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# Efficient Semi-Lagrangian Vlasov-Maxwell Simulations of High Order Harmonic Generation from Relativistic Laser-Plasma Interactions

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**Abstract.** We describe a relativistic semi-Lagrangian scheme for the numerical solution of the relativistic Vlasov-Maxwell system. The implementation strategy on a modern non-unified memory access (NUMA) architecture using the OpenMP framework is discussed. We demonstrated that close to perfect scaling can be obtained on modern many-core, multi-socket systems. Application of this code to the problem of relativistic generation of high-harmonic laser radiation is demonstrated. The results are compared to particle-in-cell (PIC) simulations, indicating in particular that for warm plasma the Vlasov simulation is superior. We discuss the impact of plasma temperature on the radiation spectrum and show that the efficiency of harmonic generation depends on the plasma temperature.

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**Key words**: Relativistic Vlasov-Maxwell system, semi-Lagrange, multi-threading, relativistic laser-plasma interaction, high-harmonic radiation generation.

## 1 Introduction

The maximum achievable laser intensity has made significant progress in the last decade, allowing intensities on the order of $10^{19}-10^{22}\mathrm{W/cm^2}$. At the same time the produced pulses are very short, typically on the order of some 10 fs or even below. The interaction of such intense laser pulses with plasmas exhibits complex phenomena, many of which have their origin in the relativistic mass variation that charged particles experience in these fields [1].

One of the most widely used set of equations to model relativistic laser-plasma interaction are the Vlasov-Maxwell equations. This system describes the interaction between

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a relativistically strong laser and a warm, collisionless plasma. In general collisions between particles tend to become negligible at high laser intensities, thus this set of equations describes a major part of laser-plasma interaction scenarios studied today.

Numerical solution of the Vlasov-Maxwell equations is mostly done via particle-in-cell (PIC) methods. These methods cluster a large number of real particles into artificial macro-particles and by doing so introduce a coarse-graininess of the plasma. The macro-particle trajectories follow the characteristics of the Vlasov equation, while the electromagnetic fields are calculated on a grid. By gathering the charge and current densities on the grid, the fields are updated. Typically the introduction of macro-particles introduces noise to the simulation results, with the noise level depending on the ratio of real to artificial particles. Since the works of Birdsall and Langdon [2], and Hockney and Eastwood [3] much effort has been dedicated to improving the noise level of these methods and there are now massively parallel implementations available that treat over $10^{10}$ macro-particles in full three-dimensional geometry (i.e. six-dimensional phase-space). Such codes are widely used today to study many phenomena in the field of laser-plasma interaction [1]. Nevertheless, full three-dimensional simulations of systems with large spatial extension still require considerable computational resources.

For applications where particles in the tail of the distribution function play a role, or where noise is an issue, direct solution of the Vlasov equation may give better results than PIC methods. These direct simulations of the advection of the particle distribution function in phase-space, without introducing an artificial graininess of the plasma, yields virtually noise-free results. Over the last ten years Vlasov solvers have been developed in the context of relativistic laser-plasma interaction [4–11].

The classical methods for the non-relativistic Vlasov equation, starting with [12], use a time splitting scheme to update the phase-space distribution on a Eulerian grid. Later works formulated finite volume schemes and combined them with flux limiting methods to guarantee the positivity of the distribution function, see e.g. [13]. For the relativistic case it was however shown that time splitting schemes introduce a numerical instability [6] and that semi-Lagrangian schemes are preferable. Semi-Lagrangian schemes have a long history in the treatment of advective problems, especially in climate and weather simulations [14]. These methods aim to combine an Eulerian and a Lagrangian fluid description approach. In the Eulerian picture, all quantities evolve over time on a fixed grid in space. This leads to limitations in the size of the time-step due to stability considerations. In the Lagrangian picture, the observer moves along with a certain fluid element, thus the quantity stays constant, but the position of the element changes. Usually, this allows for larger time-steps. However, an initially uniformly spaced grid usually evolves into a non-equidistant distribution of grid points.

Semi-Lagrangian schemes have been used previously in the context of Vlasov-Maxwell systems to study different aspects of laser-plasma interaction, such as for example wakefield acceleration [8], ion-acceleration [15], relativistic modulational instabilities [5] or self-induced-transparency [7, 16].