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The Plane Wave Discontinuous Galerkin Method Combined with Local Spectral Finite Elements for the Wave Propagation in Anisotropic Media

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Abstract. In this paper we are concerned with plane wave discretizations of nonhomogeneous and anisotropic time-harmonic Maxwell's equations. Combined with local spectral element method, we design a plane wave discontinuous Galerkin method for the discretization of such three dimensional nonhomogeneous and anisotropic Maxwell's equations. The error estimates of the approximation solutions generated by the proposed discretization method are derived in one special case of the TE mode scattering. In the error estimates, some dependence of the error bounds on the condition number of the anisotropic matrix is explicitly given. Numerical results indicate that the resulting approximate solutions generated by the new method possess high accuracy and verify the validity of the theoretical results.

AMS subject classifications: 65N30, 65N55
Key words: Electromagnetic wave, plane wave discontinuous Galerkin, local spectral elements, anisotropic media, plane-wave basis, error estimates.

1. Introduction

The plane wave method, which falls into the class of Trefftz methods [26], was first introduced to solve Helmholtz equations and was then extended to solve Maxwell's equations and time-harmonic elastic wave propagation. Examples of this approach include the variational theory of complex rays (VTCR) [23, 24, 29], the ultra weak variational formulation (UWVF) [3, 4, 6, 14–16, 28], the plane wave discontinuous Galerkin (PWDG) method [7, 9, 30], the plane wave least-squares (PWLS) method [11, 12, 22, 27, 31] and the plane wave least-squares combined with local spectral finite element (PWLS-LSFE) method [13]. The plane wave methods have an important advantage over linear, quadratic and quartic Lagrange finite elements for discretization of the Helmholtz equation and time-harmonic Maxwell equations [8–10, 16, 17]: the resulting approximate solutions have higher accuracies by using only a small number of DOFs, owing mainly to the choice of the basis functions satisfying the governing differential equation without boundary conditions.

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Since plane wave basis functions on each element are solutions of the *homogeneous* Helmholtz equation or time-harmonic Maxwell equations without boundary condition, the plane wave methods can not be directly applied to discretizations of the *nonhomogeneous* Helmholtz equation and time-harmonic Maxwell equations. Recently, the plane wave method combined with local spectral elements (PWLS-LSFE) for the discretization of such nonhomogeneous equations was firstly proposed in [13]. This method contains two steps: a series of nonhomogeneous local problems on auxiliary smooth subdomains are solved by the spectral element method, and then the plane wave method are employed to discretize the resulting (locally homogeneous) residue problem on the global solution domain. The numerical results show that the resulting approximate solution is more accurate than that generated by the PWLS method [12].

Recently, the UWVF method was extended to solve homogeneous Maxwell's equations in isotropic media in [14] and in anisotropic media in [15], respectively. The studies [14] were devoted to computing the electric and magnetic fields in a non-absorbing medium or within the PML. The studies [15] were devoted to approximating the Robin-type trace of the the electric and magnetic fields in an anisotropic medium or within the PML, and focus on the numerical test and convergence analysis in TM mode scattering, which can result in a Helmholtz equation in two dimensions with an anisotropic coefficient. Moreover, the anisotropic perfectly matched layer method [18] and the penalty discontinuous Galerkin method [32, 33] are proposed for Helmholtz scattering problems.

In this paper we are mainly interested in extending the PWDG method in an isotropic medium to the nonhomogeneous case in an anisotropic medium for three-dimensional system of Maxwell's equations. For convenience, we call the new method as PWDG-LSFE. Moreover, we derive error estimates of the approximate solutions generated by the PWDG-LSFE in one special case. The same procedure can also be generalized to the PWLS method developed in [13].

Numerical experiments show that the approximate solutions generated by the PWDG-LSFE for three-dimensional Maxwell's equations possess almost the same and satisfactory accuracy, and verify the validity of the theoretical results in the TE mode case. Particularly, the approximate solution generated by the PWDG method is clearly more accurate than that generated by the PWLS method for the homogeneous wave propagation problem in anisotropic media.

The paper is organized as follows. In Section 2, we describe the proposed PWDG-LSFE method for nonhomogeneous problems in anisotropic media associated with triangulation. In Section 3, we explain how to discretize the variational problem. In Section 4, we discuss the case of wave propagation in an orthotropic medium and give error estimates for the corresponding approximate solutions. Finally, we report some numerical results to confirm the effectiveness of the new method.

2. Local-global variational formulation for nonhomogeneous Maxwell's equations

In this section we shall recall the first-order system of Maxwell equations and derive the corresponding second-order system based on triangulation. Then we introduce local-