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Application of Homotopy Analysis Method for Solving Systems of Volterra Integral Equations

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Abstract. In this paper, we prove the convergence of homotopy analysis method (HAM). We also apply the homotopy analysis method to obtain approximate analytical solutions of systems of the second kind Volterra integral equations. The HAM solutions contain an auxiliary parameter which provides a convenient way of controlling the convergence region of series solutions. It is shown that the solutions obtained by the homotopy-perturbation method (HPM) are only special cases of the HAM solutions. Several examples are given to illustrate the efficiency and implementation of the method.

AMS subject classifications: 54A40, 26E50

Key words: Homotopy analysis method, homotopy perturbation method, systems of Volterra integral equations.

1 Introduction

Differential equations, integral equations or combinations of them, integro-differential equations, are obtained in modeling of real-life engineering phenomena that are inherently nonlinear with variable coefficients. Most of these types of equations do not have an analytical solution. Therefore, these problems should be solved by using numerical or semi-analytical techniques. In numeric methods, computer codes and more powerful processors are required to achieve accurate results. Acceptable results are obtained via semi-analytical methods which are more convenient than numerical methods. The main advantage of semi-analytical methods, compared with other methods, is based

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on the fact that they can be conveniently applied to solve various complicated problems. Several analytical methods including the linear superposition technique [14], the exp-function method [16], the Laplace decomposition method [8], the matrix exponential method [15], the homotopy perturbation method [7], variational iteration methods [2] and the Adomian decomposition method [12] have been developed for solving linear or nonlinear non-homogeneous partial differential equations. One of these semi-analytical solution methods is the Homotopy analysis method (HAM). In 1992, Liao [9] employed the basic ideas of the homotopy in topology to propose a general analytic method for nonlinear problems, namely the Homotopy analysis method, [5, 6, 9–11]. In recent years, homotopy analysis method has been used in obtaining approximate solutions of a wide class of differential, integral and integro-differential equations. The method provides the solution in a rapidly convergent series with components that are elegantly computed. The main advantage of the method is that it can be used directly without using assumptions or transformations. In this work, we aim to implement this reliable technique to solving systems of Volterra integral equations. A system of integral equations of the second kind can be presented as

$$f(t) = g(t) + \int_a^t K(s,t,(f(s))ds,$$

where

$$f(t) = \big(f_1(t),\cdots,f_n(t)\big)^T, \qquad g(t) = \big(g_1(t),\cdots,g_n(t)\big)^T,$$
$$K(s,t,(f(s)) = \big(K_1(s,t,(f(s)),\cdots,K_n(s,t,(f(s))\big)^T.$$

2 Basic idea of HAM

We consider the following differential equation

$$\mathcal{N}[u(\tau)] = 0, \tag{2.1}$$

where $\mathcal{N}$ is a nonlinear operator, τ denotes independent variable, $u(\tau)$ is an unknown function, respectively. For simplicity, we ignore all boundary or initial conditions, which can be treated in the similar way. By means of generalizing the traditional homotopy method, Liao [11] construct the so-called zero-order deformation equation

$$(1-p)\mathcal{L}[\phi(\tau;p) - u_0(\tau)] = p\hbar\mathcal{H}(\tau)\mathcal{N}[\phi(\tau;p)], \tag{2.2}$$

where $p \in [0,1]$ is the embedding parameter, $h \neq 0$ is a non-zero auxiliary parameter, $\mathcal{H}(\tau) \neq 0$ is an auxiliary function, $u_0(\tau)$ is an initial guess of $u(\tau)$ and $\phi(\tau;p)$ is an unknown function and $\mathcal{L}$ an auxiliary linear operator with the property

$$\mathcal{L}[f(\tau)] = 0 \quad \text{when} \;\; f(\tau) = 0. \tag{2.3}$$