Multi-Frequency Contrast Source Inversion for Reflection Seismic Data

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Abstract. In the contrast source inversion (CSI) method, the contrast sources (equivalent scattering sources) and the contrast (parameter perturbation) are iteratively reconstructed by an alternating optimization scheme. Traditionally integral equation CSI method is formulated for transmission tomography using analytic Green's function in homogeneous background. To extend the method to the case of reflection seismology, in this paper, we use WKBJ method to compute the Green's function of depth dependent background media and the solving method of equations to initialize the contrast source of different frequencies, resulting in an efficient method to invert multi-frequency reflection seismic data – multi-frequency contrast source inversion method (MFCSI). Numerical results for the Marmousi model show that MFCSI method can obtain good results even when low frequency data are missing, in which case the conventional FWI fails.

1 Introduction

Full waveform inversion (FWI) was introduced to the exploration seismology community by Lailly in [12] and Tarantola in [18]. They regarded seismic inversion as a minimization of the misfit between recorded and modeled data. In [5], Bunks et al. developed FWI in the time domain and proposed successive inversion of multi-frequency band inversion. Pratt et al. studied the FWI in frequency domain in [15] and Shin extended FWI to the

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Laplace domain in [16]. Shin and Cha combined the frequency domain and Laplace domain and studied the FWI method in the Laplace-Fourier domain in [17]. Readers can be referred to [20] for a detailed review. On the other hand, the FWI can be considered as an inverse scattering problem. The inverse scattering problem has been studied for many years and it has numerous applications. For seismic imaging and inversion, see [22] for a review. In addition the approach has been developed in other fields, such as target identification, non-destructive testing and medical imaging. In [21], Wang and Chew (1989) proposed Born iterative method (BIM) where the Green's function remains unchanged in the iterative procedures for solving inverse scattering problem. In [6], Chew and Wang (1990) proposed the distorted Born iterative method (DBIM), where the Green's function is updated in every iteration step. In [11], Kleinman and van den Berg (1992) introduced the modified gradient method where the cost functional consists of the superposition of the mismatch of the measured field data with the calculated scattered field and the error in satisfying the state equation. The modified gradient method formed the base of the contrast source inversion (CSI) method in [19]. In the CSI method, the contrast sources (the product of contrast and total wavefield) and the contrast (perturbation) itself, are iteratively reconstructed by an alternately updating method. This is in contrast to the modified gradient method, where the fields and the contrast are updated simultaneously. The CSI method outperforms the modified gradient method, and is computationally faster and uses less memory. In [1], Abubakar et al. introduced the finite difference contrast source inversion (FDCSI) method. Unlike the CSI method using the integral equation (IE) approach, it uses a finite difference (FD) approach as its backbone and can readily employ an arbitrary inhomogeneous medium as its background media. In [2], Abubakar et al. applied the finite difference contrast source inversion method to seismic full waveform inversion problems and extended it to three dimensional geometry in [3]. In [4] Barrière et al. applied CSI method to relatively high contrast objects, in which situation the additive regularization can get better result. CSI method bears strong relationship with the T-matrix based inversion method [8–10, 13]. However, the perturbation and T-matrix updating in the above approach was realized by matrix operation, and the CSI method is still based on the least-square error minimization.

The traditional integral equation CSI method is commonly used in tomography, where the source and receiver positions locate in the opposite sides of the object and usually only single frequency data are used in the inversion [19]. For the reflection seismology, the source and receiver positions all locate on the earth's surface, which makes the inversion strong nonlinear and more difficult. Multiple frequency data must be used in the inversion in this case. For reflection geometry, data with different frequencies correspond to different wavenumber of the medium perturbation. In addition, a variable background medium must be used. Based on these features, we expand the CSI method and make it suitable for reflection seismic data. For the variable background we use the WKBJ method in [7] to compute the Green's function. Numerical results show that the multi-frequency (MFCSI) method can achieve better results than the conventional gradient method, especially for the case of data missing low frequency.