J. Math. Study
doi: 10.4208/jms.v50n4.17.03

Vol. **50**, No. 4, pp. 323-338
December 2017

An Analysis of Complex-valued Periodic Solution of a Delayed Discontinuous Neural Networks

Yajing Wang[1] and Zhenkun Huang[1,*]

[1] *School of Sciences, Jimei University, Yinjiang Road 185, Xiamen 361021, China*

Abstract. In this paper, we investigate global stability of complex-valued periodic solution of a delayed discontinuous neural networks. By employing discontinuous, nondecreasing and bounded properties of activation, we analyzed exponential stability of state trajectory and L^1-exponential convergence of output solution for complex-valued delayed networks. Meanwhile, we applied to complex-valued discontinuous neural networks with periodic coefficients. The new results depend on $M-$matrices of real and imaginary parts and hence can include ones of real-valued neural networks. An illustrative example is given to show the effectiveness of our theoretical results.

AMS subject classifications: 37F05, 37N35, 37J45, 37K45

Chinese Library Classifications: O193

Key words: Complex-valued, Periodic solutions, Global exponential stability, Discontinuous neural networks.

1 Introduction

The global exponential stability of neural networks have been extensively studied because of their wide range of applications, such as image and signal processing, pattern recognition, optimization and automatic control, and so on (see [1-3]). An equilibrium point can be viewed as a special periodic solution of neural networks with arbitrary periods. In this sense, the analysis of periodic solutions of neural networks can be considered to be more general than that of equilibrium point. Therefore, the global exponential stability of the periodic solution received extensive concerns. In [4-7], the authors investigated the stability of periodic solutions of neural networks, where the assumptions on neuron activation functions include Lipschitz conditions, bounded and monotonic increasing properties. In [8], Du and Xu discussed the global robust exponential stability and periodic solutions for interval Cohen-Grossberg neural networks with mixed delays. The stability analysis for periodicity of BAM neural networks with discontinuous neuron activations and impulses have been studied in [9]. As shown by [10], the authors studied

*Corresponding author. *Email addresses:* `hzk974226@jmu.edu.cn` (Z. Huang)

finite time stability of periodic solution for Hopfield neural networks with discontinuous activations. We usually consider the properties of real-valued neural networks (RVNNs) in most of the papers. However, complex-valued neural networks can solve the problem that real neural networks can not solve, we can refer to [11] and the references therein. In [12], Rao and Murthy have studied global activation dynamics of a discrete CVNNs and have obtained easily verifiable sufficient conditions for global exponential stability of the unique equilibrium pattern. Due to importance of time delay in the finite speeds of the switching and transmissions of signal of neural network [14], stability criteria of real-valued or complex-valued neural networks with time delay have been reported in [13,15-18].

However, most of the results concerning the neural networks are based on the assumption that the activations are continuous or even Lipschitzian. Forti and Nistri [19] is the first one to discuss global stability of the equilibrium points for the neural networks with discontinuous neuron activations. They pointed out that a brief review of some common neural networks with discontinuous activation is important. In [20], Forti et al. introduced some new sufficient conditions for the global exponential stability of recurrently connected neural networks with (possibily) discontinuous and unbounded activation functions. Recently, there have been extensive results on the dynamical behaviors of neural networks with discontinuous activations [21-23].

Based on the previous scholar's research, we will study the stability of the periodic solution of a delayed neural network with discontinuous activations. In particular, we drop the assumption of Lipschitz continuity on the activation functions, which is usually required in most of the papers. Meanwhile, since the complex-valued neural network is more general than the real-valued neural network, it can solve the problem that the real neural network can not solve. Therefore, the stability of the periodic solution of a delayed complex-valued neural network with discontinuous activations has theoretical valued and practical application valued.

The organization of this paper is as follow. In Section 2, we introduce some definitions and preliminary lemmas. In Section 3, under suitable assumptions, we prove a result on the continuability and the uniqueness of the solution of any associated initial output problems (IOP) and give an estimation on the difference between the states and the outputs of the solutions of two different IOP. Our main results are contained in section 4, some sufficient conditions are given to guarantee the existence and exponential stability of a unique complex-valued periodic solution. Finally, our results are illustrated by an example.

2 Preliminaries

In this paper, we extend the work in [24] to the following model of a delayed complex-valued neural network with periodic coefficients

$$\dot{Z}(t) = -C(t)Z(t) + D(t)f(Z(t)) + E(t)f(Z(t-\tau)) + H(t), \quad t \geq 0, \tag{2.1}$$