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Effect of Nonuniform Grids on High-Order Finite Difference Method

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Abstract. The finite difference (FD) method is popular in the computational fluid dynamics and widely used in various flow simulations. Most of the FD schemes are developed on the uniform Cartesian grids; however, the use of nonuniform or curvilinear grids is inevitable for adapting to the complex configurations and the coordinate transformation is usually adopted. Therefore the question that whether the characteristics of the numerical schemes evaluated on the uniform grids can be preserved on the nonuniform grids arises, which is seldom discussed. Based on the one-dimensional wave equation, this paper systematically studies the characteristics of the high-order FD schemes on nonuniform grids, including the order of accuracy, resolution characteristics and the numerical stability. Especially, the Fourier analysis involving the metrics is presented for the first time and the relation between the resolution of numerical schemes and the stretching ratio of grids is discussed. Analysis shows that for smooth varying grids, these characteristics can be generally preserved after the coordinate transformation. Numerical tests also validate our conclusions.

AMS subject classifications: 65N06, 65N22

Key words: Finite difference method, nonuniform grids, coordinate transformation, Fourier analysis.

1 Introduction

The FD method is historically old and plays an important role in the computational fluid dynamics [1]. In recent 20 years, due to the efficiency and simplicity, various high-order

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schemes based on the FD method have been proposed and widely used in direct numerical simulations (DNS), computational aeroacoustics (CAA) and large eddy simulations (LES), in which the high resolution is needed. At present, the high-order FD method has been successfully applied in the simulations of incompressible, compressible and hypersonic flows [2–4] and several other practical applications [5, 6].

In the studies of high-order FD method, different discretization techniques for the spatial derivative are developed. The common one is the explicit scheme which is directly derived from the Taylor series expansion. For steadiness, the numerical dissipation should be introduced by different ways, such as using upwind schemes. However, a shortage of such schemes is that a long stencil is needed to achieve the desired order of accuracy, which makes the boundary schemes difficult to design [7]. To reduce the stencil width, the compact scheme becomes another choice. Compact FD schemes with spectral-like resolution are first systematically studied by Lele [8] and gain a quick development. Recently, Rizzetta et al. [5] carried out a high-order compact scheme with compact filter, which has been demonstrated to produce accurate and stable results in large eddy simulations. A family of hybrid dissipative compact schemes is proposed by Deng et al. [9] and suitable for simulations in aeroacoustics. To make compact schemes possess the shock-capturing capabilities, many efforts have been devoted [10–12], which were well summarized by Shen and Zha [13]. However, as stated by Tam [14], the Taylor series truncation cannot be used to quantity the wave propagation errors which are dominant in CAA and this issue results in the development of optimized schemes, where the order of accuracy is lowered to reduce errors over a range of wavenumbers [15]. One of the classical optimized schemes is the dispersion-relation-preserving (DRP) scheme developed by Tam and Webb [16], which is capable to accurately resolve harmonic components with few points-per-wavelength [17].

In most cases, the development of the FD schemes is based on the uniform Cartesian grids. However, in the simulations of practical problems, the use of the nonuniform or curvilinear grids is inevitable for adapting to the complex configurations. For solving this issue, some schemes are especially designed. Gamet et al. [18] modified the original compact scheme to approximate the first derivative on the nonuniform meshes. Cheong and Lee [19] developed the GODRP finite difference scheme to locally preserve the same dispersion relation as the original partial differential equations on the nonuniform mesh. Moreover, Zhong et al. [20,21] used the polynomial interpolation to derive arbitrary high-order compact schemes on nonuniform grids, which have been adopted for simulations of hypersonic boundary-layer stability and transition. Although these schemes can be directly applied on the nonuniform grids, the distribution of grids is still relatively simple (for example, the grids are only stretched along each direction of the Cartesian grids), making the schemes difficult to extend to practical conditions. Another method of dealing with the nonuniform or curvilinear grids in the FD schemes is to employ the coordinate transformation (or named Jacobian transformation), which is the most popular way. Using this method, the original schemes can be applied in the computational space where the grid is uniform Cartesian one, but as a result, the metrics and Jacobian are involved