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An Adaptive Data-Fitting Model for Speckle Reduction of Log-Compressed Ultrasound Images

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Abstract. A good statistical model of speckle formation is useful to design a good speckle reduction model for clinical ultrasound images. We propose a new general distribution to describe the distribution of speckle in clinical ultrasound images according to a log-compression algorithm of clinical ultrasound imaging. A new variational model is designed to remove the speckle noise with the proposed general distribution. The efficiency of the proposed model is confirmed by experiments on synthetic images and real ultrasound images. Compared with previous variational methods which assign a designated distribution, the proposed method is adaptive to remove different kinds of speckle noise by estimating parameters to find suitable distribution. The experiments show that the proposed method can adaptively remove different types of speckle noise.

AMS subject classifications: 65K10, 68U10, 94A08

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1 Introduction

It is well known that medical ultrasound imaging is a low-cost, no side-effect, noninvasive, and real time imaging modality. However, medical ultrasound images suffer from speckle noise which appears in all coherent imaging systems such as synthetic aperture radar (SAR), sonar (SAS), ultrasound and laser imaging. Image information under coherent waves results in a granular pattern that is speckle [1]. Generally, speckle is an undesirable property as it can mask small but perhaps diagnostically significant image

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features [2]. Therefore, speckle noise should be removed for the sake of further image processing task. In the literature, numerous methods for speckle suppression were proposed. There are adaptive filters such as filters based on MAP estimator [2, 3] and nonlocal means filter [6].Wavelet method or wavelet MAP combined method are reported in the despeckling problems [7]. Anisotropic diffusion methods [8, 9] and nonlinear diffusion equation based method [4, 5] are also proposed for speckle reduction. Unlike additive Gaussian noise, speckle noise is much more difficult to be removed from the corrupted images, mainly because of not only their multiplicative nature, but also their statistical characteristic [10]. A multiplicative form:

$$f=uz, \tag{1.1}$$

has been generally used to model speckle noise. f is the observed noisy image, u is the echogenicity or ground truth of the image, and z is the noise term [11]. For an ultrasound image, u represents the echo envelop, and f is the noisy version of u. Recently, several methods have been proposed for multiplicative noise removal. In [12], a convex image restoration model was introduced for simultaneously structured and multiplicative noise removal. In [13], a convex variational model based on generalized Kullback-Leibler distance was proposed to remove the speckles in ultrasound images. For the multiplicative Gamma noise removal, a nonlinear diffusion equation with smooth solution was proposed in [14].

Methods based on total variation (TV), which have outstanding performance in the removal of additive Gaussian noise and the preservation of edges, such as Rudin-Osher-Fatami (ROF) model [15], have been introduced in the literature of speckle reduction. Thus, in this paper, we will concentrate on the despeckling methods based on TV regularization. As far as we know, a variational approach devoted to speckle noise was first proposed by Rudin, Lions and Osher [16], which reads as

$$\min_u J(u), \tag{1.2}$$

s.t.

$$\begin{aligned}
&\int_\Omega \frac{f}{u}=1 \quad \text{(mean 1)},\\
&\int_\Omega \Big(\frac{f}{u}-1\Big)^2=\sigma^2 \quad \text{(known variance)},
\end{aligned}$$

where $J(u)=\int_\Omega |Du|$ is the TV regularization term. These constraints come from the assumption that the speckle noise z in (1.1) satisfies the Gaussian distribution with mean 1 and known variance.

Another TV model based on Rayleigh distribution was presented by using Markov Random Fields (MRFs) [17]:

$$\min_u J(u)+\lambda\int_\Omega \Big[\frac{f^2}{u^2}+2\log(u)\Big], \tag{1.3}$$