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A Legendre-Laguerre-Galerkin Method for Uniform Euler-Bernoulli Beam Equation

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Abstract. We consider a Galerkin method based on Legendre and Laguerre polynomials and apply it to the Euler-Bernoulli beam equation. The matrices of the method are well structured, which results in substantial reduction of computational cost. Numerical examples demonstrate the efficiency and a high accuracy of the algorithm proposed.

AMS subject classifications: 65M70, 33C45

Key words: Euler-Bernoulli beam equations, Galerkin-method, Legendre polynomials, Laguerre polynomials.

1. Introduction

The beam models describe transversely vibrating beams with external forcing functions. They have been vigorously studied in literature — cf. [26]. In particular, the Euler-Bernoulli beam model considers the undamped transverse vibrations of a flexible straight beam with no support contribution to the strain energy of the corresponding system [6, 28, 37]. In spite of being one of the simplest models, it provides reasonable results in numerous engineering problems. To find the solution of the Euler-Bernoulli problem, various algorithms have been developed, including finite difference methods [23, 27], alternating direction implicit methods [5], alternating group explicit iterative method [24], Adomian decomposition method [38, 39], sinc-Galerkin method [21, 22, 36], and various spline methods [6, 10, 28, 31, 32, 34].

Note that spectral methods play an important role in applications such as fluid dynamics, weather prediction and ocean dynamics — cf. Refs. [11, 25, 29, 33]. They allow to find approximate solutions of integral and differential equations arising in chemistry, physics, biology, engineering, astrophysics and space sciences [35]. The main feature of such methods

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consists in the representation of the solution as a linear combination of orthogonal polynomials, the coefficients of which are determined by a suitable method. Among the most popular are collocation, tau- and Galerkin methods [11]. Collocation methods demonstrate good results for nonlinear problems [7–9], whereas Galerkin methods are more handy in linear ones. In particular, the Galerkin methods can use finite combinations of various orthogonal polynomials satisfying initial or boundary conditions. For example, they have been utilised to find approximate solutions of boundary and initial value problems for high order linear differential and singular Lane-Emden equations — cf. Refs. [1–4, 17–19].

Here, we apply a Legendre-Laguerre-Galerkin algorithm to the uniform Euler-Bernoulli beams. To the best of authors' knowledge, this version of the Galerkin method has not been previously considered for the Euler-Bernoulli beam equation. In Section 2 we recall properties of Legendre and Laguerre polynomials. Section 3 deals with a Galerkin method for the uniform Euler-Bernoulli beam model. Numerical examples are presented in Section 4 and our concluding remarks are in Section 5.

2. Preliminaries

2.1. Legendre polynomials

The polynomials $P_n(x), n = 0, 1, 2, \cdots$, defined on the interval $(-1, 1)$ and satisfying the orthogonality relation

$$\int_{-1}^{1} P_k(x)P_j(x)\,dx = \frac{2}{2k+1}\,\delta_{kj},$$

where δ_{kj} is the Kronecker delta, are called Legendre polynomials. We note the relations

$$P_k(\pm 1) = (\pm 1)^k, \quad \text{and} \quad \frac{d^q P_k(\pm 1)}{dx^q} = \frac{(\pm 1)^{k+q}(k+q)!}{2^q q!\,(k-q)!}, \tag{2.1}$$

which will be used later on.

Lemma 2.1. *If k is a nonnegative integer, then*

$$(1-x^2)^2 P_k(x) = \sum_{r=0}^{4} q_r(k) P_{k-2r+4}(x), \tag{2.2}$$