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On the Selection of a Good Shape Parameter of the Localized Method of Approximated Particular Solutions

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Abstract. In this paper, we propose a new approach for selecting suitable shape parameters of radial basis functions (RBFs) in the context of the localized method of approximated particular solutions. Traditionally, there are no direct connections on choosing good shape parameters and choosing interior and boundary nodes using the local collocation methods. As a result, the approximations of derivative functions are less accurate and the stability is also an issue. One of the focuses of this study is to select the interior and boundary nodes in a special way so that they are correlated. Furthermore, a test differential equation with known exact solution is selected and a good shape parameter for the given differential equation can be selected through a good shape parameter for the test differential equation. Three numerical examples, including a Poison's equation and an eigenvalue problem, are tested. Uniformly distributed node arrangement is compared with the proposed cross knot distribution with Dirichlet boundary conditions and mixed boundary conditions. The numerical results show some potentials for the proposed node arrangements and shape parameter selections.

AMS subject classifications: 65M70, 35J05, 65N99

Key words: Method of approximated particular solutions, shape parameter, radial basis functions, collocation methods, Kansa's method.

1 Introduction

The radial basis function (RBF) collocation method or the so-called Kansa's method [9] was proposed in early 1990's and has became very popular for solving various types of

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problems in science and engineering. The main attractions of the Kansa's method are its simplicity and high accuracy. Due to its simplicity, the Kansa's method is especially useful for solving high-dimensional problems with complicated domains. To alleviate the difficulty of dense and ill-conditioning system of linear equations in the formulation of the global RBF collocation methods, a number of localized RBF methods [13, 15, 17] were proposed for solving more challenging problems, which these methods can solve a system involves large number of RBF centers. As a result, the linear system created through collocation is sparse which allows us to solve large-scale problems in science and engineering. Despite all the favorable features of the newly developed RBF collocation methods, the accuracy of the approximated solutions heavily depends on the value of the shape parameters of RBFs. It is known that the determination of the optimal shape parameters of RBFs is still an outstanding research topic. There is still no theory or recipe for selecting the optimal shape parameters that can consistently apply to various applications. This issue has been studied by several authors such as Hardy [8], Franke [6], Foley [5], Carlson and Foley [1], Golberg et al. [7], Rippa [14], Kansa and Hon [10] and Larson and Fornberg [12], to name just a few. Most of the proposed approaches were given through experiments or statistics. Each proposed technique has its advantages and drawbacks. In his paper, Rippa [14] believes that the shape parameter should depend on a number of factors such as the number of grid points, distribution of grid points, RBF functions, condition number and computer precision.

The purpose of this short paper is to propose another approach for choosing a good shape parameter for solving partial differential equations using localized RBF collocation methods. The proposed method for selecting a good shape parameter is suitable for many methods that involve RBF collocation. In particular, we implement the proposed approach in the context of the localized method of approximated particular solutions (LMAPS) [17]. In most of the RBF collocation methods, the number and the distribution of the interior and boundary points are selected in an arbitrary way and there is not close relationship between them. It is known that RBF collocation methods can produce accurate solution but less accurate for the corresponding derivative functions' approximations. To achieve a better accuracy, it is important to find a way to more accurately approximate the derivatives using RBF collocation methods.

This paper builds upon several observations. We first observe that a better approximation of derivative functions can be achieved if the boundary points and interior points are all uniformly lined up in each axis direction such as the point distribution used in the finite difference method. Next, for selecting the shape parameter, we propose to choose a test function which is a solution of a differential equation with the same differential operator as the given differential equation. As we shall see, a good shape parameter of the given differential equation can be chosen through the test function.

The structure of the paper is as follows. In Section 2, we give a brief review of the LMAPS. In Section 3, we propose a new approach to distribute the boundary and the interior nodes. In Section 4, three numerical examples are given to demonstrate the effectiveness of the proposed method. In Section 5, some concluding remarks are given.