Commun. Comput. Phys.
doi: 10.4208/cicp.OA-2017-0189

Vol. **24**, No. 5, pp. 1300-1325
November 2018

A Direct ALE Multi-Moment Finite Volume Scheme for the Compressible Euler Equations

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Received 5 September 2017; Accepted (in revised version) 5 December 2017

Abstract. A direct Arbitrary Lagrangian Eulerian (ALE) method based on multi-moment finite volume scheme is developed for the Euler equations of compressible gas in 1D and 2D space. Both the volume integrated average (VIA) and the point values (PV) at cell vertices, which are used for high-order reconstructions, are treated as the computational variables and updated simultaneously by numerical formulations in integral and differential forms respectively. The VIAs of the conservative variables are solved by a finite volume method in the integral form of the governing equations to ensure the numerical conservativeness; whereas, the governing equations of differential form are solved for the PVs of the primitive variables to avoid the additional source terms generated from moving mesh, which largely simplifies the solution procedure. Numerical tests in both 1D and 2D are presented to demonstrate the performance of the proposed ALE scheme. The present multi-moment finite volume formulation consistent with moving meshes provides a high-order and efficient ALE computational model for compressible flows.

AMS subject classifications: 76M12, 76N15, 35L55

Key words: Compressible Euler equations, multi-moment finite volume method, direct ALE, Roe Riemann solver, HLLC Riemann solver, shock waves.

1 Introduction

Moving boundary problems exist widely in the practical applications of computational fluid dynamics (CFD), such as airfoil oscillations, flapping wings or fluid-structure interactions. When the boundary of fluid domain moves or undergoes deformations, the Arbitrary Lagrangian Eulerian method (ALE) [1,2] becomes a desirable numerical methodology to solve fluid dynamics in a moving and deforming grid which can fit the moving boundary of computational domain and get more accurate numerical solutions. In the

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ALE formulations, the grids of the computational domain can move arbitrarily independent of the fluid motions. This flexibility of mesh motion makes it more robust compared to the purely Lagrangian framework which may encounter difficulties when the mesh cells tangle or experience other topological changes due to flow fields of large distortions.

Efforts have been made so far to develop practical ALE algorithms. The existing works may be divided into two classes, i.e. indirect method and direct method. The indirect method was firstly proposed by Hirt et al. [1], which consists of three phases: (1) a Lagrangian phase where the solution and the grid are updated; (2) a rezoning phase that regularizes the tangled or heavily distorted mesh cells; (3) a remapping phase in which the Lagrangian solution is transferred to the rezoned mesh adjusted in step (2). However, rezoning and remapping procedures can be computationally expensive especially for two and three dimensional realistic calculations. The direct method [3, 4] does not need the remapping as a separate step because the mesh velocity is already taken into account in the numerical formulations that are consistent with the governing equations in the moving mesh framework. Both types of methods are widely implemented in the simulations of fluid dynamics.

In ALE schemes, the mesh geometry changes in time, which requires the geometrical quantities, such as volumes, boundary surfaces and vertices of moving cells, to be updated at each time step. Some existing works started from purely Lagrangian framework for developing mesh moving problems. Munz et al. [5] devised a cell-centered Godunov-type scheme using Roe [6] and HLL [7] flux solvers for Lagrangian hydrodynamics equations. Maire et al. [8–10] built a relationship between nodal displacement and flux formulation and presented a robust cell-centered Lagrangian method on multi-dimensional meshes with first and second order accuracy. To achieve high order accuracy, Cheng et al. [11] developed a class of Lagrangian type schemes based on high order essentially non-oscillatory (ENO) [12] reconstruction and obtained third order accuracy with curved mesh in [13]. Dumbser et al. [14] proposed a one-dimensional high order Lagrangian ADER (arbitrary high-order accurate) finite volume method. Besides, the Discontinuous Galerkin (DG) method has also been used to solve the gas dynamics equations in the total Lagrangian formulation for high-order accuracy [15]. These purely Lagrangian methods move mesh cells with the flow velocity so that the advection terms are removed from the governing equations. Particular care must be paid when the mesh is distorted to an unacceptable extent. A remedy is to improve the mesh quality with subsequent rezoning and remapping steps, which results in the indirect ALE method.

These Lagrangian methods can also be extended to direct ALE methods. High order discontinuous Galerkin (DG) direct ALE method has been implemented in [3, 16, 17] for compressible Euler equations. In the conventional finite volume framework, the direct ALE scheme can be constructed upon cell-centered Lagrangian solver completed with an edge-based upwinded formulation of the numerical fluxes as presented in [18]. Moreover, Boscheri et al. developed high-order direct ALE ADER finite volume schemes in [4, 19] with WENO (weighted essentially non-oscillatory) reconstruction and MOOD (multi-dimensional optimal order detection) approach. In this paper, we focus on direct