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NUMERICAL TESTS ON CONVERGENCE OF THE RANDOM CHOICE METHOD*[1)]

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The random choice method (RCM) has been successfully used for computing very complicated combustion problems in [1], which shows its robustness. In this paper, we shall observe its convergence through numerical tests.

The problem computed in this paper is the ignition problem. The formulation of the problem and the RCM method can be found in [1].

We have computed this problem using five different meshes and two different sequences of random numbers, and estimated the error of pressure obtained by RCM in L_2 norm, defined by

$$\sigma=\left[\frac{1}{b-a}\int_a^b[p(x,\ t_0)-p^*(x,\ t_0)]^2\,dx\right]^{1/2},$$

where $p(x, t_0)$ denotes the approximate pressure at time $t=t_0$ and $p^*(x, t_0)$ the exact one, and $[a, b]$ is the computational interval in the x-direction. In our computation, the numbers of mesh points in the x-direction are 81, 161, 321, 641, 1281. In Tables 1 and 2 the values of σ for $t=3$ and 11 are given. We can find from the tables that the results possess strong randomness. The values of σ change by $20\%\sim50\%$ when different sequences of random numbers are adopted. Moreover, the error is not a monotonic decreasing function, though the general trend of error is on the decrease while Δt decreases.

In what follows we shall make a rough estimate of convergence rate using the data in the tables. Suppose that the rate of convergence is $O(\Delta t^a)$. Therefore between the error and the parameter a there is the following approximate relation

$$\frac{\Delta t_1^a}{\Delta t_2^a}=\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)},$$

Table 1 Errors of RCM for $t=3.0$

Numbers of mesh points	Errors σ_1 (sequence 1)	Errors σ_2 (sequence 2)
81	0.180	0.234
161	0.176	0.127
321	0.127	0.055
641	0.065	0.0635
1281	0.078	0.0408

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Table 2 Errors of RCM for $t=11.0$

Numbers of mesh points	Errors σ_1 (sequence 1)	Errors σ_2 (sequence 2)
81	4.48	5.50
161	4.67	5.21
321	2.36	3.96
641	3.43	2.01
1281	3.27	3.08

Because this problem is very complicated, no analytical solution has been obtained. The solution obtained by using the Singularity-Separating Method is quite accurate. It was taken as the exact solution while we computed σ. This substitution will not have an essential influence on the correctness of the values in Tables 1 and 2 since the error of RCM is much larger than that of the Singularity-Separating Method.

which can be rewritten as

$$a=\frac{\log\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)}}{\log\frac{\Delta t_1}{\Delta t_2}}=\frac{\log\frac{\sigma(\Delta t_1)}{\sigma(\Delta t_2)}}{\log\frac{\Delta x_1}{\Delta x_2}}.$$

Here Δt denotes the step size in the t-direction and Δx the step size in the x-direction. In our computation $\Delta x/\Delta t$ is unchanged as Δt changes. That is, we always take $\Delta t_1/\Delta t_2=\Delta x_1/\Delta x_2$, where Δt_1, Δx_1 are the two increments for a net and Δt_2, Δx_2 for the other. Generally speaking, σ depends on Δx and Δt. In our case, $\Delta x/\Delta t$ is fixed; so we could think that σ depends just on Δt. This is why we use the symbol $\sigma(\Delta t)$ instead of $\sigma(\Delta x, \Delta t)$. From Table 1 we know that while 161 points are taken in the x-direction and the first sequence of random numbers is used, $\sigma=0.176$ for $t=3$. And $\sigma=0.127$ if 321 points are taken. Therefore we have

$$a=\frac{\log\frac{0.176}{0.127}}{\log\frac{1/160}{1/320}}\approx 0.48.$$

It can be easily found that we shall obtain another approximate value of a if taking two other nets. Therefore we should compute its average. In the case $t=3$, its average is 0.47. According to Table 2, the average of a is 0.16 in the case $t=11$. Therefore, it seems that for the problem considered the convergence rate of RCM is less than $O(\Delta t^{1/2})$.

Table 3 CPU times of RCM (from $t=0$ to $t=12$)

Numbers of mesh points in the x-direction	81	161	321	641	1281
CPU times (sec)	14	36	119	367	1511

In Table 3 we list the CPU times of RCM for five different nets. As is well-known, the CPU time spent on solving a problem can be roughly divided into two parts. One part (for example, the time spent on compilation) does not depend on the total number of mesh points and the other part does. For explicit schemes, the latter is directly proportional to the total number of mesh points. In our computation while the net size is reduced by $\frac{1}{2}$ in the x-direction, the same thing is done in the t-direction. Therefore in this case the total number of net points is four times the original one. The RCM is an explicit scheme, so the second part of CPU time should approximately be four times the original one. But the first part of CPU time will not increase so fast. This means the following. If the number of net points is quite large, then the CPU time will increase to four times the original one while the size is reduced by $\frac{1}{2}$. The data in Table 3 supports such an opinion. Therefore for the problem considered, the computing time is approximately equal to $c\cdot\Delta t^{-2}$, where c is a constant. Once we have the approximate convergence rate and the estimate of CPU time, we can estimate how high a "price" is needed in order to make error reduced to 10^{-k} times the original one. Obviously, to reduce the error to 10^{-k} times the original one, Δt should be reduced to $10^{-k/a}$ times the original one and the CPU time will increases approximately by $10^{2k/a}$. In the case here $a<\frac{1}{2}$, so the CPU time needed will increase by more than 10^4 times in order to raise the accuracy of the results by 10 times. Therefore when RCM is used to solve this problem, a very high price will be paid in order to obtain a quite accurate result.

Table 4 The moment of transition from deflagration to detonation

	Numbers of mesh points in the x-direction		81	161	321	641	1281
RCM	moment of transition	sequence 1	9.90	9.90	10.16	10.36	10.34
		sequence 2	9.32	9.45	9.99	10.20	10.30
SSM	moment of transition		10.17				

In what follows we can observe how the result of RCM converges to the exact solution (we can take the result of the Singularity-Separating Method (SSM) as the exact one because it is quite accurate). In Table 4 we list the values of the moment of transition from deflagration to detonation. This table shows the following: If we observe the general trend, then we can say that the result of RCM is increasingly closer to the exact solution while the net size becomes smaller and smaller. However the result possesses some randomness. In Figs. 1(a) and 1(b) the pressure curves for different net sizes are given. We can easily find from the figures that the result of RCM tends to the result of SSM in the interval $x=8\sim40$. However in the interval $x=0\sim8$, all the three pressure curves of RCM are quite far away from the result of SSM. The physical picture of RCM in the interval $x=0\sim8$ for $t=11$ is similar to that of SSM for $t=8$ (see Fig. 9 in [2]). That is, according to the result of SSM, the "peak" of the result of RCM near $x=0$ should reach the wall ($x=0$) earlier. The "peak" near $x=0$ for a fine net (see Fig. 1(b)) is closer to the wall than that for a coarse net (see Fig. 1(a)). From this fact, we can see that the result of RCM in interval $x=0\sim8$ also possesses the trend towards the result of SSM.

Therefore we think that the Random Choice Method is the kind of method which possesses strong robustness and by which we can obtain results even for very

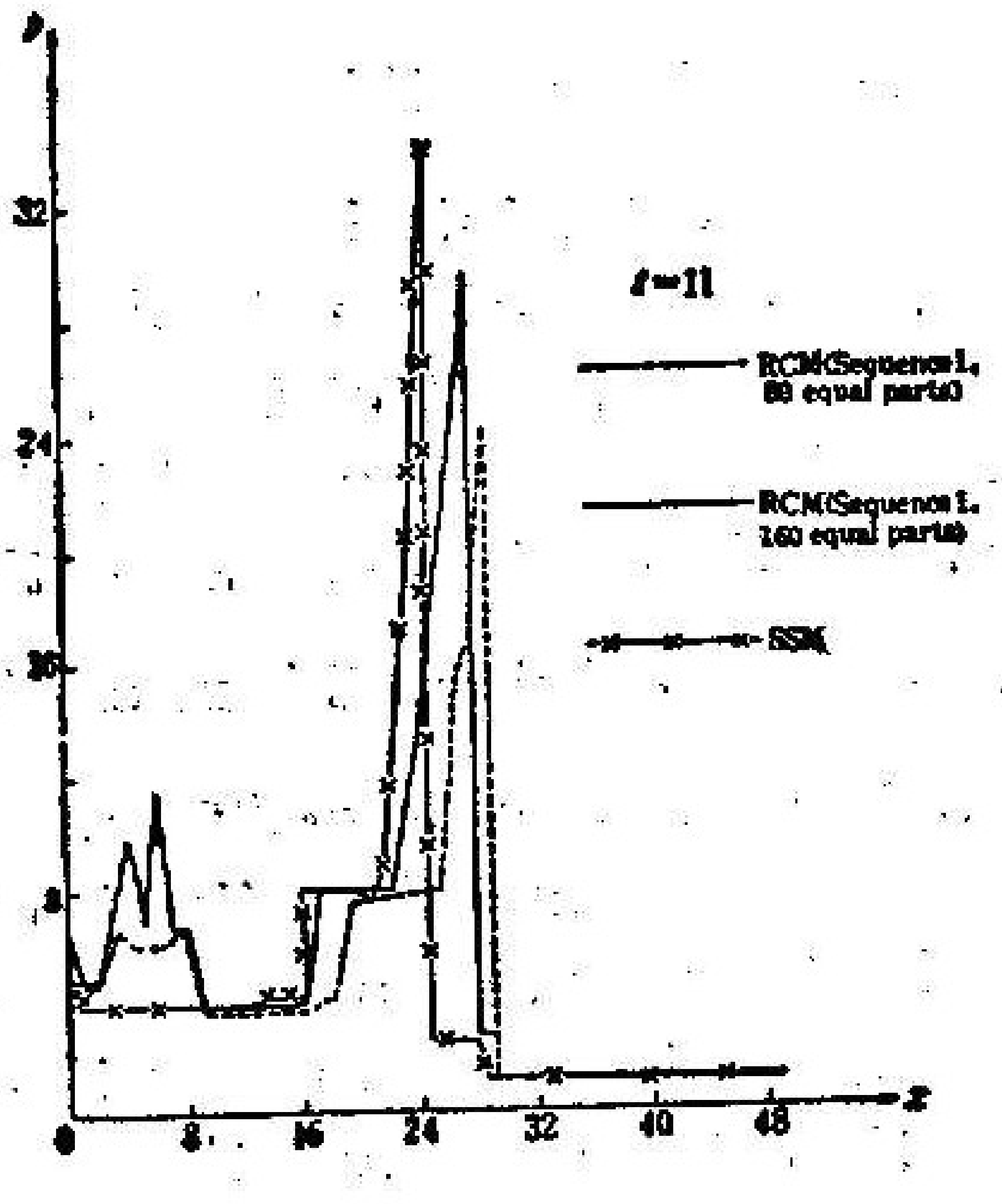

Fig. 1(a)

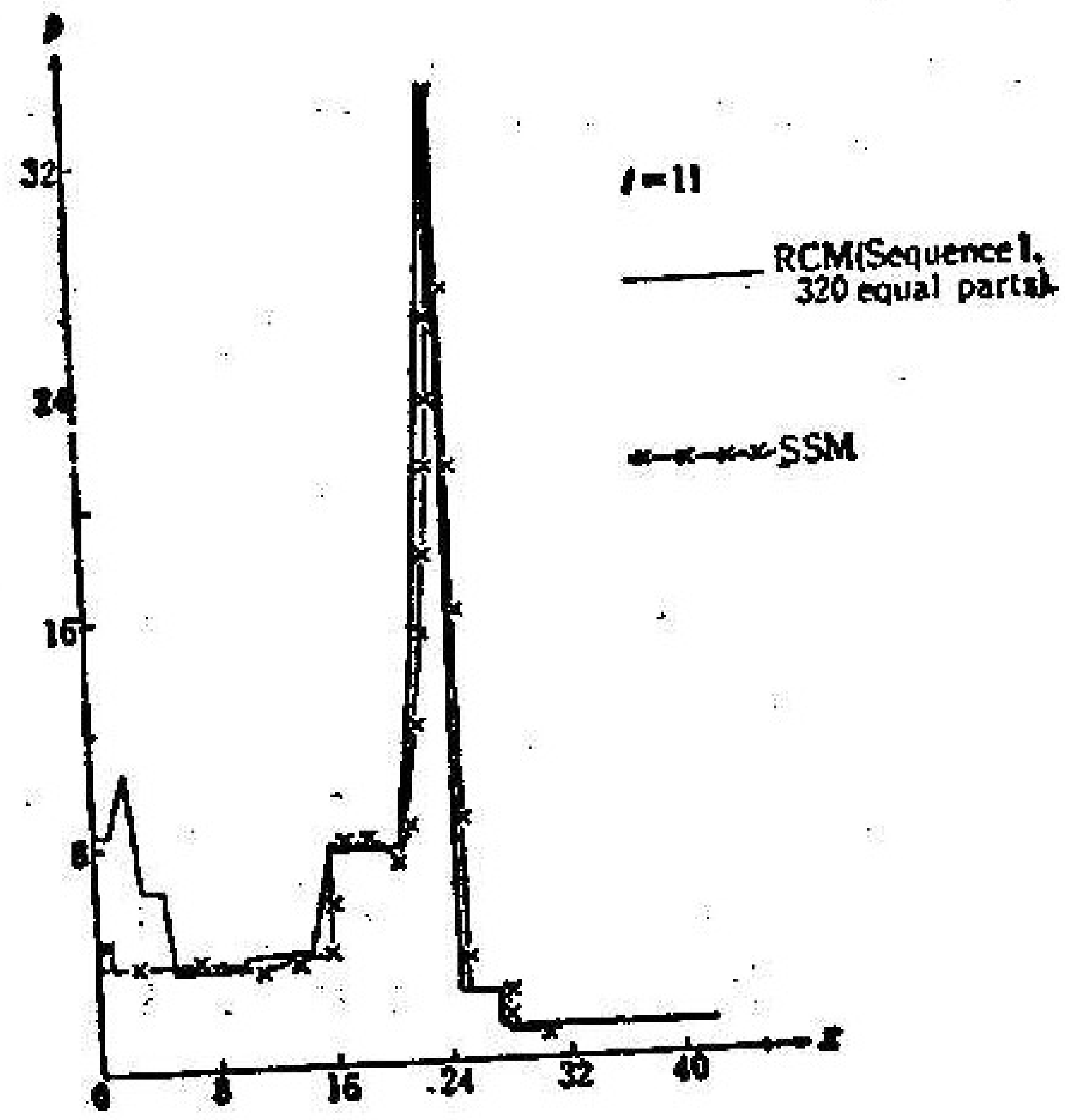

Fig. 1(b)

complicated problems. However, the accuracy is quite low and it is extremely difficult to raise the accuracy of the results.

All the results in this paper are obtained by using the code of Professor Z. h. Teng from Department of Mathematics, Peking University. We think that this paper also contains his labour, and would like to express our heartfelt thanks to him for his help and cooperation.

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