Commun. Comput. Phys.
doi: 10.4208/cicp.171214.210715s

Vol. **18**, No. 4, pp. 1012-1049
October 2015

High-Order Conservative Asymptotic-Preserving Schemes for Modeling Rarefied Gas Dynamical Flows with Boltzmann-BGK Equation

Manuel A. Diaz[1], Min-Hung Chen[2] and Jaw-Yen Yang[1,3,*]

[1] *Institute of Applied Mechanics, National Taiwan University, Taiwan, Taipei 10167.*
[2] *Department of Mathematics, National Cheng-Kung University, Taiwan, Tainan 701.*
[3] *Institute of Advanced Study in Theoretical Science, National Taiwan University, Taiwan, Taipei 10167.*

Received 17 December 2014; Accepted (in revised version) 21 July 2015

Abstract. High-order and conservative phase space direct solvers that preserve the Euler asymptotic limit of the Boltzmann-BGK equation for modelling rarefied gas flows are explored and studied. The approach is based on the conservative discrete ordinate method for velocity space by using Gauss Hermite or Simpsons quadrature rule and conservation of macroscopic properties are enforced on the BGK collision operator. High-order asymptotic-preserving time integration is adopted and the spatial evolution is performed by high-order schemes including a finite difference weighted essentially non-oscillatory method and correction procedure via reconstruction schemes. An artificial viscosity dissipative model is introduced into the Boltzmann-BGK equation when the correction procedure via reconstruction scheme is used. The effects of the discrete velocity conservative property and accuracy of high-order formulations of kinetic schemes based on BGK model methods are provided. Extensive comparative tests with one-dimensional and two-dimensional problems in rarefied gas flows have been carried out to validate and illustrate the schemes presented. Potentially advantageous schemes in terms of stable large time step allowed and higher-order of accuracy are suggested.

AMS subject classifications: 35Q20, 76P05

Key words: Rarefied gas dynamics, Boltzmann-BGK equation, asymptotic preserving, conservative discrete ordinate method, weighted essentially non-oscillatory, correction procedure via reconstruction, artificial viscosity.

*Corresponding author. *Email addresses:* `f99543083@ntu.edu.tw` (M. A. Diaz), `mhchen@math.ncku.edu.tw` (M.-H. Chen), `yangjy@iam.ntu.edu.tw` (J.-Y. Yang)

1 Introduction

Discontinuous polynomial approximations have become a popular mean to obtain high-order solutions for a broad range of engineering applications. Arguably the most popular method in the literature are the discontinuous Galerkin (DG) methods [8, 10]. However, it is noted that the flux reconstruction (FR) method of Huynh [13] have gained popularity as this method can recover the nodal DG, spectral volume (SV) and spectral difference (SD) schemes under a single unified framework. Wang and Gao [27] extended the FR formulation to unstructured meshes and the resulting formulation was termed correction procedure via reconstruction (CPR) scheme. Meanwhile, Vincent et al. [26] showed that the FR schemes are energy stable and their connections to filtered nodal DG methods were established. To date, although all discontinuous polynomial methods perform favourably for problems with smooth data, the techniques for capturing discontinuities with DG and FR schemes remain a challenging and computationally intensive task.

In classical gas dynamics, several approaches inspired by finite volume methodology have been proposed for shock capturing with high-order discontinuous polynomial representation methods. The most straightforward approach consists of using some form of shock sensor to identify the elements lying in the shock region and then limiting/reducing the order of the interpolation polynomial in those elements flagged by the sensor [34]. Reducing the order of the interpolation polynomial degrades the order of accuracy of the overall scheme and increases the inter-element jumps and hence the dissipation of the algorithm. Limiter techniques are commonly used and have been demonstrated for moderate order ($p \leq 3$). Despite its simplicity, this approach yields satisfactory results especially when combined with adaptive mesh refinement procedures. More sophisticated approaches exist for the reconstruction of a smooth interpolating polynomial such as those based on weighted essentially non-oscillatory (WENO) concepts. These methods allow for stable discretization of conservative schemes. Although having attractive features, they appear to have very high cost when the degree of the approximating polynomial is increased. Other interesting alternatives are based on applying filters to the solution [14].

Persson and Peraire [20], in the context of DG methods, proposed the *viscous shock capturing* as an alternative to the limiting strategy. Their approach was to use sufficient high-order and sufficient high viscosity such that the shock is resolved within the span of a single element. In classical gas dynamics the contributions of Barter [3], Atkins and Pampell [2] and Klöckner et al. [16] indicate that providing a minimum amount of diffusion to the interpolation polynomials of the elements in the vicinity of a discontinuity can produce stable, robust, and accurate solutions. By not reducing the polynomial order, tiny oscillations remain present in the system; however, the computation is stable and can be carried out with high-order resolution polynomial in coarser grids and larger temporal steps.

Following the work of Xu [31], explorations of the Boltzmann model equation using discontinuous polynomial representations have been reported. Recently, Xiong et al. [29]