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A High Order Compact Scheme for a Thermal Wave Model of Bio-Heat Transfer with an Interface

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Abstract. In this paper, a high order compact difference scheme has been developed for second order wave equations with piecewise discontinuous coefficients. The idea presented here can be used to solve a wide variety of hyperbolic models for non-homogenous inner structures. Thermal wave model of bio-heat transfer in a multilayered skin structure with different thermal and physical properties is investigated subject to constant, linear, exponential and sinusoidal heating. The success of the new procedure is demonstrated by solving test problem as well as by application to the triple layered skin structure.

AMS subject classifications: 65M06

Key words: Compact finite difference scheme, thermal wave bio-heat model, immersed interface method, jump condition, discontinuous coefficients.

1. Introduction

Skin is the largest organ of human body which plays an important role such as defense, sensory and thermoregulation etc. Skin burns are one of the most devastating injuries encountered in human's daily life. These injuries usually result from heat, electricity, radiation, or chemicals. To understand and accurately predict tissue damage following a burn, bio-heat transfer based mathematical model [1–3] of the skin were developed. Investigation of such bio-heat transfer models is always demanding. For bio-heat transfer, Pennes [2] proposed a model based on classsic Fourier's Law which is

$$q(x,t) = -k\nabla T(x,t), \tag{1.1}$$

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where q is the heat flux vector representing heat flow per unit time per unit area, T is the temperature and κ is the thermal conductivity. Several authors [4–7] discussed temperature distribution in biological tissue described by Pennes bio-heat equation.The models based on Pennes bio-heat equation is most frequently adopted for its simplicity and validity. Pennes bio-heat equation may inherit some questionable physical aspects especially in process with extremely short time or at very low temperature. Considering the concept of finite heat propagation effects for more realistic predictions, Vernott [8] and Cattaneo [9] reported a modified unsteady heat conduction equation as

$$q(x,t)+\tau\frac{\partial q(x,t)}{\partial t}=-k\nabla T(x,t), \tag{1.2}$$

which differs from the classical Fourier's Law by additional term consisting of multiplication of time rate of change of heat flux and thermal relaxation time. The thermal relaxation time is defined as $\tau=\alpha/c^2$, where α is the thermal diffusivity and c is the speed of thermal wave in the medium. Using the above equation along with Pennes bio-heat equation, Liu [10] reported a thermal wave model of bio-heat transfer (TWMBT) in living biological tissues as

$$\begin{aligned}&\nabla.(k\nabla T(x,t))+w_bc_b(T_b(x,t)-T(x,t))+Q_m+Q_r+\tau\left(-w_bc_b\frac{\partial T(x,t)}{\partial t}+\frac{\partial Q_m}{\partial t}+\frac{\partial Q_r}{\partial t}\right)\\ =&\rho c\left(\tau\frac{\partial^2 T(x,t)}{\partial t^2}+\frac{\partial T(x,t)}{\partial t}\right),\end{aligned} \tag{1.3}$$

where ρ and c denote density and specific heat of the tissue; c_b is the specific heat of the blood; w_b blood perfusion rate; Q_m and Q_r are volumetric heat due to metabolism and spatial heating, respectively; T_b is the arterial blood temperature.

Note that the above equation reduces to Pennes bio-heat equation when τ is zero. The importance and necessity of considering the thermal wave effects in bio-heat transfer problems involving high heat flux incident on the skin surface with a short duration has been studied by Liu et al. [11].

In most of the earlier studies, single layer model with constant thermal properties was used for investigations. More realistically, skin tissue has a complicated multilayer structure; the complex boundary and interfacial conditions raise the difficulty to solve the problem analytically. The numerical methods serve as tool to understand the fundamental behaviour of the model. This motivates to develop high accurate numerical methods to solve TWMBT in a multilayered skin structure. Liu et al. [11] obtained one dimensional solution of skin subjected to constant surface temperature heating in a finite medium using separation of variables. Solution of TWMBT using Numerical Green's function was investigated by Loureiro et al. [12]. The effects of thermal physical properties on wave like behaviour of bio-heat transfer are investigated by Liu et al. [13, 14] using Laplace transformation.

The immersed interface method for elliptic equations with discontinuous coefficients and singular sources was introduced by LeVeque and Li [15]. Feng et al. [16] developed high order compact difference scheme for the Helmholtz equation with discontinuous coefficients. Following the approach of [15,17], we develop a high order compact difference