SPACE-TIME TRANSFER FUNCTION-NOISE MODELING OF RAINFALL-RUNOFF PROCESS*

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Abstract

An explanatory model class belonging to the family of Space-Time Transfer Function-Noise (STTFN) processes is presented. The paper develops a three-stage iterative procedure for building STTFN models of the rainfall-runoff process. Four precipitation and runoff stations located in a watershed in southern Ontario, Canada, sampled at 15-day intervals are used for the numerical analysis. Three STTF models are identified. The model parameters are estimated by the polytope technique, a nonlinear optimization algorithm. Two of the developed space-time models proved adequate in describing the spatio-temporal characteristics of precipitation and runoff time series.

§ 1. Introduction

In recent years a large number of stochastic models have been adapted to represent different aspects of the rainfall-runoff process. The most extensively used approach has been the Box-Jenkins (Box and Jenkins, 1976) transfer function-noise (TFN) modeling of hydrologic time series. In particular, linear input-output models are developed with a stochastic part (Strupczewski and Budzianowski, 1984). In this way an increase in accuracy can be achieved within the class of linear models. This modeling procedure relates the output (runoff) of a hydrologic system to the input (rainfall) of the system by adding a noise series. These empirical models have proven very useful in hydrologic analysis and modeling (Salas et al., 1980) and can be used in long-term, as well as short-term hydrologic forecasting.

Besides single-input systems, this class of TFN models has also been used in multi-input systems (Tiao and Box, 1979; Chow et al., 1983). For example, TFN models have been developed relating river flow to precipitation, groundwater levels and temperature. Similarly, the transformation of precipitation series to modified processes has been examined based on cross-correlations of the original and the modified series and by accounting for evaporation and soil moisture storage. Chang et al. (1982) have also developed a TFN model of daily rainfall-runoff process and applied it to five Indiana watersheds. Moreover, a TFN model was developed (Adamowski and Hamory, 1983) relating groundwater levels (output) to streamflow (input).

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There is an increasing interest in hydrology to develop empirical spatio-temporal models of rainfall-runoff in the context of regional hydrologic analysis. Since rainfall and runoff series are correlated in space and time, the Box-Jenkins TFN modeling procedure is extended to a multivariate input-output hydrologic system (Cooper and Wood, 1982; Mohamed, 1985). This results in a general model class of space-time transfer function-noise (STTFN) models. The STTFN model is extended into the spatial domain by using a hierarchical ordering of the spatial neighbors of each rainfall and runoff gage site. The purpose of this study is to develop space-time models of the rainfall-runoff process from the general class of space-time transfer function-noise (STTFN) processes suitable for regional hydrologic analysis and forecasting. In Section 2 the three-stage iterative procedure of identification, parameter estimation and diagnostic checking of the STTFN models is discussed. Section 3 presents an application of the space-time model class to actual rainfall-runoff data for a selected watershed located in southern Ontario, Canada (Fig. 1).

§ 2. Space-Time TFN Model Development

In STTFN modeling, the output y_{it} from $i=1, 2, \cdots, N$ zones over $t=1, 2, \cdots, T$ time periods is assumed to be linearly dependent upon the input series $X_1, X_2, \cdots$, etc. in time and space. The STTFN model may take the form

$$y_{it}=\frac{\sum_{s=0}^{l}\sum_{k=1}^{p}\omega_{sk}B^kL_s}{\left(1-\sum_{s=0}^{m}\sum_{k=1}^{q}\delta_{sk}B^kL_s\right)}B^bL_jx_{it}+a_{it}, \tag{1}$$

where l and m are the spatial orders, p and q are the temporal orders, b and j define an initial period of pure delay or dead time before the response to a given input change begins to take effect, a_{it} is the output noise series independent of x_{it}, B is the backward shift operator in time defined as $B^kY_{it}=Y_{i(t-k)}$, L_s is the spatial lag operator and ω_{sk} and δ_{sk} are parameters.

The spatial lag operator L_s is defined such that

$$L_sy_{it}=\sum_{j=1}^{N}w_{ijs}y_{jt}, \quad \text{for } s>0, \tag{2}$$

where w_{ijs} are a set of weights scaled so that

$$\sum_{j=1}^{N}w_{ijs}=1 \tag{3}$$

for all i and w_{ijs} nonzero only for i and j sites being sth order neighbors. For $s=0$, equation (2) becomes $L_0Y_{it}=y_{it}$. The weights follow a hierarchical ordering of spatial neighbors based on distances between the observation sites in the watershed and may reflect physical characteristics of the observed time series.

2.1. Identification of the STTFN Model

The space-time cross-correlation function (STCCF) between y_{it} and x_{it} series at spatial lag s and time lag k is given by

$$r_{sk}(v, z)=\frac{\sum_{t=k+1}^{T}\sum_{i=1}^{N}\left[v_{it}\left(\sum_{j=1}^{N}(w_{ijs}z_{j(t-k)})\right)\right]}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N}v_{it}^{2}\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left(\sum_{j=1}^{N}(w_{ijs}z_{jt})\right)^{2}\right]^{1/2}}, \tag{4}$$

where $v_{it}=y_{it}-\bar{y}$ and $z_{it}=x_{it}-\bar{x}$ with $\bar{y}$ and $\bar{x}$ being respectively estimates of the space-time grand means given by

$$\bar{y}=\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}y_{it}$$

and (5)

$$\bar{x}=\frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N}x_{it}.$$

The identification of STTFN models is based on the estimation of the STCCF between the rainfall series x_{it} and the runoff series y_{it}. From the physical understanding of the hydrologic cycle there should be at least one value of the STCCF significantly different from zero. These values can explain the lagging of runoff with respect to rainfall referred to as the delay parameter.

The estimated STCCF could lead to spurious correlations due to autocorrelations which are present in the rainfall and runoff series. The STCCF can also have high variance and at different time lags can be highly correlated with one another. To minimize these errors and to develop white noise series a model from the general family of space-time autoregressive moving average (STARMA) models (Martin and Oeppen, 1975; Cliff and Ord, 1975; Pfeifer and Deutsch, 1980; Mohamed, 1985) can be identified for the input rainfall series x_{it}. This model may be used to transform the correlated input series x_{it} to the uncorrelated or prewhitened z_{it} series. The same model or another one can be used to transform the output y_{it} series to the v_{it} series. The STCCF is then computed between the new z_{it} and v_{it} series.

The general family of STARMA models is given by

$$y_{it}=a_{it}+\sum_{s=0}^{l}\sum_{k=1}^{p}\phi_{sk}L_{s}y_{i(t-k)}-\sum_{s=0}^{m}\sum_{k=1}^{q}\theta_{sk}L_{s}a_{i(t-k)}, \tag{6}$$

where p is the autoregressive (AR) order, q is the moving average (MA) order, l and m are the spatial orders of AR and MA, respectively and ϕ_{sk} and θ_{sk} are parameters to be estimated. The identification of STARMA model for the rainfall series x_{it} is based on the inspection of the space-time autocorrelation function (STACF) given at spatial lag s and time lag k by

$$r_{sk}=\frac{\sum_{t=k+1}^{T}\sum_{i=1}^{N}\left[z_{it}\left(\sum_{j=1}^{N}(w_{ijs}z_{j(t-k)})\right)\right]}{\left[\sum_{t=1}^{T}\sum_{j=1}^{N}z_{it}^{2}\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left(\sum_{j=1}^{N}(w_{ijs}z_{jt})\right)^{2}\right]^{1/2}} \tag{7}$$

and the space-time partial autocorrelation function (STPACF) given for the same spatial and time lags by

$$r_{hgsk}=\frac{\sum_{t=v+1}^{T}\sum_{i=1}^{N}\left[\left[\sum_{j=1}^{N}(w_{ijh}z_{j(t-g)})\right]\left[\sum_{j=1}^{N}(w_{ijs}z_{j(t-k)})\right]\right]}{\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\sum_{j=1}^{N}(w_{ijh}z_{jt})\right]^{2}\right]^{1/2}\left[\sum_{t=1}^{T}\sum_{i=1}^{N}\left[\sum_{j=1}^{N}(w_{ijs}z_{jt})\right]^{2}\right]^{1/2}}, \tag{8}$$

where $v = \max(g, k)$.

Following the prewhitening of x_{it} to z_{it} series and transforming y_{it} to v_{it} series by using the identified STARMA model for the rainfall series x_{it}, the STTFN model (l, p, m, q, b, n, r) may be written as

$$v_{it} = \frac{\sum_{s=0}^{l}\sum_{k=1}^{p} \omega_{sk} B^k L_s}{\left(1-\sum_{s=0}^{m}\sum_{k=1}^{q} \delta_{sk} B^k L_s\right)} B^b L_j Z_{it} + N_{it}, \tag{9}$$

where N_{it} is the transformed noise series defined by

$$N_{it} = \frac{\sum_{s=0}^{l}\sum_{k=1}^{p} \phi_{sk} B^k L_s}{\sum_{s=0}^{m}\sum_{k=1}^{q} \theta_{sk} B^k L_s} a_{it}. \tag{10}$$

The STCCF is estimated from equation (4). The impulse response weights v_{sk} (the coefficients of (B)) are given by

$$v_{sk} = r_{sk}(v, z)\frac{SD_v}{SD_z}, \tag{11}$$

where SD_z and SD_v are the standard deviations of the z_{it} and v_{it} series, respectively.

The STCCF $r_{sk}(v, z)$ and the coefficients are used to identify the orders of l, p, m, q and the pure delay parameters b and j using the following rules (Martin and Oeppen, 1975): zero or near zero correlation values up to spatial lag $j-m$ and time lag $b-p$ followed by irregular or rising values up to spatial lag $j+1-m$ and time lag $b+p-q$ and correlation $r_{sk}(v, z)$, $s > j+1-m+1$, $k > b+p-q+1$, which decay exponentially in time and space. It should be mentioned that no such rising correlation values occur if $l < m$ and $b < p$. Once the values of these parameters are determined the initial values of the coefficients ω_{sk} and δ_{sk} can then be estimated. In this way the STTF model is identified for the input-output system.

2.2. Estimation of the STTFN Model

Estimates of the parameters ω_{sk} and δ_{sk} of the tentative STTF model can be obtained by minimizing the residual sum of squares:

$$s(\omega, \delta, \phi, \theta) = \sum_{i=1}^{N}\sum_{t=1}^{T} a_{it}^2. \tag{12}$$

Given any initial values for the parameters ω_{sk} and δ_{sk} the errors n_{it} can be estimated from

$$n_{it} = y_{it} - \frac{\sum_{s=0}^{l}\sum_{k=1}^{p} \omega_{sk} B^k L_s}{\left(1-\sum_{s=0}^{m}\sum_{k=1}^{q} \delta_{sk} B^k L_s\right)} L_j X_{i(t-b)} \tag{13}$$

and the transformed noise series n_{it} is then defined by

$$n_{it} = \frac{\sum_{s=0}^{l}\sum_{k=1}^{p} \phi_{sk} B^k L_s}{\sum_{s=0}^{m}\sum_{k=1}^{q} \theta_{sk} B^k L_s} a_{it}. \tag{14}$$

Equation (13) is a modified version of the following equation:

$$y_{it}=\frac{(\omega_0-\omega_1B^l-\cdots-\omega_nB^n)}{(1-\delta_1B-\delta_2B^2-\cdots-\delta_rB^r)}x_{i(t-b)}, \tag{15}$$

where n and r are orders to be determined.

The adopted Box-Jenkins modeling procedure requires that the errors a_{it} should be pure white noise. Since the STTFN model is nonlinear in form the polytope method (Nelder and Mead, 1965) is employed for the estimation of the parameters. This method is one of several nonlinear optimization techniques included in a nonlinear optimization package (Birta, 1983). The polytope method is found to offer the advantage of not requiring gradient information about the objective function to be minimized and not having a dependence on a linear search subproblem. In addition, this method is considered an appropriate technique when the objective function to be minimized is subject to random errors. The computations start by setting up a regular simplex in n-dimensional space and evaluating the objective function at the vertices. The simplex then proceeds by reflecting the maximum vertex in the centroid of the other vertices. If at any stage the new vertex has the largest value then it proceeds by reflecting the next largest value and so on.

2.3. Diagnostic Checking of the STTFN Model

Diagnostic checking is performed to examine whether there is any inadequacy in the selected STTF model. The residuals STACF and the Port Manteau tests are used to examine the whiteness of the residuals. The cumulative periodogram test is also used to investigate the presence of any periodicities in the residuals. A pattern of nonzero STCCFs between the residuals of the STTFN model and the prewhitened series z_{it} would indicate any inadequacy in the tentative model.

Port Manteau Test: The whiteness of the estimated residuals of the fitted model is tested using the equation:

$$Q=N\sum_{k=1}^{k}r_k^2(\hat{a}), \tag{16}$$

where $r_k^2(\hat{a})$ are the ACF of the residuals a_{it}, N is the sample size and k is the maximum time lag. The statistic Q is approximately chi-square distributed with $k-p-q$ degrees of freedom, where p and q are the AR and MA orders, respectively. The adequacy of the selected space-time model may be checked by comparing the statistic Q with the chi-square value $X^2(k-p-q)$ of a given significance level. If $Q<X^2(k-p-q)$, a_{it} is an independent series and the model is adequate, otherwise the model is inadequate.

Anderson's Test: The ACF of the residuals are used to test the whiteness of the noise series. If the residuals are approximately white noise the ACFs should all be approximately equal to σ^2I_N. If any of the residuals ACF is significantly different from zero another model is selected.

Cumulative Periodogram Test: If the residuals contain any periodicities, the cumulative periodogram would show significant deviation from the lines of the confidence limits at the specified level of significance, otherwise the residuals should lie within the confidence limits and should be considered white noise.

Space-Time Cross Correlation Test: The residual STCCF could indicate inadequacy of the selected STTFN model. This test is performed by obtaining the

STCCF between the prewhitened input series z_{it} and the estimated residuals from the fitted model. The standard errors associted with the STCCF are estimated from

$$ER=\frac{1}{\sqrt{N-k}}, \tag{17}$$

where N is the sample size and k is the number of lags in time. The pattern of the nonzero STCCFs would indicate inadequacy of the selected space-time model.

If any of the above diagnostic checks reveal inadequacy in the selected space-time model, the model-building procedure is repeated and a new model is selected. Alternatively, a STARMA model could be identified for the noise series, which could be then combined with the original space-time model to form a STTFN model.

§ 3. Space-Time TFN Model of the Rainfall-Runoff Process

The data used in this study consist of precipitation and runoff time series from four gaging stations located within the Grand River basin in southern Ontario, Canada (Fig. 1). Data is available for the period of July 1966 to June 1980. A time step of 15-day precipitation totals is used in this study, which allows the time series to preserve the characteristics of the storm events. Similarly, the runoff series are sampled every 15 days. A portion of the data, i.e., 192 values, is used in the model-building procedure and the remaining 144 values are available for the evaluation and forecasting phase. There were a few missing values in the precipitation time series and the normal-ratio method (Linsley et al., 1982) was used to estimate these data points. The watershed is divided into four polygonal subareas using the Thiessen method. The four runoff gage stations correspond to the same subareas where the four raingage stations are located (Fig. 1). Equal weights are selected for this hierarchical weighting scheme of the space system with a maximum spatial order of 2 (Table 1).

The STACF and STPACF of the original precipitation series show a lack of structure, which suggests that the space-time system is nonstationary. First temporal differencing is applied to the precipitation system to achieve stationarity. Initial identification of the differenced precipitation series results in an STMA (1_2) model of order one in time and two in space. The parameters of the STMA (1_2) precipitation model are estimated using the polytope algorithm. Diagnostic checking of the tentative model indicates that the STMA (1_2) model adequately describes the observed precipitation series. The STMA (1_2) model is expressed by the following form:

$$x_{it}=x_{i(t-1)}+a_{it}-0.9448a_{i(t-1)}+0.0672W_1a_{i(t-1)}-0.004W_2a_{i(t-1)}. \tag{18}$$

This model is used in the STTFN model-building procedure of the rainfall-runoff process to determine the prewhitened series z_{it} and the transformed series v_{it}.

The STACFs of the original and deseasonalized runoff series are shown in Tables 2 and 3, respectively. The original data are nonstationary in time since the STACF fails to tail off quickly at all spatial lags. The deseasonalized runoff series indicate stationarity in time and the decay with spatial lag is much steeper than that with time lag. The STCCF between the z_{it} and v_{it} series given in Table 4 shows that there are relatively significant correlations at time lag one ($k=1$) for all the spatial lags $s=0, 1, 2$. Similarly, the estimated impulse response function u_{sk} of equation (11) given in Table 5 indicates that there are values significantly different from zero only at time lag one ($k=1$) for all the spatial lags $s=0, 1, 2$. In other words, runoff is lagged 15 days or one time step behind precipitation, which suggests that the memory of the rainfall-runoff process b is 15 days or one time lag.

Preliminary identification of the STTF(m, q, l, p, b, n, r) model of equation (13) or (15) based on the Box-Jenkins procedure suggests three tentative models. The first model is an STTF(0, 0, 2, 0, 1, 0, 0) given by

$$y_{it}=\sum_{s=0}^{2}\omega_{0s}W_s x_{i(t-1)}+a_{it}. \tag{19}$$

The second model is an STTF(1, 2, 2, 0, 1, 0, 0) expressed by

$$y_{it}=\frac{\sum_{s=0}^{2}\omega_{0s}W_s}{1-\sum_{s=0}^{2}\delta_{sk}B^kW_s}\,x_{i(t-1)}+a_{it} \tag{20}$$

and the third is an STTF(0, 0, 2, 1, 1, 1, 0) model given by

$$E_{it}=\sum_{s=0}^{2}\sum_{l=1}^{1}(\omega_{0s}-\omega_{1s}B)W_sX_{i(t-1)}+a_{it}. \tag{21}$$

The parameters of the STTF(0, 0, 2, 0, 1, 0, 0) model of equation (19) are estimated by using the polytope algorithm and the results are summarized in Table 6. In the diagnostic checking stage, the residuals STACFs are computed to check the adequacy or not of the identified model and the results are presented in Table 7. The residuals are generated by incorporating the parameter estimates of the developed STTF(0, 0, 2, 0, 1, 0, 0) model into the appropriate form of equation (19) for the model-building period. The Anderson's test of Table 7 shows a lack of srtucture among the STACFs, which suggests that the generated residuals are uncorrelated and consequently white noise. Similarly, the Port Manteau test of Table 8 indicates that the generated residuals are uncorrelated at the 0.05 level of significance. Finally, the cumulative periodograms of the generated residuals at the 0.05 confidence level suggest that the residuals contain no periodicities. These tests summarize that the STTF(0, 0, 2, 0, 1, 0, 0) model is considered adequate.

Estimation of the STTF(1, 2, 2, 0, 1, 0, 0) model parameters (equation (20)) using the polytope method results in the following estimates: $\omega_{00}=0.05725$, $\omega_{01}=0.01699$, $\omega_{02}=-0.02132$, $\delta_{02}=0.06290$, $\delta_{12}=0.02156$ and $\delta_{22}=0.00502$ with the residual sum of squares being $S=0.234\times10^6$. The Anderson's and Port Manteau tests indicate that the generated residuals are white noise. However, the cumulative periodograms show a marked departure from linearity, since they fall outside the confidence limits at the 0.05 level of significance. As a result the residuals contain periodicities and the STTF(1, 2, 2, 0, 1, 0, 0) model is rejected. In the case of the STTF(0, 0, 2, 1, 1, 1, 0) model, the parameters are again estimated by the polytope method and the model has the following form:

$$E_{it} = -0.0568\nabla X_{i,t-1} - 0.0046\nabla X_{i,t-2} + 0.0180 W^{(1)}\nabla X_{i,t-1} + 0.0095 W^{(1)}\nabla X_{i,t-2} - 0.0112 W^{(2)}\nabla X_{i,t-1} + 0.0232 W^{(2)}\nabla X_{i,t-2} + a_{it} \quad (22)$$

with the residuals sum of squares being $S=0.235\times10^6$. The Anderson's, the Port Manteau and the cumulative periodogram tests indicate that the generated residuals are uncorrelated without periodicities, which suggests the residuals are white noise. Based on the above findings the STTF(0, 0, 2, 1, 1, 1, 0) model is considered adequate. Since one of the objectives of the Box-Jenkins procedure remains the development of a parsimonious space-time model with the smallest number of parameters, the STTF(0, 0, 2, 0, 1, 0, 0) model is adopted to represent the rainfall-runoff process.

Finally, the forecasting performance of the adopted STTF (0, 0, 2, 0, 1, 0, 0) model is examined for the evaluation period of the remaining 144 values. For this purpose the mean and variance of the generated residuals are estimated for the model-building period of 192 data points. These statistical parameters are used to randomly generate normally distributed residuals for the evaluation or forecasting period. These residuals have been tested for whiteness and presence of periodicities using the cumulative periodograms (Fig. 2), the Port Manteau and the Anderson's tests and have shown to be uncorrelated and white noise. The new generated residuals are then subtracted from the observed series and the computed series are produced for the rainfall-runoff system. Fig. 3 shows plots of the observed and computed runoff series for the model development as well as the evaluation period for the Galt station. The plots indicate satisfactory performance for the developed space-time rainfall-runoff system.

§ 4. Summary and Conclusions

The comprehensive procedure presented in this paper for building a space-time transfer function noise model is useful in detecting dynamic relationships between hydrologic time series. It can be used as a forecasting tool in the field of water resources engineering. The STTF modeling technique is applied to the rainfall-runoff process of a system of four rainfall and four runoff series spatially located in the Grand River watershed in southern Ontario, Canada (Fig. 1).

The choice of a spatial lag structure to reflect the influence of one zone on another can deeply affect the form of the space-time autocorrelation and partial autocorrelation functions. This could lower the adequacy of the identified space-time model and produce spurious results and might also eliminate useful models from consideration. The runoff series have shown seasonality in the means. The seasonal components were removed. The characteristics of the theoretical STCCF model are similar to the one of the Box-Jenkins transfer function noise model. The generated noise components presented in the STTF models are found to possess the characteristics of a white noise. The performance of the generated series from the STTF models are found to compare well with the corresponding observed rainfall and runoff data. For the selected watershed, the output (runoff) lags behind the input (rainfall) and the delay parameter is 15 days or 1 lag in time.

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Table 1 Neighbors of each site for each spatial order

Spatial order		0	1	2
Site	1	1	2, 3	4
	2	2	1, 3	4
	3	3	1, 2, 4	
	4	4	3	1, 2

Table 2 STACF of the original runoff series

Spatial lag (s) / Time lag (k)	0	1	2
1	0.5236	−0.0228	−0.2168
2	0.4367	−0.0533	−0.2203
3	0.3805	−0.0588	−0.2163
4	0.4055	−0.0554	−0.2143
5	0.3760	−0.0585	−0.2113
6	0.3845	−0.0587	−0.2096

Table 3 STACF of deseasonalized runoff series

Spatial lag (s) / Time lag (k)	0	1	2
1	0.0643	0.0088	0.0003
2	−0.0505	−0.0309	−0.0033
3	−0.0297	−0.0087	−0.0012
4	0.0724	0.0115	0.0023
5	0.0504	0.0126	0.0016
6	0.1115	0.0303	0.0039

Table 4 STCCF between the z_{it} and the v_{it} series

Spatial lag (s) / Time lag (k)	0	1	2
0	−0.0186	−0.0236	−0.0076
1	0.1357	0.1000	0.1312
2	0.0275	0.0030	0.0237
3	0.0220	0.0078	0.0490
4	0.0015	−0.0073	0.0029
5	−0.0044	0.0041	−0.0058
6	0.0522	0.0421	0.0857

Table 5 Impulse response functions

Spatial lag (s) / Time lag (k)	0	1	2
0	−0.014	−0.016	−0.005
1	0.095	0.070	0.092
2	0.019	0.002	0.016
3	0.015	0.005	0.034
4	0.001	−0.005	0.002
5	−0.003	0.003	−0.004
6	0.036	0.029	0.060

Table 6 Parameter estimates of the STTF(0, 0, 2, 0, 1, 0, 0) model

Parameter	Guess	Estimate	Initial S	Final S
	0.00	−0.0457		
	0.00	0.0157		
	0.00	−0.0243	0.238×10^6	0.235×10^6

S is the residual sum of squares

Table 7 STACF of the generated residuals of the STTF (0, 0, 2, 0, 1, 0, 0) model

Spatial lag (s) / Time lag (k)	0	1	2
1	0.0646	0.0083	−0.0040
2	−0.0553	−0.0286	−0.0014
3	−0.0277	−0.0098	0.0026
4	0.0778	0.0134	0.0083
5	0.0512	0.0120	0.0009
6	0.1057	0.0214	−0.0074

Table 8 Results of Port Manteau test on generated residuals

Spatial lag (s)	Port Manteau test	Chi-square statistics =0.05	Decision
0	11.59	27.6	accepted
1	8.97	27.6	accepted
2	6.68	27.6	accepted

List of Figures

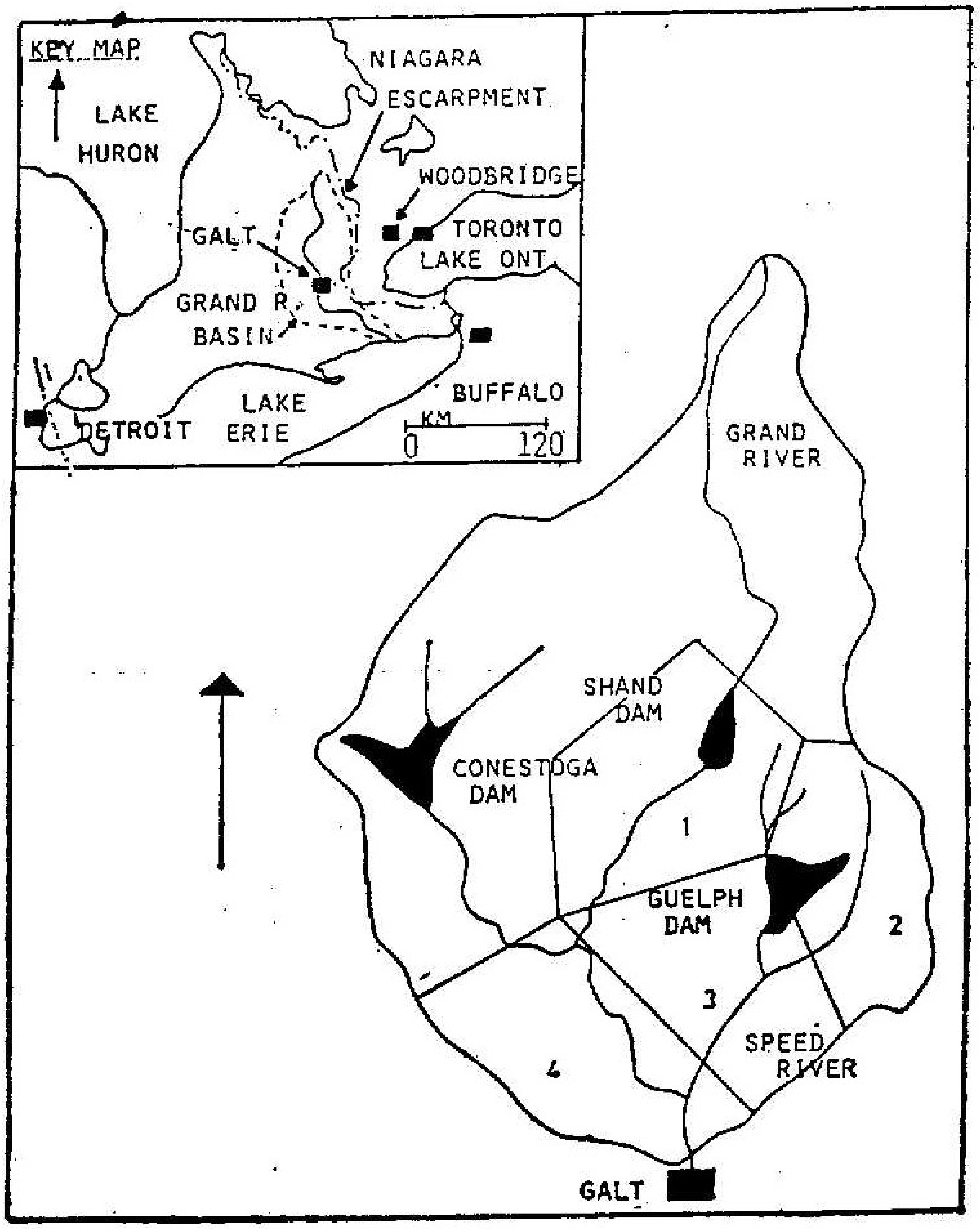

Fig. 1 The watershed and key map.

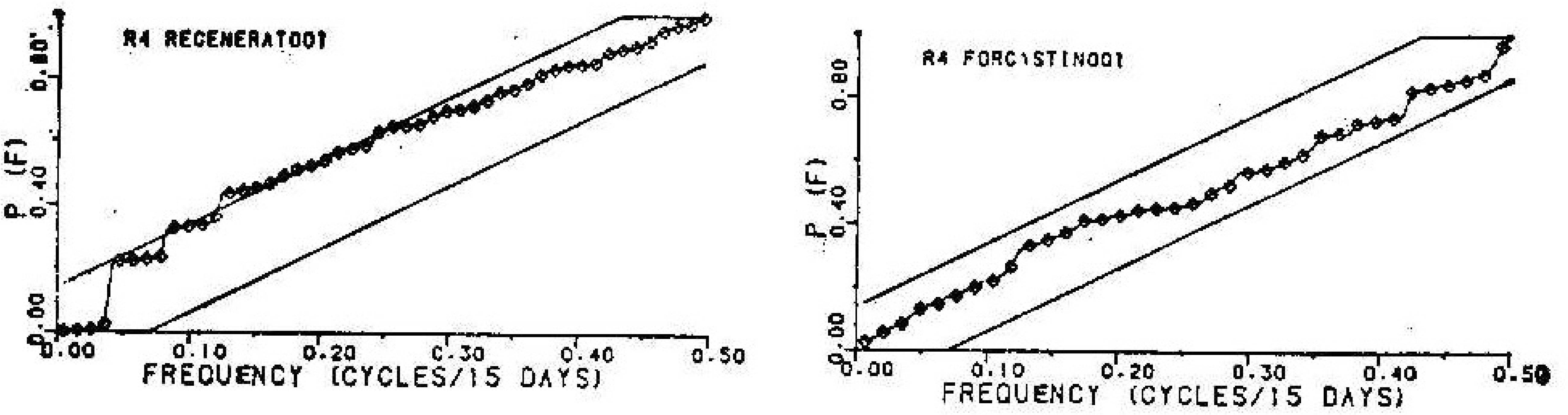

Fig. 2 Cumulative periodograms of generated residuals of the Galt runoff series.

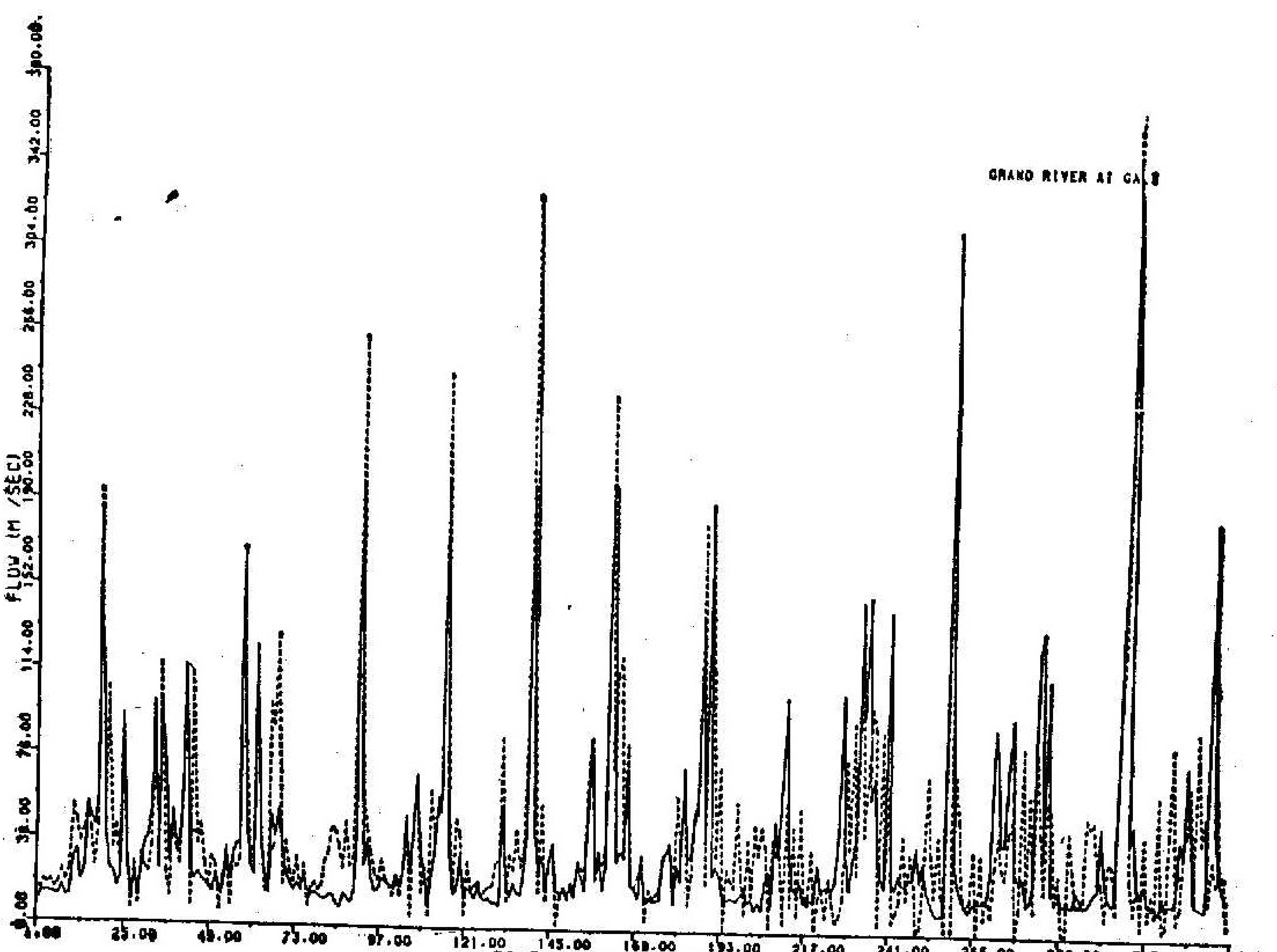

Fig. 3 Observed and generated runoff series for the Galt station.