s f + \xi \cdot \nabla_z f = \frac{1}{\varepsilon} Q(f,f), \quad (s,z,\xi) \in \mathbb{R}_+ \times \mathbb{R}^3 \times \mathbb{R}^3,$$

where $f(s,z,\xi) \geq 0$ is the distribution function of particles, $Q(f,f)$ is the collision operator with a kernel determined by the particle interaction. There are two parameters $Sh$ and $\varepsilon$ called Strouhal and Knudsen numbers respectively. Their product $Sh \cdot \varepsilon$ is $2\sqrt{\pi}$ times the ratio of the mean free time to the reference time.

There has been tremendous progress on the mathematical theories for the Boltzmann equation, see [4, 5, 7, 12, 14, 15, 17, 26, 28, 29, 36, 37, 51–53] and the references therein. Among them, the classical works of Hilbert, Chapman-Enskog revealed the close relation of the Boltzmann equation to the classical systems of fluid dynamics through asymptotic expansions when $Sh = 1$ and $\varepsilon$ is small.

On the other hand, there are some phenomena described by the Boltzmann equation, for which the time evolution of the macroscopic components are not governed by the classical fluid dynamic systems. This happens, for example, when the parameters $Sh$ and $\varepsilon$ as well as the macroscopic velocity are small while the density and temperature are of the order 1, such as the thermal creep flow phenomenon. There have been a lot of studies on this kind of phenomena which is called the "ghost effect" in the Boltzmann equation and most of the results are mainly built on the asymptotic expansions and numerical computations, see [5, 46] and the references therein. A rigorous mathematical analysis for the ghost effect was first given in [10] where the thermal creep flow was studied for the stationary linearized Boltzmann equation. Recently we justified the thermal creep flow phenomenon for the Boltzmann equation [27], whose main results were outlined as follows.

We assume both the Strouhal number and the macroscopic velocity (that is, flow velocity) is of the order of $\varepsilon$, and rewrite the Boltzmann equation under the