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DUAL BASES FOR A NEW FAMILY OF GENERALIZED BALL BASES *

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Abstract

This paper presents the dual bases for a new family of generalized Ball curves with a position parameter K, which includes the Bézier curve, generalized Said-Ball curve and some intermediate curves. Using the dual bases, the relative Marsden identity, conversion formulas of bases and control points of various curves are obtained.

Key words: Bézier curve, New generalized Ball curve, Dual basis, Marsden identity.

1. Introduction

The generalized Ball curve possesses many properties similar to the ones of the Bézier curve and an advantage that there are some more efficient recursive algorithms for computing the points on the curve [1∼8]. For a given control polygon, the positions of both the generalized Ball curve and the Bézier curve are different. A new family of generalized Ball curves with a position parameter K has been presented recently in [9,10], which includes the Bézier curve, generalized Said-Ball curve and some other intermediate curves. When the position selecting of a curve is considered as important as the efficiency for evaluating, it is suggested to use the family of curves.

To converse various bases and various curves interactively, a powerful means is to find the corresponding dual basis. The dual basis for the generalized Said-Ball basis with its Marsden identity has been discussed in the papers [11∼15]. In this paper, Section 2 gives the definition of new generalized Ball bases with a position parameter K and presents its dual bases. In Section 3, the Marsden identity about power basis expanded by new bases is obtained. Section 4 presents conversion formulas of the bases and the control points. In the end two examples of the proposed algorithm are provided.

2. Dual Basis

We adopt the notation of combination in [11]

$$\binom{\alpha}{k} = \frac{\alpha(\alpha-1)\cdots(\alpha-k+1)}{k!}$$

for real number α and positive integer k and

$$\binom{\alpha}{0} = 1, \quad \binom{\alpha}{k} = 0 \ (k < 0).$$

It is easy to verify

$$(-1)^k\binom{\alpha}{k} = \binom{k-1-\alpha}{k}$$

and

$$\binom{n}{p} = \sum_{k=0}^{p}\binom{m}{p-k}\binom{n-m}{k} \quad \text{for } m \in R;\ n,p \in Z.$$

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In the paper [9] a new family of generalized Ball basis functions of degree n with a given integer K $(0 \le K \le m)$ has been introduced:

$$\alpha_i(u;2m+1,K) = \begin{cases} \binom{m+K+i}{i} u^i(1-u)^{m+1+K}, & 0 \le i \le m-K; \\ \binom{2m+1}{i} u^i(1-u)^{2m+1-i}, & m-K+1 \le i \le m, \end{cases} \quad (0 \le u \le 1) \quad (1)$$

$$\alpha_{2m+1-i}(u;2m+1,K) = \alpha_i(1-u;2m+1,K), \quad 0 \le i \le m,$$

for odd degree $n = 2m+1$, and

$$\alpha_i(u;2m,K) = \begin{cases} \binom{m+K+i}{i} u^i(1-u)^{m+1+K}, & 0 \le i \le m-K-1; \\ \binom{2m}{i} u^i(1-u)^{2m-i}, & m-K \le i \le m, \end{cases} \quad (0 \le u \le 1) \quad (2)$$

$$\alpha_{2m-i}(u;2m,K) = \alpha_i(1-u;2m,K), \quad 0 \le i \le m-1,$$

for even degree $n = 2m$.

The basis functions $\alpha_i(u;n,K)$ possess many properties similar to the ones of the Bernstein-Bézier basis and the generalized Ball basis, such as the nonnegativity, the partition of unity, and so on. When $K = 0$, it becomes the generalized Said-Ball basis, and $K = m$ gets the Bernstein- Bézier basis.

Let $\{\boldsymbol{p}\ \}_{i=0}^{n}$ be a set of control points in R^2 or R^3. Using bases $\{\alpha_i(u;n,K)\}$, a family of curves can be constructed

$$NB(u;n,K) = \sum_{i=0}^{n} \boldsymbol{p}\ \alpha_i(u;n,K), \qquad 0 \le u \le 1;\ K = 0,1,\cdots,m. \quad (3)$$

In addition to the Bézier curve and generalized Said-Ball curve, the family of curves (3) includes some intermediate curves. The integer K can be regarded as a parameter, which determines the position of a curve. The figure 1 shows a family of curves $\{NB(u;n,K)\}_{K=0}^{3}$ of degree 7, where the Bézier curve with $K = 3$ is the nearest to the control polygon, the generalized Said-Ball curve with $K = 0$ is the farthest from the control polygon, and the other curves with $K = 1,2$ are between the Bézier curve and Said-Ball curve.

Definition 1. *Suppose that $\{b_i(u)\}_{i=0}^{n}$ is a basis of the space P_n of polynomials of degree non-exceeding n, if the linear functionals $\{\lambda_i(f)\}_{i=0}^{n}$ satisfy the conditions*

$$\lambda_i(b_j) = \delta_{ij}, \qquad i,j = 0,1,\cdots,n,$$

where δ_{ij} is the Kronecker sign, then $\{\lambda_i(f)\}_{i=0}^{n}$ are called the dual bases or dual functionals of $\{b_i(u)\}_{i=0}^{n}$.

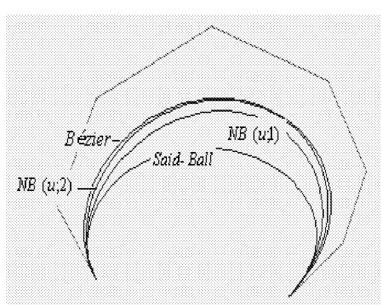

Figure 1 Bźier curve, Said-Ball curve and intermediate curves of degree 7

The following theorem is about the dual bases of the basis functions $\{\alpha_i(u;n,K)\}$.