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# A Unified Momentum Equation Approach for Computing Flow-Induced Stresses in Structures with Arbitrarily-Shaped Stationary Boundaries

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**Abstract.** This article presents a novel monolithic numerical method for computing flow-induced stresses for problems involving arbitrarily-shaped stationary boundaries. A unified momentum equation for a continuum consisting of both fluids and solids is derived in terms of velocity by hybridizing the momentum equations of incompressible fluids and linear elastic solids. Discontinuities at the interface are smeared over a finite thickness around the interface using the signed distance function, and the resulting momentum equation implicitly takes care of the interfacial conditions without using a body-fitted grid. A finite volume approach is employed to discretize the obtained governing equations on a Cartesian grid. For validation purposes, this method has been applied to three examples, lid-driven cavity flow in a square cavity, lid-driven cavity flow in a circular cavity, and flow over a cylinder, where velocity and stress fields are simultaneously obtained for both fluids and structures. The simulation results agree well with the results found in the literature and the results obtained by COMSOL Multiphysics®.

**AMS subject classifications**: 74A10, 76M12

**Key words**: Flow induced stress, unified momentum equation, monolithic approach, smeared interface, stationary boundary.

## 1 Introduction

Flow induced stress is ubiquitous in nature and is at the core of many important engineering problems, such as vascular flows [1,2], flows around wind turbine blades [3], and ship in water [4]. Due to its complexity, the numerical approach is deemed the most practical, and therefore there have been a great deal of interest in making efficient and capable

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numerical algorithms [5]. Although it is important to compute both fluid flow and stress development in a structure in many cases, a proper coupling of the fluid and the structure is challenging due to their dissimilar governing equations and physical behavior. Furthermore, complex structure geometry makes the problem even more difficult.

Generally, there are two approaches for the coupling of flow and structure: partitioned approach and monolithic approach. In the partitioned approach, different phases are solved separately using different solvers and the information is transferred across the interface enforcing the interface conditions [6, 7]. This approach has been traditionally preferred because existing codes for both phases can be used, but the numerical instabilities may occur due to the added-mass effect [8]. A strongly-coupled scheme can remedy these problems with extra iterations, which however is computationally expensive [9]. On the other hand, the monolithic approach solves the flow-structure system with a single algorithm. Although the development of a well-conditioned system is difficult due to the entirely different properties of fluids and solids, this approach is more robust than the partitioned approach. Accordingly, many monolithic methods have been developed with various coupling strategies. For example, Hübner et al. developed a monolithic method based on the space-time finite element method, where a weighted residual formulation was used for the coupling [10]. Heil proposed a monolithic method for fluid-structure interaction problems by using Newton's method [11]. He showed that block-triangular approximations of the Jacobian matrix, which was obtained by neglecting selected fluid-structure interaction blocks, provide good preconditioners for the solution of the linear systems with GMRES. Ryzhakov et al. [12] derived a displacement-based monolithic formulation by using global pressure condensation, where a matrix-free technique was used for efficiency and a free surface flow with a flexible structure was solved. Franci et al. [13] introduced a unified formulation based on a mixed velocity-pressure formulation to simulate Newtonian fluids and quasi-incompressible hypoelastic structures where the finite element method was used for structures and the particle finite element method was used for fluids.

Another important issue is how to treat complex geometries of structures. One approach is the use of a body-fitted grid where grid is aligned with interface. Then, interfacial conditions can be directly enforced and obtaining the interaction between fluid and structure is straightforward. However, the grid generation and discretization procedure can become complicated. A more efficient approach is the use of a non-body-fitted grid, such as the immersed boundary method and the fictitious domain method. The immersed boundary method which was originally devised by Peskin [14, 15] to compute blood flow in heart valves, expresses an immersed structure as a momentum forcing term. This method is suited for fiber-like structures and was effectively applied to many problems. However, there is a difficulty in applying the method to rigid structures and various numerical methods have been proposed. For example, in the direct forcing method, the representation of a structure is simplified by imposing the no-slip condition at the interface [16, 17]. The cut-cell method is proposed to satisfy the con-