Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2019-0060

Vol. **27**, No. 5, pp. 1485-1504
May 2020

# Uncertainty Quantification of Density Reconstruction Using MCMC Method in High-Energy X-ray Radiography

Xinge Li, Haibo Xu*, Na Zheng, Qinggang Jia, Tongxiang Gu and Suhua Wei

*Institute of Applied Physics and Computational Mathematics, Beijing 100094, P.R. China.*

Received 12 April 2019; Accepted (in revised version) 25 September 2019

**Abstract.** High-energy X-ray radiography is a measuring technique for quantitative measurement and diagnosis of the object and its internal structure. Tomographic reconstruction determines the geometric and physical properties of the object according to the energy distribution on the imaging plane. Considering the noise and blur in the process of radiographing, we construct a general reconstruction model for the axisymmetric single image photographic system. This inverse problem is then cast within a statistical framework in order to compute volumetric object densities from X-ray radiographs and to quantify uncertainties in the reconstruction. A hierarchical Bayesian model is developed with a likelihood based on a Gaussian noise model and with priors placed on the unknown nonnegative density profile, the precision matrix, and two scale parameters. This results in a joint posterior distribution, which can be readily sampled using the Markov chain Monte Carlo (MCMC) method. To study the role of hyperparameters and their sensitivity analysis, a wide variety of tests were conducted which led to a number of definitive conclusions. Results of the density reconstructions and pointwise uncertainty estimates are presented for simulated signals with various physical factors in the imaging process included.

**AMS subject classifications**: 65K99, 65Z05, 62F15, 62P35

**Key words**: Inverse problem, density reconstruction, uncertainty quantification, Bayesian inference, MCMC method.

## 1 Introduction

With the aid of nondestructive characterization and high transmission ability of X-rays, X-ray tomography has been widely used in many areas, including radiation medicine,

*Corresponding author. *Email addresses:* `lixinge@lsec.cc.ac.cn` (X. Li), `xu_haibo@iapcm.ac.cn` (H. Xu), `zheng_na@iapcm.ac.cn` (N. Zheng), `QGJIA_XJTU@126.com` (Q. Jia), `txgu@iapcm.ac.cn` (T. Gu), `wei_suhua@iapcm.ac.cn` (S. Wei)

 

material science, earth science and nuclear physics. High-energy X-ray radiography measures the internal structure of the object according to the X-ray energy distribution on the imaging plane. The measured spatial density distribution is of great significance for studying the compression behavior of objects subjected to powerful shocks under the effect of explosives in hydro-test experiments. In this paper, we focus on the problem of uncertainty quantification of density reconstruction for high-energy X-ray radiography.

The essence of density reconstruction is to reconstruct the spatial density distribution of objects from the projection data obtained by transmission imaging. The existing reconstruction methods can be roughly classified into three categories, namely analytical methods [1–4], iterative methods [5–8] and statistical methods [9–13], which are extensively used in medical imaging diagnosis, industrial nondestructive detection and other fields. However, it is difficult to meet the reconstruction requirements of high-energy X-ray radiography under complex imaging engineering conditions. Uncertainty is a measure of the confidence in a calculated or measured value. A frequentist approach to uncertainty estimation would take a large number of experimental measurements and determine uncertainty from the spread of results obtained. This approach is not applicable in the case of hydro-test experiments which are costly and likely to be performed only once.

Given the challenging nature of hydro-test radiography, there are limitations in the quality of radiographs obtained. For the low photon flux that occurs with a highly attenuating object, the statistical variation of photons measured by the detector forms the output noise. Another significant issue is system blur, which results from the finite size of X-ray source spot and the energy spread within the detector. Both noise and blur make it difficult to extract information about edges and density distributions of materials, particularly near the interfaces. Therefore density reconstruction technology is highly demanded. Moreover, physical factors such as cone beam light source, scattering and energy spectrum effect must also be considered. In this paper, a Bayesian approach is used to reject those solutions that are unphysical, based on our experience and understanding of the physics involved. Examples of prior knowledge typically used are density nonnegativity and smoothness.

Bayesian formulations for inverse problems have gained considerable attention in the inverse problems community for their utility in uncertainty quantification [12, 14, 16, 17]. It has been pointed out that many of the classical regularization methods for solving ill-posed problems can be viewed as constructing estimators based on the posterior distribution [12]. A large part of the literature discussing regularization techniques is devoted to the problem of selecting the regularization parameters. Moreover, the prior densities typically depend on parameters such as variance and expectation that are always assumed to be known in traditional Bayesian approaches. From the point of view of classical methods, this corresponds to knowing ahead of time the regularization parameters. In the Bayesian framework the answer to the question of how the parameters should be chosen is: If a parameter is not known, it is a part of the inference problem. This approach leads to hierarchical models, which allow the incorporation of empirical knowledge, not just about the reconstruction being sought but also about the structure of the prior [18–21].