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A SIMPLE FINITE ELEMENT METHOD FOR THE REISSNER-MINDLIN PLATE*1)

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Abstract

A simple finite element method for the Reissner-Mindlin plate model in the primitive variables is presented and analyzed. The method uses conforming linear finite elements for both the transverse displacement and rotation. It is proved that the method converges with optimal order uniformly with respect to thickness.It is simpler and more economical than the Arnold-Falk element[1].

1. Introduction

The Reissner-Mindlin model describes the deformation of a plate subject to a transverse load. This model, as well as its generalization to shells, is frequently used for plates and shells of small to moderate thickness. It is well known that many numerical schemes for this model are satisfactory only when the thickness parameter t is not too small. For a very small t, some bad behaviors (such as the locking phenomenon) might occur. In 1986, F.Brezzi and M.Fortin[2] derived an equivalent formulation of the Reissner-Mindlin plate equations by using the Helmholtz theorem to decompose the shear strain vector. The optimal error estimates for transverse displacement, rotations and shear stresses were obtained uniformly with respect to thickness. Unfortunately their method is not known to be equivalent to any discretization of the original Reissner-Mindlin model.

In [1] Arnold and Falk modified the method in [2] and obtained a finite element method for the Reissner-Mindlin problem in the primitive variables. This so-called Arnold-Falk element may be the only method with the approximate values of displacement and the rotation, together with their first derivatives, all converging at an optimal rate uniformly with respect to thickness. Recently, R.Duran et al. [3] introduced a modification of Arnold-Falk element, with the internal degrees of freedom removed.

In this paper, we present a new finite element method for the Reissner-Mindlin model which is based on a different discrete version of the Helmholtz decomposition.

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The method used here is easier to implement and optimal error estimates for displacement and the rotation are proved uniformly in the plate thickness. It is simpler and more economical than the methods in [1,3].

2. The Reissner-Mindlin Plate Model

We will use the usual L^2-based Sobolev spaces H^s. The space H^{-1} denotes the normed dual of H^1, the closure of C_0^∞ in H^1. We use a circumflex above a function space to denote the subspace of elements with mean value zero. An underline to a space denotes the 2-vector-valued analogue. The underline is also affixed to vector-valued functions and operators, and double underlines are used for matrix-valued objects. The letter C denotes a generic constant, not necessarily the same at each occurrence. Finally, we use various standard differential operators:

$$\underline{\text{grad}}\; r = (\frac{\partial r}{\partial x}, \frac{\partial r}{\partial y})^T, \quad \underline{\text{curl}}\; p = (\frac{\partial p}{\partial y}, -\frac{\partial p}{\partial x})^T, \quad \text{div}\,\underline{\psi} = \frac{\partial \psi_1}{\partial x} + \frac{\partial \psi_2}{\partial y},$$

$$\text{rot}\,\underline{\psi} = \frac{\partial \psi_1}{\partial y} - \frac{\partial \psi_2}{\partial x}, \quad \underline{\underline{\text{grad}}}\;\underline{\psi} = \begin{pmatrix} \frac{\partial \psi_1}{\partial x} & \frac{\partial \psi_1}{\partial y} \\ \frac{\partial \psi_2}{\partial x} & \frac{\partial \psi_2}{\partial y} \end{pmatrix}.$$

Let Ω denote the region in R^2 occupied by the midsection of the plate, and denote by ω and $\underline{\phi}$ the transverse displacement of Ω and the rotation of the fibers normal to Ω, respectively. The Reissner-Mindlin plate model determines ω and $\underline{\phi}$ as the unique solution to the following variational problem.

Problem RM. Find $(\omega, \underline{\phi}) \in H_0^1(\Omega) \times \underline{H}_0^1(\Omega)$ such that

$$a(\underline{\phi}, \underline{\psi}) + \lambda t^{-2}(\underline{\phi} - \underline{\text{grad}}\;\omega, \underline{\psi} - \underline{\text{grad}}\;\mu) = (g, \mu), \tag{2.1}$$

$$\forall (\mu, \underline{\psi}) \in H_0^1(\Omega) \times \underline{H}_0^1(\Omega).$$

Here g is the scaled transverse loading function, t is the plate thickness, $\lambda = Ek/2(1+\nu)$ with E Young's modulus, ν the Poisson ratio, k the shear correction factor, and the parentheses denote the usual L^2 inner product. The bilinear form $a(\cdot,\cdot)$ is defined by

$$a(\underline{\phi}, \underline{\psi}) = \frac{E}{12(1-\nu^2)} \int_\Omega \Big[\Big(\frac{\partial \phi_1}{\partial x} + \nu \frac{\partial \phi_2}{\partial y}\Big)\frac{\partial \psi_1}{\partial x} + \Big(\nu \frac{\partial \phi_1}{\partial x} + \frac{\partial \phi_2}{\partial y}\Big)\frac{\partial \psi_2}{\partial y}$$
$$+ \frac{1-\nu}{2}\Big(\frac{\partial \phi_1}{\partial y} + \frac{\partial \phi_2}{\partial x}\Big)\Big(\frac{\partial \psi_1}{\partial y} + \frac{\partial \psi_2}{\partial x}\Big)\Big].$$

By Korn's inequality, $a(\cdot,\cdot)$ is an inner product on $\underline{H}_0^1(\Omega)$ equivalent to the usual one. For simplicity of notation, we will consider the problem whose weak formulation is given by (2.1) with $\lambda = 1$, and

$$a(\underline{\phi},\ \underline{\psi}) = (\underline{\underline{\text{grad}}}\;\underline{\phi},\ \underline{\underline{\text{grad}}}\;\underline{\psi}).$$

For our analysis we shall also make use of an equivalent formulation of the Reissner-Mindlin plate equations suggested by Brezzi and Fortin[2]. Which is derived from