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Sobolev-type Fractional Stochastic Differential Equations Driven by Fractional Brownian Motion with Non-Lipschitz Coefficients

ZHAN Wentao and LI Zhi*

School of Information and Mathematics, Yangtze University, Jingzhou 434023, China.

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Abstract. In this paper, we are concerned with the existence and uniqueness of mild solution for a class of nonlinear fractional Sobolev-type stochastic differential equations driven by fractional Brownian motion with Hurst parameter $H\in(1/2,1)$ in Hilbert space. We obtain the required result by using semigroup theory, stochastic analysis principle, fractional calculus and Picard iteration techniques with some non-Lipschitz conditions.

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Key Words: Fractional Sobolev-type stochastic differential equations; fractional Brownian motion; mild solution.

1 Introduction

In the research of various fields of science and engineering, fractional differential equations have been studied by many researchers because it can efficiently describe dynamical behavior of real life phenomena more accurately than integer order equations. Fractional differential equations have been applied successfully in many areas such as viscoelasticity, electrochemistry, nonlinear oscillation of earthquake, the fluid-dynamic traffic model, flow in porous media, aerodynamics and in different branches of physical and biological science [1–3].

Recently, fractional differential equations and stochastic fractional differential equations driven by fractional Brownian motion or Brownian motion have been considered

*Corresponding author. *Email address:* lizhi_csu@126.com. (Z. Li)

greatly by the research community in various aspects due to its salient features for real-world problems. Cui and Yan [4] investigated the existence of mild solutions for neutral fractional stochastic integral differential equations with infinite delay using Sadovskii's fixed point theorem. Sakthivel et al. [5] discussed the mild solutions for fractional stochastic differential equations with infinite delay and impulses. Furthermore, they proved the existence of mild solutions for nonlocal fractional stochastic differential equations. By constructing Picard successive approximation, Wang [6] established the approximate mild solutions of fractional stochastic differential equations. The existence and asymptotic stability of neutral fractional stochastic differential equations with infinite delays were studied by Sakthivel et al. [7]. On the other hand, the Sobolev-type fractional equation appears in a variety of physical problems such as flowof fluid through fissured rocks, thermodynamics, propagation of long waves of small amplitude and so on.

There are many interesting results on the existence and uniqueness of mild solutions for a class of Sobolev-type fractional evolution equations. The existence result of mild solutions of fractional integrodifferential equations of Sobolev-type with nonlocal condition in a separable Banach space is studied by using the theory of propagation family as well as the theory of the measures of noncompactness and the condensing maps [8]. The existence and uniqueness of mild solution to Sobolev-type fractional nonlocal dynamical equations in Banach spaces is reported in [9], where a new set of conditions is obtained to achieve the required result by using fractional power of operators, a singular version of Gronwall's inequality and Leray-Schauder fixed point theorem. Benchaabanea and Sakthivel (see [10]) investigated the existence and uniqueness of mild solutions for a class of nonlinear fractional Sobolev-type stochastic differential equations using Picard's iteration in Hilbert spaces with non-Lipschitz coefficients. Sakthivel et al (see [11]) investigated the approximate controllability of fractional stochastic differential inclusions with nonlocal conditions. the existence and uniqueness of mild solutions of nonlinear fractional Sobolev-type stochastic differential equations driven by fractional Brownian motion in Hilbert spaces has not been investigated yet and this motivates our study. In order to fill this gap, in this paper, we study the existence and uniqueness of mild soulutions for the following nonlinear Sobolev-type fractional stochastic differential equation driven by fractional Brownian motion.

The purpose of this paper is to investigate the existence and uniqueness of mild solutions to the equation. The appropriate definition of mild solutions is given, and the main results are obtained by employing Picard type approximate sequences.

The paper is organized as follows. In Section 2, some basic notations and preliminary facts on stochastic integrals and stochastic analysis theory for fractional Brownian motion, fractional calculus, and some special functions are given. In Section 3, the existence result is established by using stochastic analysis techniques, fractional calculation, semi-group theory and Picard type approximate sequences under some assumptions.