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Solitary Waves of 1-Nonlinear Schrödinger Equation in the Composite Right- and Left-Handed Metamaterial

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Abstract. In this article, we analyze solitary waves in nonlinear left-handed transmission line with nonlinear diodes (Schottkys) which is an important issue, especially for soliton devices. By applying the Kirchhoffs laws and reductive direct method, the voltage in the spectral domain was obtained. Considering the Taylor series around a certain modulation frequency, we obtained one dimensional Nonlinear Schrödinger Equation (NSE), which support envelops soliton, and bright soliton solutions. Using sine-cosine mathematical method, soliton solutions of the standard Nonlinear Schröd--inger equation are obtained. The method used is straightforward and concise and can be applied to solve further of nonlinear PDEs in mathematical physics.

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1 Introduction

During these last decades, the experimental and theoretical study of artificially designed structures with electromagnetic properties (EM), known as metamaterials, have been the

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object of particular interest of mathematicians and physicists [1, 2]. Recently, scientist have succeeded in producing materials with negative magnetic permeability [3] as well as Left-handed Metamaterials [4]. Left-handed (LH) Metamaterial have attracted the attention of microwave and RF engineers [5]. Recently, nonlinear left-handed materials that combine the nonlinearity with the anomalous dispersion exhibited by the left-handed material [6] has shown to exhibit a new electromagnetic (EM), such as the harmonic generation, parametric amplification, sub wavelength imaginary and solitary wave propagation [7, 8]. Many nonlinear systems exhibit an instability that leads to modulation of the steady state as a result of interplay between the nonlinear and dispersive effects [9, 10]. The balanced CRLH transmission lines with shunt voltage-dependent capacitance and demonstrates that both bright and dark solitons can propagate [11]. Another implementation of Nonlinear Left-Handed (NLH) transmission line was introduced in [12]. NLH transmission lines are also used as the nonlinear media, which incorporate loaded nonlinear capacitance or inductance elements with the ordinary left-handed structures [13]. One of the important phenomena associated with the nonlinear left-handed transmission lines is solitary wave propagation. In that respect, solitons represent one of the most striking aspects of nonlinear phenomena. Solitons haved an important role in many physical systems and it appears in various forms: kink, pulse, envelope, bright, breather, dark and many others.

The existence of solitons in an environment that requires it to be nonlinear and dispersive, these two features can be demonstrated through experiments in nonlinear left-handed transmission lines. Recently, many methods of investigation of exact solutions of nonlinear equations and particularly nonlinear Schrödinger equation have made enormous progress in mechanics, quantum mechanics, engineering, chemistry, biology and many others field of physics. In this fact, the scenario described by the nonlinear Schr dinger like equation (NSE), have been demonstrated in LH TL [14–21]. The achievement of Bright soliton, Dark soliton and envelop soliton have been the subject of important results [19–21] and which doubtless occupy a place of choice in the advanced technology. It is through well known and sometimes complex mathematical methods that these results are obtained. To achieve similar and convincing results, it will be presented the properties that characterize the CRLH. Thereafter, by adopting the reductive direct method it will be established the standard Schrödinger equation which leads us through the application of the sine-cosine method to soliton solutions.

The rest of the paper will be organized as follow: Sec. 2, will described the model and equation of motion. With the presence of the nonlinear element in the NLH TL, the nonlinear Schrödinger like equation is obtained. Then, Sec. 3 will be used mathematical method namely sine-cosine to derive the nonlinear Schrödinger equation obtained. The last section concluded the work.