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Cross-Kink Wave Solutions and Semi-Inverse Variational Method for $(3+1)$-Dimensional Potential-YTSF Equation

Jalil Manafian[1,*], Onur Alp İlhan[2], Karmina K. Ali[3] and Sizar Abid Mohammed[4]

[1]*Department of Applied Mathematics, Faculty of Mathematical Sciences, University of Tabriz, Tabriz, Iran.*
[2]*Department of Mathematics, Faculty of Education, Erciyes University, 38039-Melikgazi-Kayseri, Turkey.*
[3]*Department of Mathematics, Faculty of Science, University of Zakho, Zakho, Iraq.*
[4]*Department of Mathematics, College of Basic Education, University of Duhok, Zakho Street 38, 1006 AJ Duhok, Iraq.*

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Abstract. Periodic wave solutions of $(3+1)$-dimensional potential-Yu-Toda-Sasa-Fukuyama (YTSF) equation are constructed. Using the bilinear form of this equation, we chose ansatz as a combination of rational, trigonometric and hyperbolic functions. Density graphs of certain solutions in 3D and 2D situations show different cross-kink waveforms and new multi wave and cross-kink wave solutions. Moreover, we employ the semi-inverse variational principle (SIVP) in order to study the solitary, bright and dark soliton wave solutions of the YTSF equation.

AMS subject classifications: 35K20, 65M06, 65M12

Key words: Potential-Yu-Toda-Sasa-Fukuyama equation, Hirota bilinear operator method, semi-inverse variational principle, cross-kink wave solution, existence conditions.

1. Introduction

Many nonlinear phenomena, which play an important role in applied sciences and engineering are modeled by nonlinear partial differential equations (NPDEs). Numerous examples of such equations can be found in plasma physics, elastic media, optical fibers, fluid dynamics, quantum mechanics, chemical physics, biotechnology, signal processing, solid state physics, and shallow water wave theory. However, their explicit analytic solutions are rarely available. Therefore, finding localised solutions and, more specifically, solitary wave solutions [1, 6, 7, 28–30, 33–36, 41, 55], lump-type solutions [5, 14–17, 19, 26–28, 32,

*Corresponding author. *Email addresses:* j_manafianheris@tabrizu.ac.ir (J. Manafian), oailhan@erciyes.edu.tr (O.A. İlhan), karmina.ali@uoz.edu.krd(K.K. Ali), sizar@uod.ac (S.A. Mohammed)

33, 37, 38, 40, 44, 45, 49], and also describing the interactions soliton-soliton, soliton-kink, kink-kink [9, 18, 46], as well as the interaction between solitary waves, lumps [47, 53] and periodic wave solutions [2, 8, 31] is an interesting problem. The approaches used in these studies include exp-function method [4, 28], homotopy perturbation technique [3], and inverse scattering method [39].

The nonlinear (3 + 1)-dimensional potential-Yu-Toda-Sasa-Fukuyama equation has the form

$$-4u_{xt} + u_{xxxz} + 4u_x u_{xz} + 2u_{xx} u_z + 3u_{yy} = 0. \tag{1.1}$$

It appears in fluid dynamics, plasma physics, weakly dispersive media and other physical applications. Various powerful methods for solving (3+1)-dimensional YTSF equation, such as G'/G-expansion method [43], generalized projective Riccati equation method [51], symmetry method [48], Korteweg-de Vries equation-based sub-equation method [42], extended homoclinic test technique [50], homoclinic test approach and three-wave method [13] have been considered. Applying the dependent variable transformation

$$\eta = x + \omega z, \quad u = 2(\ln f)_\eta, \quad f = f(\eta, y, t), \tag{1.2}$$

one can transform (1.1) into the nonlinear equation

$$-4u_{\eta t} + \omega u_{\eta\eta\eta\eta} + 6\omega u_\eta u_{\eta\eta} + 3u_{yy} = 0,$$

and consequent application of the mapping

$$u = 2(\ln f)_\eta, \quad f = f(\eta, y, t)$$

leads to the Hirota bilinear form

$$\left(-4D_\eta D_t + \omega D_\eta^4 + 3D_y^2\right) f \cdot f = 0 \tag{1.3}$$

with a bilinear operator D and an unknown function $f = f(x, y, t)$, which has to be determined later on.

Suppose the Hirota derivatives for functions f and g can be written as

$$\prod_{i=1}^{3} D_{\jmath_i}^{\beta_i} f.g = \prod_{i=1}^{3} \left(\frac{\partial}{\partial \jmath_i} - \frac{\partial}{\partial \jmath_i'}\right)^{\beta_i} f(\jmath) g(\jmath')\bigg|_{\jmath'=\jmath},$$

where

$$\jmath = (\jmath_1, \jmath_2, \jmath_3) = (\eta, y, t), \quad \jmath' = (\jmath_1', \jmath_2', \jmath_3') = (\eta', y', t')$$

and $\beta_1, \beta_2, \beta_3$ are arbitrary nonnegative integers. The corresponding bilinear formalism for the Eq. (1.3) is

$$-4f f_{\eta t} + 4f_\eta f_t + \omega\left(f f_{\eta\eta\eta\eta} - 4f_\eta f_{\eta\eta\eta} + 3f_{\eta\eta}^2\right) + 3f f_{yy} - 3f_y^2 = 0. \tag{1.4}$$

For simplicity, we change η to x, so that the Eq. (1.4) takes the form

$$-4f f_{xt} + 4f_x f_t + \omega\left(f f_{xxx} - 4f_x f_{xxx} + 3f_{xx}^2\right) + 3f f_{yy} - 3f_y^2 = 0. \tag{1.5}$$