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Temporal Evolution and Scaling of Mixing in Turbulent Thermal Convection for Inhomogeneous Boundary Conditions

Yikun Wei[1], Hua-Shu Dou[1,*], Zuchao Zhu[1], Zhengdao Wang[2], Yuehong Qian[2] and Haihong Xue[3]

[1] *Faculty of Mechanical Engineering and Automation, Zhejiang Sci-Tech University, Hangzhou, Zhejiang 310023, China*
[2] *Shanghai Key Laboratory of Mechanics in Energy Engineering, Shanghai University, Shanghai 200444, China*
[3] *Department of Pediatric, Xinhua Hospital, School of Medicine, Shanghai Jiaotong University, Shanghai 200240, China*

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Abstract. Numerical simulations of two-dimensional (2D) turbulent thermal convection for inhomogeneous boundary condition are investigated using the lattice Boltzmann method (LBM). This study mainly appraises the temporal evolution and the scaling behavior of global quantities and of small-scale turbulence properties. The research results show that the flow is dominated by large-scale structures in the turbulence regime. Mushroom plumes emerge at both ends of each heat source, and smaller plumes increasingly rise. It is found that the gradient of root mean-square (rms) vertical velocities and the gradient of the rms temperature in the bottom boundary layer decreases with time evolution. It is further observed that the temporal evolution of the Kolmogorov scale, the kinetic-energy dissipation rates and thermal dissipation rates agree well with the theoretical predictions. It is also observed that there is a range of linear scaling in the 2nd-order structure functions of the velocity and temperature fluctuations and mixed velocity-temperature structure function.

AMS subject classifications: 65M10, 78A48

Key words: Thermal convection, Turbulence, inhomogeneous boundary conditions, Lattice Boltzmann method.

1 Introduction

Heat transfer has been widely studied in some thermal engineering applications since it could strongly affect the performance of the devices and applications. For flow in the

*Corresponding author.
Email: huashudou@yahoo.com (H.-S. Dou)

atmosphere, thermal convection (e.g., see Hartmann et al. [1]) is relevant on both smaller length scales and time scales for weather predictions and on larger scales for climate calculations. In the ocean, Marshall et al. [2] have studied a main driving mechanism of deep-ocean circulation. Cardin and Olson [3] have investigated that thermally driven convection takes place both in Earth' s outer core. In all of these flows, rotational effects, phases changes, complex boundary conditions and nonlinear equations of state might be involved. Lohse and Xia [4] and Chilla and Schumacher [5] have reviewed on turbulent Rayleigh-Bénard (RB) convection. Turbulent RB convection has been investigated widely in a controlled manner, but it still has enough complexity to contain the key features of convective turbulence in the examples just mentioned [6,7]. RB convection in cylindrical cells has been studied intensely over the last few years in several laboratory experiments, mostly in slender cells of aspect ratio smaller than or equal to unity in order to reach the largest possible Rayleigh numbers or to resolve the detailed mechanisms of turbulent heat transport close to the walls [8,9].

Although homogeneous RB convection is an important physical problem, in engineering devices and in nature, inhomogeneities are also aconspicuous feature. Results on inhomogeneities convection are scant, which is instark contrast to the homogeneous case. While the boundary layers are strongly dominated by the presence of mean gradients of the temperature and velocity fields, the bulk of the convection layer is well mixed by the turbulence such that mean gradients of the involved turbulent fields remain subdominant compared to the local fluctuations. Seiden et al. [10]; Freund, Pesch and Zimmermann [11]; Weiss, Seiden and Bodenschatz [12] have investigated the onset of convection and the transition to pattern formation in RB convection with periodic temperature modulation on one plate with and without vertical inclination of the cell in theoretical, numerical and experimental studies. Ripesi et al. [13] have investigated the stability and dynamics of thermal convection in two dimensions, subject to inhomogeneous boundary conditions.

Although turbulent RB convection for inhomogeneous boundary conditions is of great importance and has been studied for many decades [10–12], there are still some open issues. Specifically, for the recent years, many studies have focused on small-scale turbulent fluctuations [14]. Chertkov [15] proposed a phenomenological theory in two dimensions, which predicts a Bolgiano-Obukhov-like (BO59) [4] scaling for the cascades of both the velocity and temperature fields by assuming equi-partition of the buoyancy and inertial forces at all scales. Temperature is regarded as a passive scalar and thus the Kolmogorov-like (K41) phenomenology was predicted in turbulent RB convection [4].

Previous work [13] have been executed by investigating the stability and dynamics of turbulent thermal convection for inhomogeneous boundary conditions in the two-dimensional (2D) space. In this paper, special attention is paid to investigate the time evolution and the scaling behavior of the global quantities and of small-scale turbulence properties for inhomogeneous boundary conditions. Attraction of studying is the scaling properties in a turbulent system in a BO59 scaling phenomenology, which has long been believed to characterize the cascades of the velocity and temperature fluctuations in