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AN ADAPTIVE MULTI-SCALE CONJUGATE GRADIENT METHOD FOR DISTRIBUTED PARAMETER ESTIMATION OF 2-D WAVE EQUATION*

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Abstract *An adaptive multi-scale conjugate gradient method for distributed parameter estimations (or inverse problems) of wave equation is presented. The identification of the coefficients of wave equations in two dimensions is considered. First, the conjugate gradient method for optimization is adopted to solve the inverse problems. Second, the idea of multi-scale inversion and the necessary conditions that the optimal solution should be the fixed point of multi-scale inversion method is considered. An adaptive multi-scale inversion method for the inverse problem is developed in conjunction with the conjugate gradient method. Finally, some numerical results are shown to indicate the robustness and effectiveness of our method.*

Key words *Multi-scale; conjugate gradient method; wave equation.*
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1 Introduction

Designing effective numerical methods for the nonlinear inverse wave equation is a very significant challenge. Especially for large-scale inverse problems, the number of parameters to be estimated is mesh-dependent. As we know, there are some reasons:

First, conventional nonlinear solvers such as Newton's method entail difficulties for large-size problems. The Hessian matrices for Newton's method are formally dense and are of the order of the number of the inversion parameters. Their construction involves numerous solutions of the direct problem. Therefore, the straightforward implementations based on grid require prohibitive memory and computational cost. The denseness, large scale, and construction of the

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linearized inverse operators rule out matrix-based iterative solvers.

Secondly, the objective functions of the nonlinear inverse problems are often non-convex. In other words, Newton's method will diverge if globalization is neglected. Even worse, for many nonlinear inverse problems, the objective functions to be minimized may have many local minima. Newton's method as well as other local methods will be trapped at a local minimum unless the objective function can be convexified[1].

Given the above difficulties faced by the Newton-like methods, it is not surprising that many large-scale PDE inverse methods are based on gradients alone in order to avoid Hessian matrices. The simplest of such techniques are the steepest decent-type methods, in which the current solution is improved by moving in the direction of the negative gradient of the objective function. Better directions can be constructed based on gradient information, resulting in the nonlinear conjugate gradient (CG) method. All of these gradient-based methods avoid the use of Newton Hessians.

Indeed, any large-scale nonlinear inverse problems defined in physical space can have an approximated description at any given scale of the same space. The multi-scale algorithm recursively constructs a sequence of such descriptions with increasingly larger (coarser) scales, and combines local processing at each scale with various inter-scale interactions. As a result of such multilevel interactions, the fine scales can be employed efficiently. Moreover, the inter-scale interaction can eliminate various kinds of difficulties, such as: slow convergence in minimization processes; ill-posedness of inverse problems; large-scale attraction basin traps in global optimization; etc[1].

The multi-scale algorithms have been applied to the inverse 2-D Wave Equation; for example, E. Gelman and J. Mandel[2], J. Thomas King[9], U. M. Ascher and E.Haber[3] performed the analysis of multi-scale inversion algorithms. Y. M. Chen[4], W. H. Chen[5], J.L. Qian etc[6,7,8] improved the efficiency of the generalized pulse-spectrum technique (GPST) inversion algorithm by using multi-scale algorithms. Some researchers combined the multi-scale idea with various iterative solvers for solving inverse problems[9]. Another popular approach is the multi-resolution wavelet theory[10,11,12,13,14,15].

Once the scale has been chosen, the actual level of refinement is difficult to choose: if the scale is too poor, the inversion parameter will not honor properly the data. If it is too rich, it will lead to overparameterization. The adaptability of multi-scale algorithms is hence of crucial importance.

The existing multi-scale methods often employ adjoint equations at each scale[1], which increase the computational work. In the paper, an adaptive multi-scale conjugate gradient method