JOURNAL OF PARTIAL DIFFERENTIAL EQUATIONS
J. Part. Diff. Eq., Vol. **32**, No. 3, pp. 207-228

doi: 10.4208/jpde.v32.n3.2
September 2019

Bifurcation and Stability Analysis in Complex Cross-Diffusion Mathematical Model of Phytoplankton-Fish Dynamics

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Received 25 May 2018; Accepted 4 August 2019

Abstract. In this paper, we propose a nonlinear reaction-diffusion system describing the interaction between toxin-producing phytoplankton and fish population. We analyze the effect of cross-diffusion on the dynamics of the system. The mathematical study of the model leads us to have an idea on the existence of a solution, the existence of equilibria and the stability of the stationary equilibria. Finally, numerical simulations performed at two-dimensions allowed us to establish the formation of spatial patterns and a threshold of release of the toxin, above which we talk about the phytoplankton blooms.

AMS Subject Classifications: 65P30, 68Q85, 68U20.

Chinese Library Classifications: O175.27

Key Words: Toxin effect; populations dynamics; predator-prey model; reaction-diffusion system; bifurcation; pattern formation.

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1 Introduction

The major concern in population and community ecology is to understand how a population of a given species influences the dynamics of population of other species, which are members of the same interaction network, [1–4]. The problem of the management of bio-diversity resources in general and particularly, the halieutic resources management, interest many researchers. Interaction networks in natural ecosystems can be visualized

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as consisting of simple units known as food-chains or food-webs that consists of a number of species linked by tropic interactions.

Researchers have focused a great deal of their attention to analyze the dynamical behavior of model food chain. Two species continuous time models of interacting species have been extensively studied in literature, [5–8]. These models exhibit two basic patterns: approach to equilibrium (stable focus) or to a limit cycle. Three species continuous time and space models are reported to have more complicated patterns, [9]. These models form dissipative dynamical systems which can possess three distinct dynamical possibilities like stable focus, limit cycle and chaos in the phase space. The research of the last two decades demonstrates that very complex dynamics can arise in three or more species food chain models, [6, 10], while similar results are obtained for multi-species food web models, [11, 12]. May [13] reviewed the literature and concluded that the study of nonlinear systems in marine environment are indispensable as far as understanding about nature is concerned. Although it has been seen in the models quite a bit, yet there are very few examples from the laboratory as well as from the field. Therefore, it can be understood that no unambiguous evidence of chaos exists till date. In [4], the investigations into reason why chaos had been rarely observed in natural populations concluded that natural terrestrial ecosystems are not suitable candidates for the exploration of chaotic dynamics. This is paradoxical, since ecological systems have all the necessary characteristics (nonlinearity, high-dimensions, etc.) to be able to support chaotic dynamics. The existence of chaos in almost all the physical systems motivates to study the same in natural population, [4, 9, 14–17]. Since almost all form of scientific inquiry have found application of ideas from nonlinear dynamics and chaos, there is a natural curiosity and urge to explore the possibility of aquatic systems evolves on strange chaotic attractor or not?

Recent studies on ecological systems indicate that chaotic dynamics may play an important role in reaction diffusion models, [6, 9, 11, 18]. There are some evidences that the real time evolution of species involved in two or three food chains could be characterized by chaotic attractors as observed in many natural food chains. Now, the more challenging issue is the observation that natural systems seems to have no difficulty switching from one state into the other, from chaos to order and from order to chaos. In aquatic ecosystem, most aquatic life relies on plankton and the toxin producing phytoplankton may act as controlling factor for such dynamics. The role of toxin producing phytoplankton for reduction of grazing pressure of zooplankton is well known, [19]. Toxicity may be a strong mediator of fish feeding rate as shown by field studies, [9, 20, 21]. Areas rich in some phytoplankton organisms are unaccepted or avoided by zooplankton and fish due to dense concentration of phytoplankton or the production of toxic substances released by phytoplankton. This phenomena are well explained by "Exclusion principle", [4]. The role of toxin producing phytoplankton for the termination of planktonic blooms are investigated in [20]. In [22], it is considered the complex patterns in a predator prey model to analyze the diffusion-driven instability and stability, as well as cross-diffusion of the predator, under the influence of prey in the spatial model. These