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G^1 SMOOTHING SOLID OBJECTS BY BICUBIC BEZIER PATCHES*

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Abstract

A general and unified method is presented for generating a wide range of 3-D objects by smoothing the vertices and edges of a given polyhedron with arbitrary topology using bicubic Bezier patches. The common solution to the compatibility equations of G^1 geometric continuity between two Bezier patches is obtained and employed as the foundation of this new method such that this new solid and surface model is reliable and compatible with the solid modeling and surface modeling systems in the most common use. The new method has been embeded in an algorithm supported by our newly developed solid modeling system MESSAGE. The performance and implementation of this new algorithm show that it is efficient, flexible and easy to manipulate.

§1. Introduction

In recent years, much effort has been put to developing more reliable and flexible solid modeling systems and surface modeling systems to meet the needs in industry. Combining the surface modeling and solid modeling is a new trend in computer graphics, CAD/ CAM and their applications. The application of surface modeling techniques within a solid modeling system requires a general and unified method to generate a wide range of 3-D objects bounded by planar and Bezier patches. Especially, in shape design, it is very common and important to creat a wide range of objects from polyhedra to free form shape in one system.

Quite a number of solid and surface modeling systems have adopted more flexible mathematical models such as B-reps, CSG, Bezier patches, B-spline surfaces, Coons patches and so on. Recently, the topic of integration of surface modeling with solid modeling has received much attention [1]-[2]. For the description of many objects, both the flexibility of the shape controlling of free-form surfaces and compatible representation of solid modeling techniques must be provided. Unfortunately, how to generate a wide range of 3-D objects from a polyhedron such that the model is consistent with the most solid modeling system is still a problem.

Some useful methods for smoothing vertices and edges of polyhedra have been proposed by Doo et al.[3]-[4], Lu et al.[5], H.Chiyokura and F.Kimura et al.[6]-[7] and J.R. Rossignac and A.A.G. Requicha et al.[8]-[9]. In [3], [4], extraordinary points are represented by a lot of subdivided patches, and globally rounded surfaces are generated from polyhedra. But it is difficult to round off a solid locally or generate sharp edge curves, and it is difficult to do any analysis because it is not described in mathematical expression and analytic form. In [6] a method is proposed for rounding off corners and edges of polyhedra using Gregory patches

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and applied to MODIF system successfully. The Gregory patches are neither represented by Bezier nor B-spline surfaces and are inconsistent with most solid and surface modeling systems. In [8], [9] some blending methods were proposed, but they cannot be used for rounding off corners. The main difficulties in solving this problem come from the smooth joint between free-form surfaces. Unfortunately, the conditions of geometric continuity and its solutions are still a considerable problem.

In this paper the common solutions to the compatible equations of geometric continuity of first order (denoted by G^1) between two Bezier patches are obtained from which we drive the condition of G^1 geometric continuity (Fig. 1)

$$r(1,v) = \bar{r}(\bar{u},0), \quad 0 \le \bar{u} = v \le 1.$$

It is readily shown that if S and $\bar{S}$ meet with G^1 along Γ, then there exist the following rational polynomials

$$p(v) = \sum_{i=0}^{5} e_i v^i \Big/ \sum_{i=0}^{5} d_i v^i, \tag{7}$$

$$q(v) = \sum_{i=0}^{6} f_i v^i \Big/ \sum_{i=0}^{5} d_i v^i, \tag{8}$$

such that

$$\left.\frac{\partial \bar{r}}{\partial \bar{u}}\right|_{\bar{v}=1} = p(v)\left.\frac{\partial r}{\partial u}\right|_{u=0} + q(v)\left.\frac{\partial r}{\partial v}\right|_{u=0}, \quad 0 \le v = \bar{u} \le 1. \tag{9}$$

Substituting (7) and (8) into (9) and comparing the coefficients on both sides yield

$$M_d^1(\bar{Q}_{10},\bar{Q}_{11},\bar{Q}_{12},\bar{Q}_{13})^T = M_e^1(\bar{P}_{20},\bar{P}_{21},\bar{P}_{22},\bar{P}_{23})^T + M_f^1(\bar{P}_{31},\bar{P}_{32},\bar{P}_{33})^T, \tag{10}$$

$$M_d^2(\bar{Q}_{10},\bar{Q}_{11},\bar{Q}_{12},\bar{Q}_{13})^T = M_e^2(\bar{P}_{20},\bar{P}_{21},\bar{P}_{22},\bar{P}_{23})^T + M_f^2(\bar{P}_{31},\bar{P}_{32},\bar{P}_{33})^T, \tag{11}$$

where

$$M_d^1 = \begin{bmatrix} d_0 & & & \\ d_1 & d_0 & & \\ d_2 & d_1 & d_0 & \\ d_3 & d_2 & d_1 & d_0 \end{bmatrix}, \quad M_e^1 = \begin{bmatrix} e_0 & & & \\ e_1 & e_0 & & \\ e_2 & e_1 & e_0 & \\ e_3 & e_2 & e_1 & e_0 \end{bmatrix}, \quad M_f^1 = \begin{bmatrix} f_0 & & \\ 2f_1 & 3f_0 & \\ f_2 & 2f_1 & 3f_0 \\ f_3 & 2f_2 & 3f_1 \end{bmatrix},$$

$$M_d^2 = \begin{bmatrix} d_4 & d_3 & d_2 & d_1 \\ d_5 & d_4 & d_3 & d_2 \\ & d_5 & d_4 & d_3 \\ & & d_5 & d_4 \\ & & & d_5 \end{bmatrix}, \quad M_e^2 = \begin{bmatrix} e_4 & e_3 & e_2 & e_1 \\ e_5 & e_4 & e_3 & e_2 \\ & e_5 & e_4 & e_3 \\ & & e_5 & e_4 \\ & & & e_5 \end{bmatrix}, \quad M_f^2 = \begin{bmatrix} f_4 & 2f_3 & 3f_2 \\ f_5 & 2f_4 & 3f_3 \\ f_6 & 2f_5 & 3f_4 \\ & 2f_6 & 3f_5 \\ & & 3f_6 \end{bmatrix}.$$

From (10) we obtain $(\bar{Q}_{10},\bar{Q}_{11},\bar{Q}_{12},\bar{Q}_{13})$. Substituting it into (11) we have some conditions on the coefficients d_i, e_i and f_i which are called shape parameters. Notice that the shape parameters must be independent of the control points. Thus we have

$$M_e^2 = M_d^2(M_d^1)^{-1}M_e^1, \tag{12}$$

$$M_f^2 = M_d^2(M_d^1)^{-1}M_f^1. \tag{13}$$

Denote

$$M_d^1)^{-1} = \begin{bmatrix} a_0 & & & \\ a_1 & a_0 & & \\ a_2 & a_1 & a_0 & \\ a_3 & a_2 & a_1 & a_0 \end{bmatrix} \tag{14}$$

where

$$a_0 = 1/d_0, \quad a_i = -a_0 \sum_{k=0}^{i-1} a_k d_{i-k}. \tag{15}$$

of an umbrella which is a set of patches in an umbrella (see the definition in Section 2) and obtain the common solutions to the compatible equations of the umbrella. Then we develop a new method for generating a wide range of 3-D objects which are obtained by local modifications of polyhedra. The algorithm for generating this new 3-D model has been designed and applied in our newly developed solid modeling system MESSAGE which is capable of designing complex solid objects bounded by planar and free-form surfaces. The performance and pictures produced by this new algorithm are illustrated in Section 5.

§2. Compatible Equations of Geometric Continuity of First Order

Recently, the topic of geometric continuity between two free-form patches has received much attention from CAD, CAGD and computer graphics community. A number of papers on this subject have been published [10]–[14]. We have obtained the common solution to the compatible equations of geometric continuity of nth order between two regular patches [15]–[17]. In this paper we confine ourselves to bicubic Bezier patches only.

2.1. The Condition of G^1 Geometric Continuity Between Bezier Patches

Let S and $\bar{S}$ be bicubic Bezier patches

$$S : r(u,v) = \sum_{i,j=0}^{3} P_{ij} B_{i3}(u) B_{j3}(v), \qquad 0 \le u \le 1, 0 \le v \le 1, \tag{1}$$

$$\bar{S} : \bar{r}(\bar{u},\bar{v}) = \sum_{i,j=0}^{3} Q_{ij} B_{i3}(u) B_{j3}(v), \qquad 0 \le \bar{u} \le 1, 0 \le \bar{v} \le 1 \tag{2}$$

where $B_{i3}(i = 0,1,2,3)$ are Beinstein base functions. Rewrite (1) and (2) into the following forms:

$$r(u,v) = \sum_{i,j=0}^{3} \bar{P}_{ij} u^i v^j, \tag{3}$$

$$r(u,v) = \sum_{i,j=0}^{3} \bar{Q}_{ij} (1-\bar{u})^i \bar{v}^j, \tag{4}$$

$$(\bar{P}_{ij}) = M_1 (P_{ij}) (M_1)^T, \tag{5}$$

$$(\bar{Q}_{ij}) = M_2 (Q_{ij}) (M_1)^T, \tag{6}$$

where M_1, M_2 are matrices of transformations from base functions $\{u^3, u^2, u, 1\}$ and $\{(1-u)^3, (1-u)^2, 1-u, 1\}$ to Beinstein base functions respectively.

Assume that the two patches S and $\bar{S}$ meet along their common boundary Γ. Hence from (12)–(15),

$$e_i = a_0 e_0 d_i, \quad i = 0, 5; \quad f_{i+1} = a_0 f_0 d_{i+1} + (a_1 f_0 + a_0 f_1) d_i, \quad i = 1, 5. \tag{16}$$

Denote

$$\alpha = a_0 e_0, \quad \beta = a_0 f_0, \quad \beta' = -(a_0 f_0 + a_0 f_1 + a_1 f_0). \tag{17}$$

Thus we obtain

Theorem 1. *If the shape parameters are independent of the control points, then the common solution to the condition of G^1 continuity (9) is*

$$p(v) = \alpha, \quad q(v) = \beta(1-v) - \beta' v \tag{18}$$

where α, β and β' are arbitrary constants and are called shape parameters of Γ.

The assumption of the independence of the shape parameters of the control points is essential, because in shape design the designers need to modify the shapes by changing their shape parameters while keeping the control points invariable. In this sense, we can say that (18) is the common solution to the condition of G^1 continuity. From Theorem 1 we have

Theorem 2. *The necessary and sufficient condition for S and $\bar{S}$ to meet with G^1 continuity along Γ is that there exist $\alpha(>0), \beta$ and β' such that the following equations hold:*

$$T_2 = -\alpha T_0 + \beta T_1, \tag{19}$$

$$V_2 = -\alpha V_1 + (1 + \alpha - \frac{2}{3}\beta - \frac{1}{3}\beta') T_1 + \frac{2}{3}\beta(E + T_1'), \tag{20}$$

$$T_2' = -\alpha T_0' + \beta' T_1', \tag{21}$$

$$V_2' = -\alpha V_1' + (1 + \alpha - \frac{2}{3}\beta' - \frac{1}{3}\beta) T_1' + \frac{2}{3}\beta'(E' + T_1) \tag{22}$$

where

$$T_0 = P_{20} - P_{30}, \qquad T_1 = Q_{01} - Q_{00}, \qquad T_2 = Q_{10} - Q_{00}, \qquad V_1 = P_{21} - P_{30},$$
$$V_2 = Q_{11} - Q_{00}, \qquad E = Q_{03} - Q_{00}$$

and similarly for T_i', V_i' and E' (Fig. 2).

In Theorem 2, the necessary and sufficient condition for S and $\bar{S}$ to meet with G^1 continuity along Γ is expressed in terms of the shape parameters, tangent vectors and twist vectors.

2.2. G^1 Continuity of an Umbrella

To apply surface design techniques to solid design, it is necessary to establish the concept of geometric continuity among Bezier patches which meet at a common vertex and form an umbrella in shape.

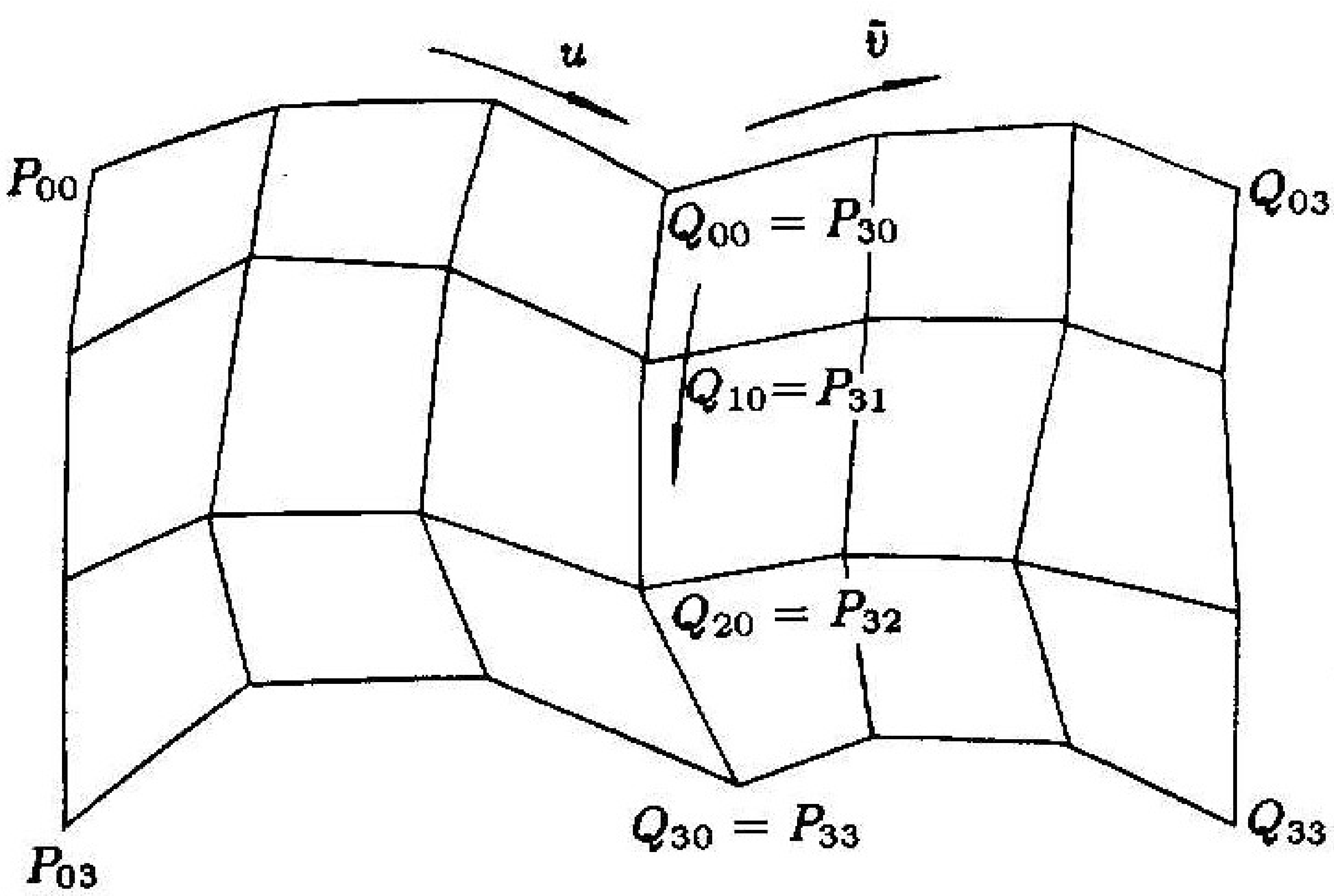


Fig. 1. Two adjacent bicubic Bezier patches

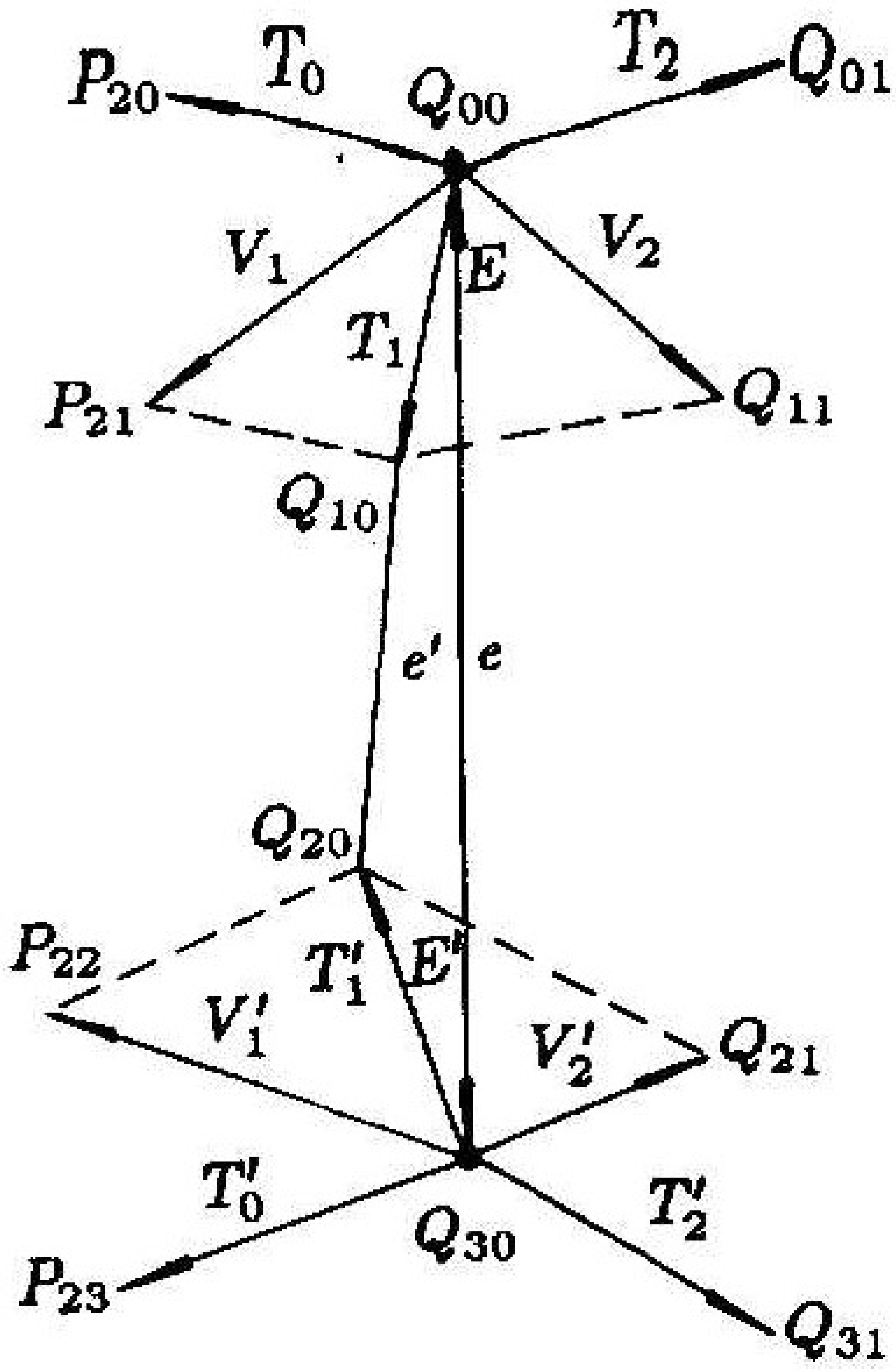


Fig. 2. Bicubic Bezier patches with G^1 continuity

Definition 1. *Let $U = \{S_i\}_0^n (S_n = S_0)$ be a set of Bezier patches or polygons. U is called an umbrella or in umbrella shape if all S_i meet at a common corner P, S_{i-1} and $S_i (i = 1, 2, \cdots, n)$ meet regularly along their boundary edges, denoted by e_i, and all e_i are different (see [19], [20]). The corner P is called the vertex of U, e_i is called the edge of U, n is called the degree of the vertex P, and the other end point of e_i denoted by Q_i is called the end point of U (Fig.3).*

Definition 2. *The umbrella $U = \{S_i\}_0^n$ is said to be with G^1 continuity if all S_{i-1} and S_i meet along e_i with G^1 continuity*

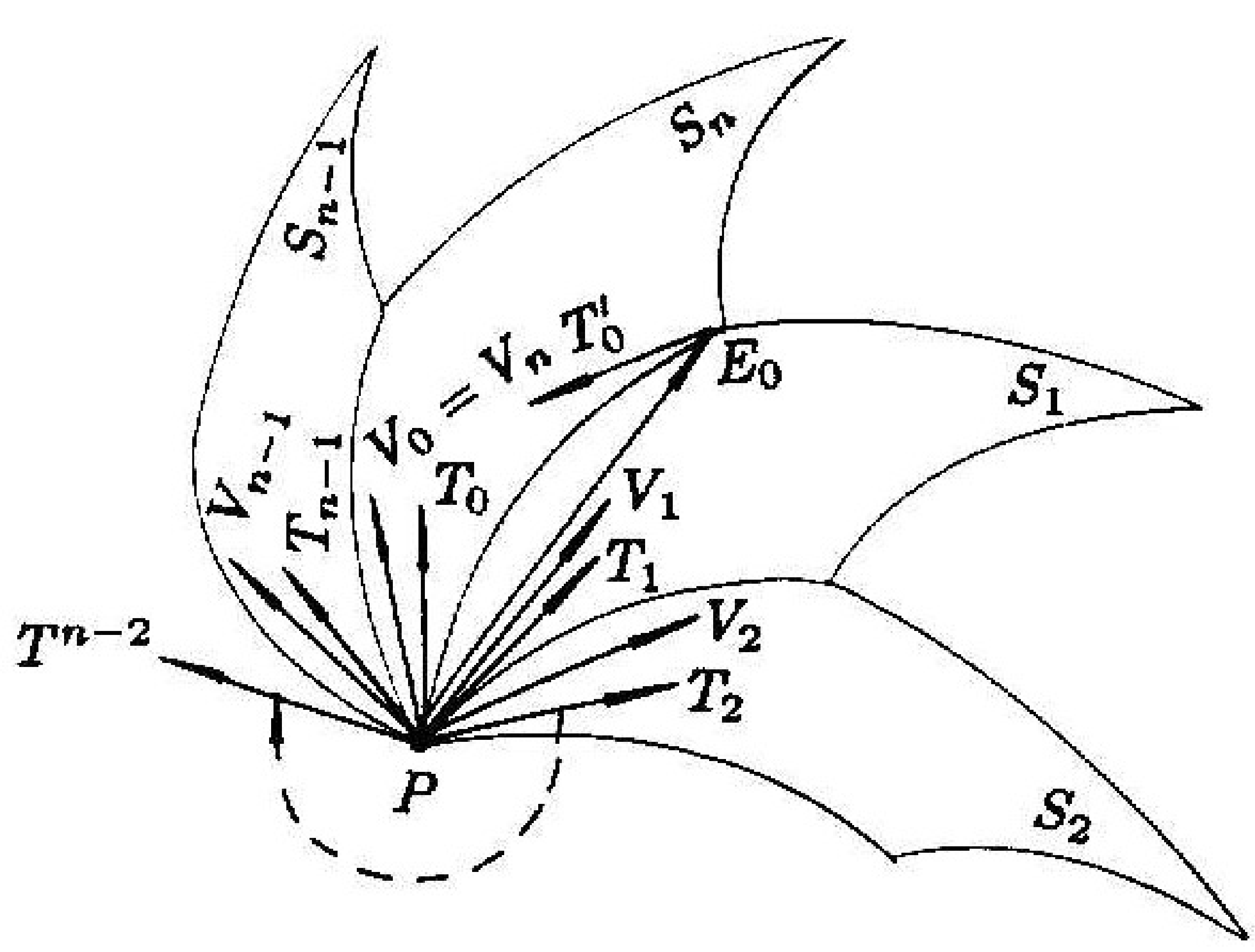


Fig.3. Bezier patches in an unmbrella shape and their tangent and twist vectors

Let $U = \{S_i\}_0^n (S_n = S_0)$ be given, α_i, β_i and β'_i be the shape parameters of edge e_i, $\{T_{i-1}, T_i\}$ and V_i be tangent vectors and twist vectors of the vertex P on S_i respectively, and $E_i = Q_i - P$ (Fig. 3). From (19)-(22) and Definition 2, we can conclude that the necessary and sufficient condition for umbrella $U = \{S_i\}_0^n$ to be with G^1 continuity is that the condition (19)-(20) of end points Q_i of U is satisfied and the following equations hold:

$$T_{i+1} = -\alpha_i T_{i-1} + \beta_i T_i, \qquad i = \overline{1, n}, \tag{23}$$

$$V_{i+1} = -\alpha_i V_i + (1 + \alpha_i - \frac{2}{3}\beta_i - \frac{1}{3}\beta'_i) T_i + \frac{2}{3}\beta_i (E_i + T'_i), \quad i = \overline{1, n}, \tag{24}$$

$$T_{n+i} = T_i, \quad i = 0, 1, \tag{25}$$

$$V_n = V_0. \tag{26}$$

$\{T_0, T_1, \cdots, T_{n-1}\}$ and $\{V_1, \cdots, V_n\}$ are called tangent vectors and twist vectors at P respectively.

2.3. Compatible Condition of the Umbrella of Vertex P

Let $l_i = /T_i/, \theta_i =< T_{i-1}, T_i >$. From (23) and (25) we have

$$\alpha_0\alpha_1 \cdots \alpha_{n-2}\alpha_{n-1} = 1, \tag{27}$$

$$l_{i+1} = \alpha_i l_{i-1} \sin(\theta_i)/\sin(\theta_{i+1}), \tag{28}$$

$$\beta_i = l_{i+1} \sin(\theta_i + \theta_{i+1})/[l_i \sin(\theta_i)]. \tag{29}$$

Obviously, (27), (28) and (29) are equivalent to (23) and (25). From (24) we have

$$\begin{aligned} V_n &= (-1)^n V_0 + \sum_{i=0}^{n-1} (-1)^{n-i+1} \alpha_{n-1} \cdots \alpha_{i+1}((1 + \alpha_i - \frac{2}{3}\beta_i - \frac{1}{3}\beta_i')T_i + \frac{2}{3}\beta_i(E_i + T_i')) \\ &= (-1)^n V_0 + W. \end{aligned} \tag{30}$$

From (26) we conclude that, when n is odd, (30) turns out to be

$$V_0 = W/2; \tag{31}$$

otherwise

$$W = \sum_{i=0}^{n-1} (-1)^{n-i+1} \alpha_{n-1} \cdots \alpha_{i+1} \left[(1 + \alpha_i - \frac{2}{3}\beta_i - \frac{1}{3}\beta_i')T_i + \frac{2}{3}(E_i + T_i')\right] = 0. \tag{32}$$

(32) is called the compatible condition of the umbrella at vertex P.

The value α_i depends on the direction in which we determine neighboring patches with G^1 geometric continuity. If we choose the opposite direction, α_i will be changed into $1/\alpha_i$. By symmetry, we have

$$\alpha_i = 1, \quad i = \overline{0, n} - 1.$$

§3. Local Smoothing Operation

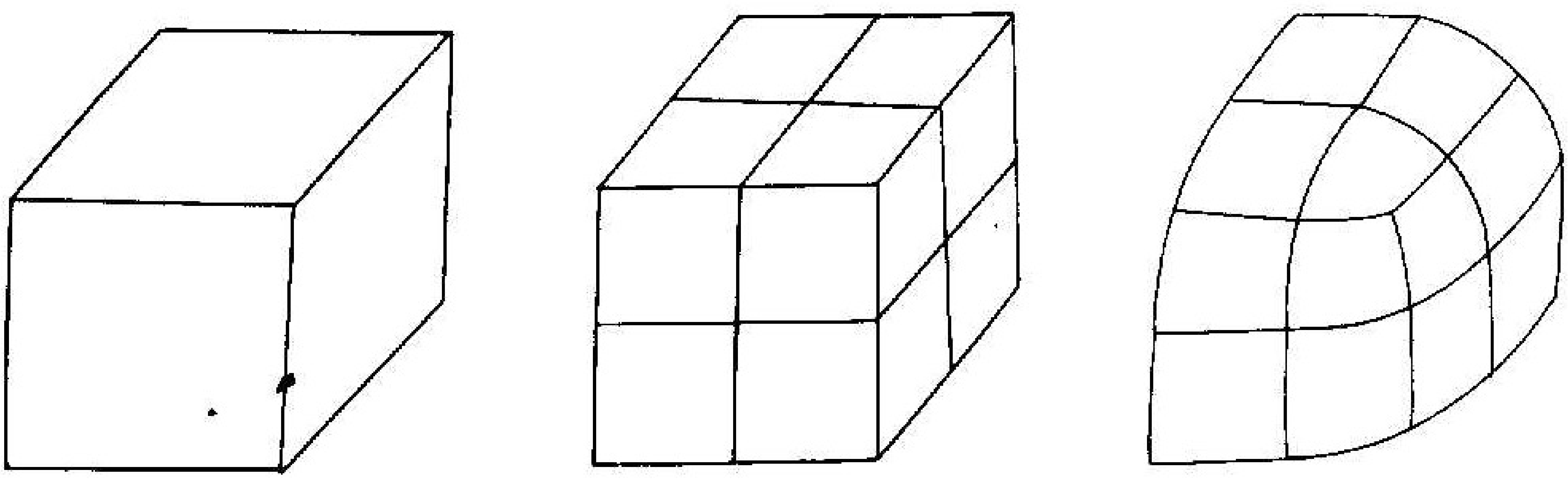
Generally, there are two kinds of local smoothing operations to modify a polyhedron: edge smoothing operation and vertex smoothing operation. The edge smoothing operation is simple and omitted here.

Definition 3. *Let V be a vertex of a given polyhedron, $U = \{X_i\}_0^n$ be an umbrella with V as its vertex. Vertex smoothing operation on V is defined as the operation of determining a surface S with G^1 continuity which is composed of a set of umbrellas $\{U_i\}_0^n$ and a set of polygons $\{Y_i\}_0^{n-1}$, where $U_1, U_2, \cdots, U_n$ are umbrellas with the end points of U_0 as their vertices and $Y_i \in X_i$. S is called a smoothing surface, and the vertex P of U_0 is called the center of the smoothing surface S. The Bezier patches in U_0 are called smoothing patches* *of vertex P, the Bezier patches in $U_i - U_0$ are called smoothing patches of edge e_i, and Y_i is called the residue polygon of X_i.*

From Definition 3, we have

$$S = \sum_{i=0}^{n} U_i + \sum_{i=0}^{n-1} Y_i. \tag{33}$$

The topological information of the smoothing surface is obtained by using Euler operations, and the geometric information is obtained from the solution of equations (23)–(26). The geometric description is given in the next section. Fig. 4 (a), (b) and (c) show the change of topology and the new object which is obtained by using vertex smoothing operation.



(a) (b) (c)

Fig.4. The change of topology ((a), (b)) and the new object obtained by using vertex smoothing operation on a cube ((c))

Generally, let umbrella $U = \{X_i\}_0^n$, where X_i are polygons. Let $\{S^i\}_0^{n-1}$ be the smoothing patches of vertex P, $\{S_0^i, S_1^i\}$ be the smoothing patches of edge e_i, Y_i be the residue polygon of X_i, P_i be a point in X_i, and Q_i be a point on a certain curve with P_{i-1} and P_i as end points, as shown in Fig.5. From Theorem 2 and Definition 3, we have

Theorem 3. *Assume the following conditions:*

(1) *S_0^i is tangent to Y_{i-1}, S_1^i is tangent to Y_i, S_0^i and S_1^i meet with G^1 continuity along their common boundary.*

(2) *The tangent vectors and twist vectors of the vertex $P, P_i, Q_i (i = \overline{0, n} - 1)$ satisfy the corresponding condition of* (23) *and* (24).

(3) *The tangent vectors and twist vectors of P_i are all on the plane of X_i.*

(4) *The tangent vectors and twist vectors of Q_i satisfy the compatibility condition* (32).

(5) *When n is even, the tangent vectors and twist vectors of P satisfy the compatible equation* (31)*; otherwise they satisfy condition* (30).

Then the smoothing surface

$$S = \sum_{i=0}^{n-1} (S^i + S_0^i + S_1^i + Y_i) \tag{34}$$

is of G^1 continuity.

Obviously, the smoothing operation is local, and the new object is obtained by changing the neighbor of P and keeping the rest of the object unchanged. Examples of the change of topology of a polyhedron and the new object generated by using vertex smoothing operation on two different vertices of the polyhedron are shown in Fig. 6 (a), (b) and (c) respectively.

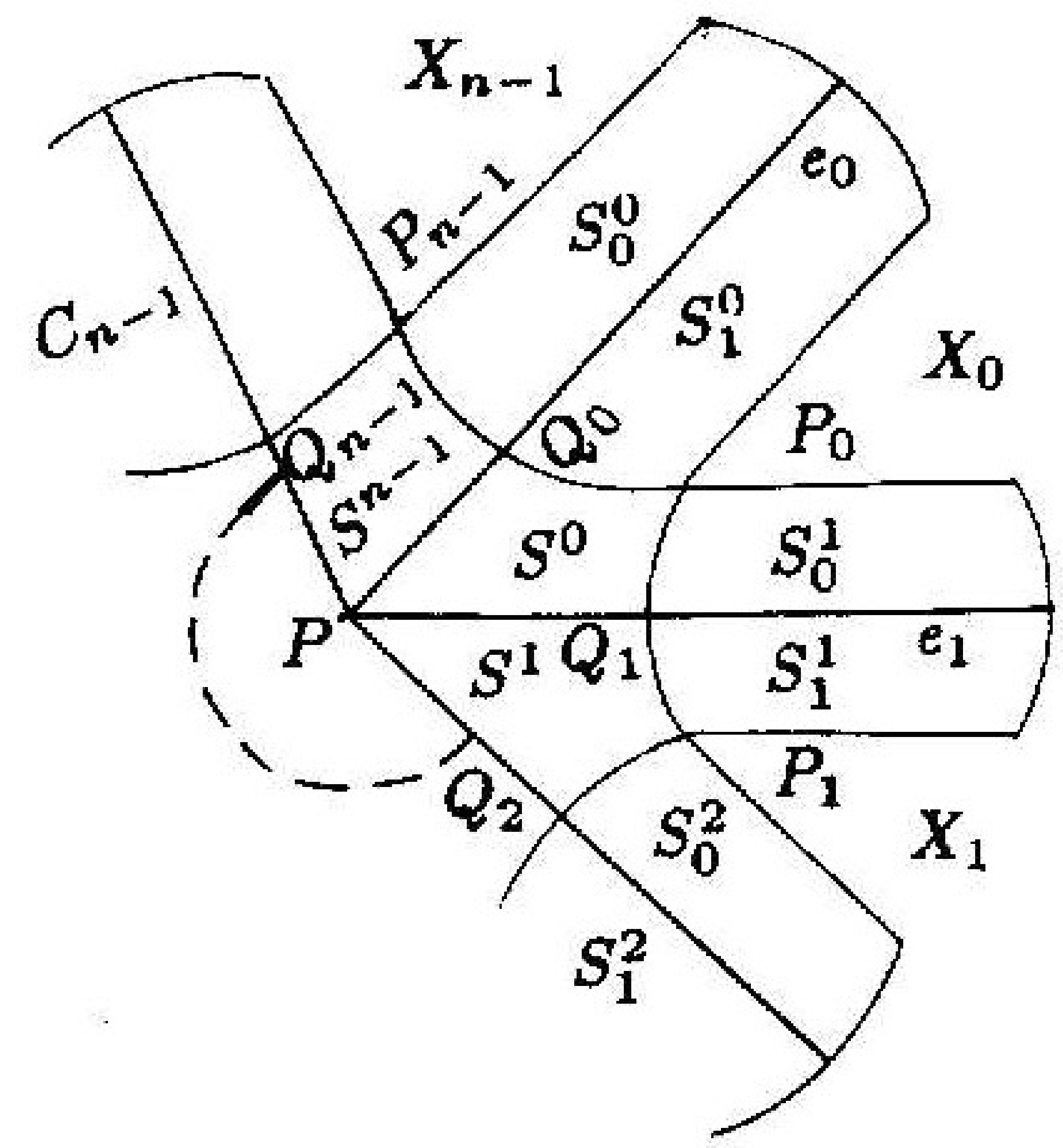


Fig.5. The umbrellas and smoothing patches of vertex smoothing operation

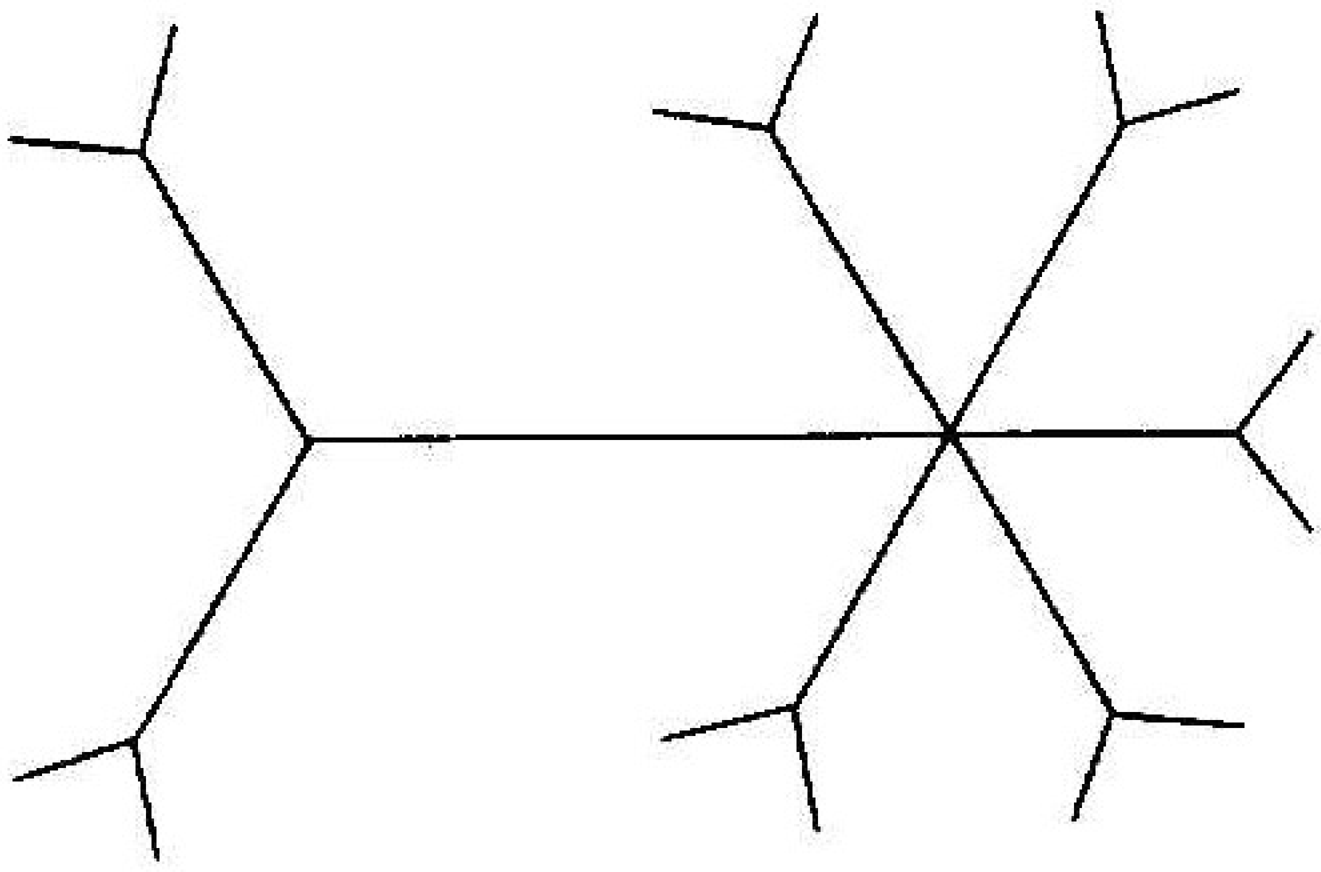


Fig.6. (a) The origin topology of a polyhedron

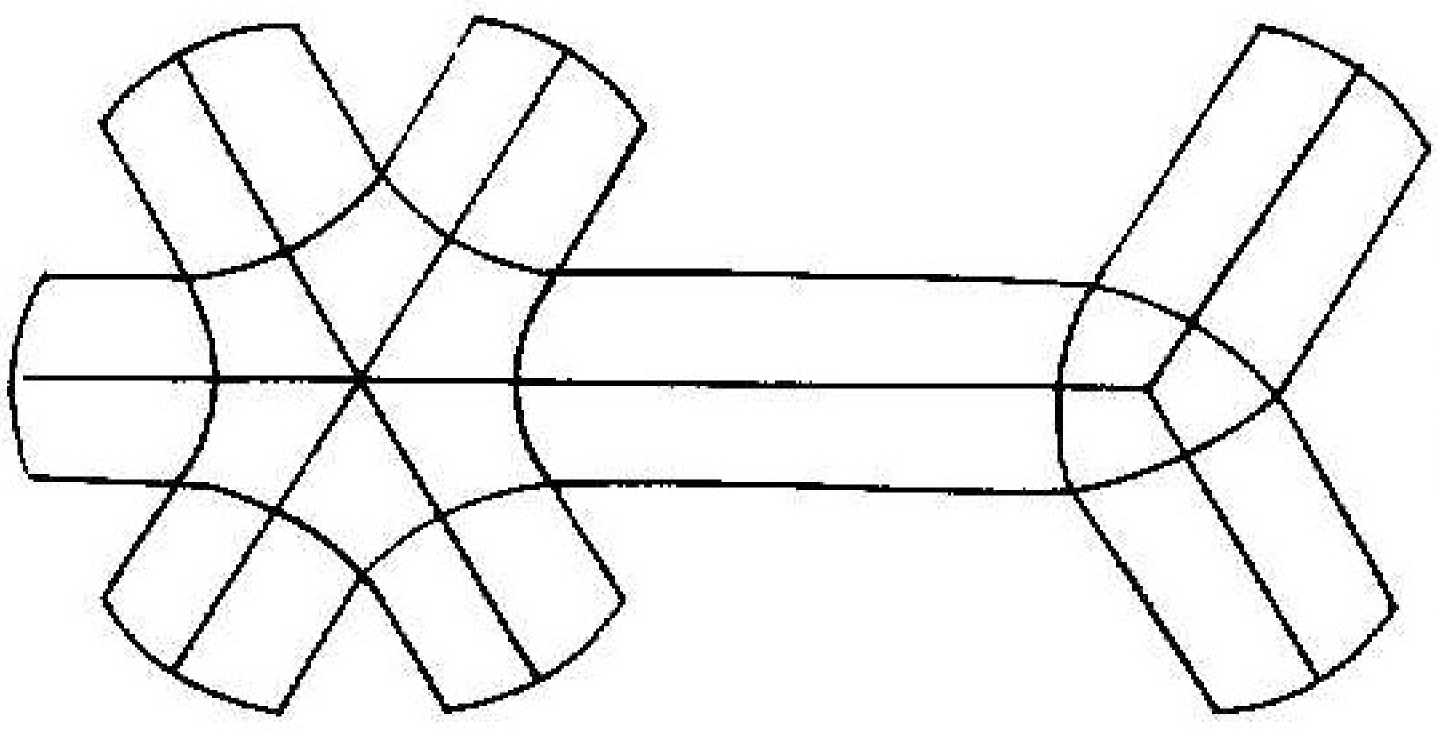


Fig.6. (b) The topology obtained by using vertex smoothing operations to Fig.6 (a)

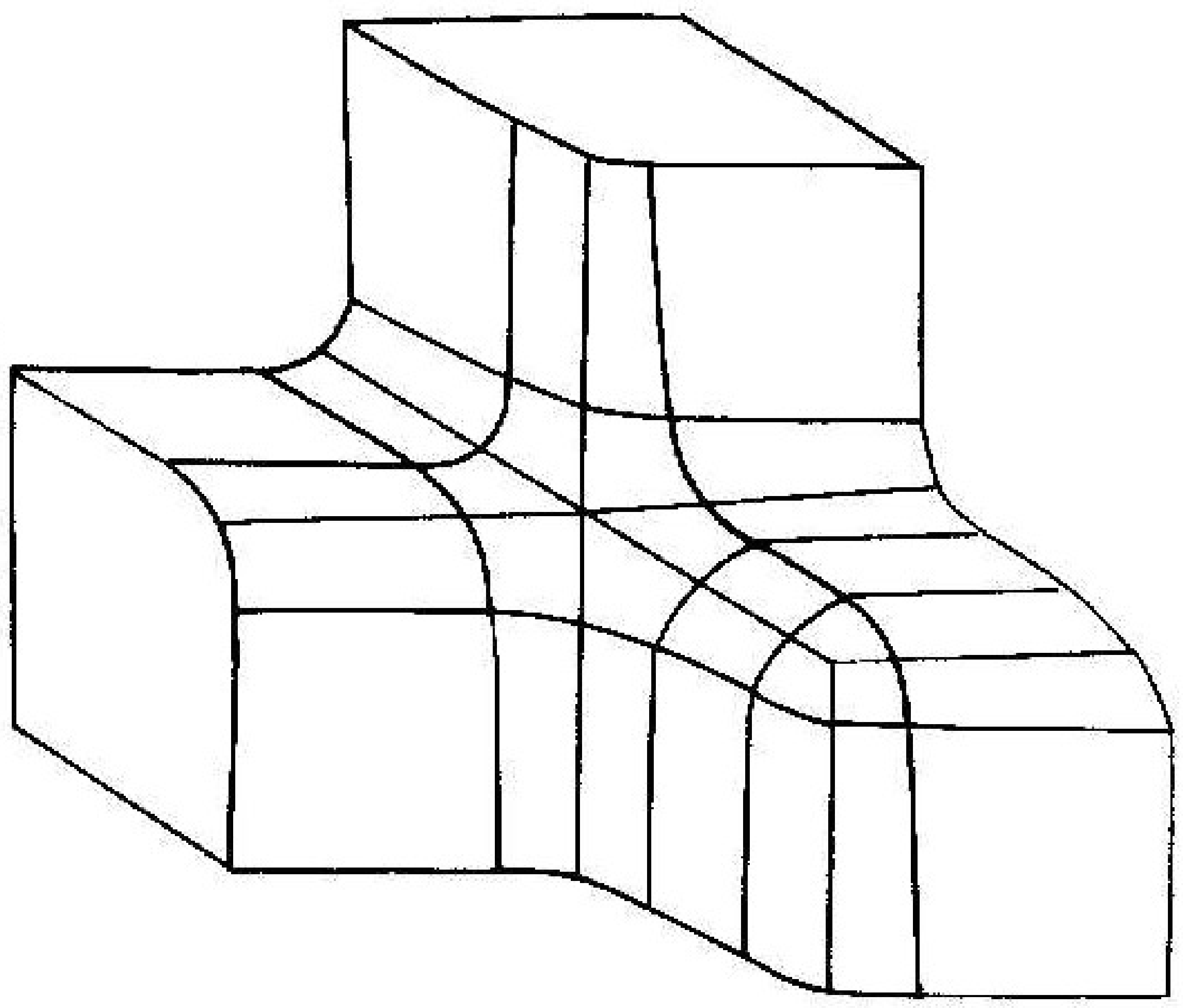


Fig.6. (c) The object corresponding to the topology of Fig.6 (b)

§4. The Geometric Description of a Smoothing Surface

Let n be the degree of center P. The main points of the geometric description are as follows:

4.1. Determination of $P_i(i = \overline{0, n-1})$ and its Tangent and Twist Vectors

A simple way to determine P_i and its tangent and twist vectors are shown in Fig. 7 (a) and (b), where d_1 is a shape parameter.

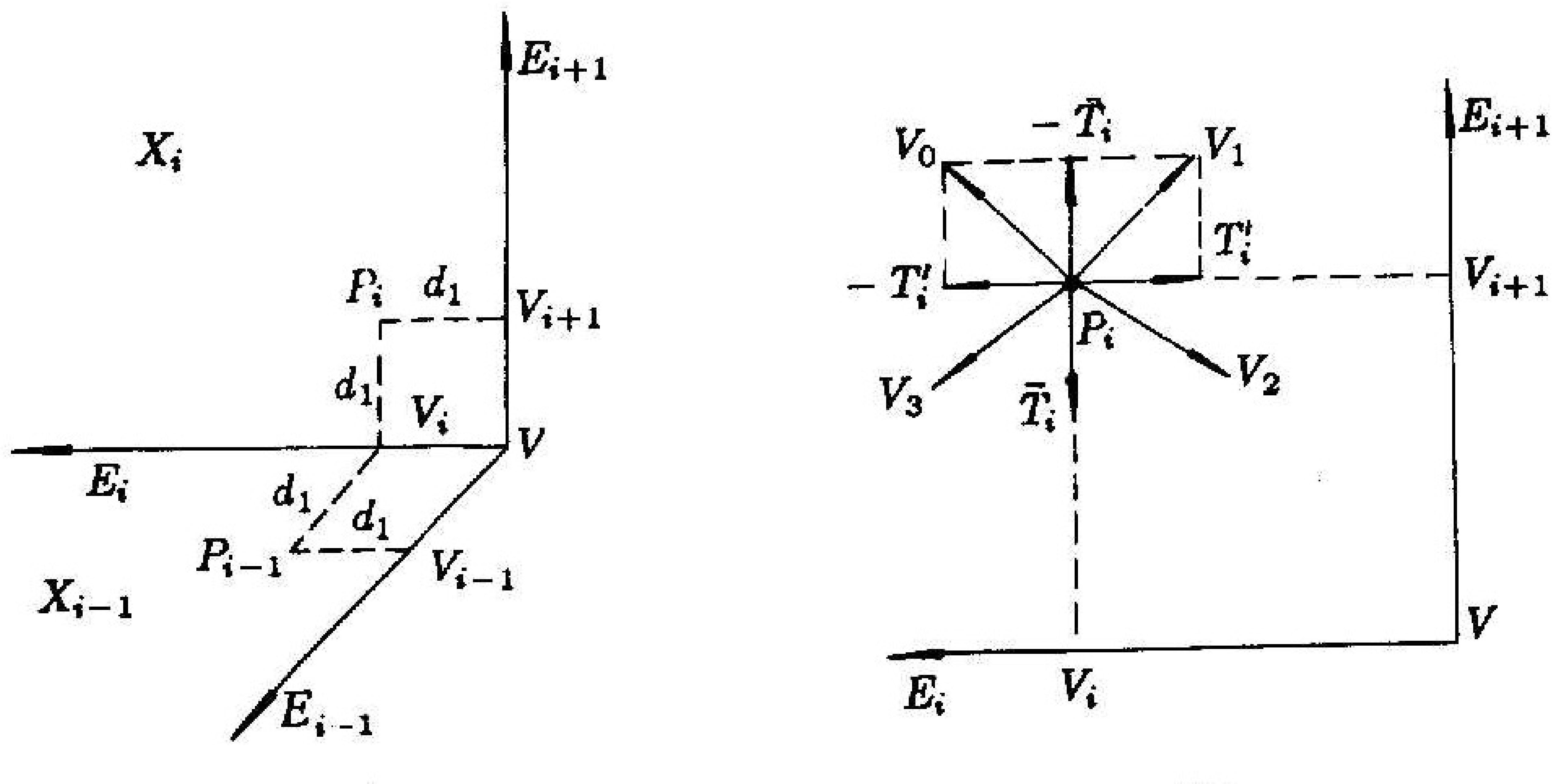


Fig.7. The geometric description of P_i (a) and its tangent and twist vectors (b).

4.2. Determination of $Q_i(i=\overline{0,n-1})$ and Its Tangent and Twist Vectors

Take the middle point of the quadric Bezier curve with control points $\{P_{i-1}, V_i, P_i\}$ as Q_i, where V_i is shown in Fig.7 (a). From the compatibility condition (32) satisfied by the tangent vectors and twist vectors of Q_i we have (Fig. 8)

$$T_1^i = (-\beta^i T_0^i + 2d_2(P_i + \bar{T}_i) - (P_{i-1} - T_{i-1}'))/(d_2(\bar{\beta}^i - 10))$$

where $T_0^i//(V_i - V)$, d_2 is a shape parameter and $d_2 \neq 0$, and $\alpha = 1, \beta = d_2, \beta' = \bar{\beta}^i$ are the shape parameters of the edge with center P of the smoothing surface and Q_i as end points.

4.3. Determination of Center P and Its Tangent and Twist Vectors

Set

$$P = (1-d_4)V + d_4 \sum_{i=0}^{n-1} P_i/n, \tag{36}$$

$$N = \sum_{i=0}^{n-1}(P_{i+1} - P) \times (P_i - P) \Big/ \Big| \sum_{i=0}^{n-1}(P_{i+1} - P) \times (P_i - P)\Big|, \tag{37}$$

$$T_1 = d_3(\sin(2\pi/n)(T_0 \times N) + \cos(2\pi/n)T_0), \tag{38}$$

$$T_{2j} = \sin(4\pi j/n)(T_0 \times N) + \cos(4\pi j/n)T_0, \tag{39}$$

$$T_{2j+1} = \sin(4\pi j/n)(T_1 \times N) + \cos(4\pi j/n)T_1, \quad j = 1, 2, \cdots, (n/2) - 1 \tag{40}$$

where d_3 is a shape parameter and $d_3 = 1$ if n is odd otherwise $d_3 \neq 1$; T_0 is a certain vector.

Obviously, $T_i(i = \overline{0, n-1})$ determined by (38)-(40) satisfy the conditions (23) and (25). From (28) and (29) we have

$$\beta_i = \bar{\beta}^i = \begin{cases} 2d_3\cos(2\pi/n), & \text{if } n \text{ is even,} \\ 2d_3^{-1}\cos(2\pi/n), & \text{otherwise.} \end{cases} \tag{41}$$

If n is even, then we obtain V_0 from (31); otherwise set $V_0 = T_0 + T_{n-1}$ and notice that

$$\sum_{j=0}^{n/2-1} T_{2j} = \sum_{j=0}^{n/2-1} T_{2j+1} = 0. \tag{42}$$

From compatibility condition (32) (Fig.9) we reset

$$P = 2\sum_{j=0}^{n/2-1} [d_3^2 - (Q_{2j} + T_2^{2j}) - (Q_{2j+1} + T_2^{2j+1})]/(d_3^2 - 1) \tag{43}$$

where $T_2^i (i = \overline{0, n} - 1)$ are shown in Fig. 8.

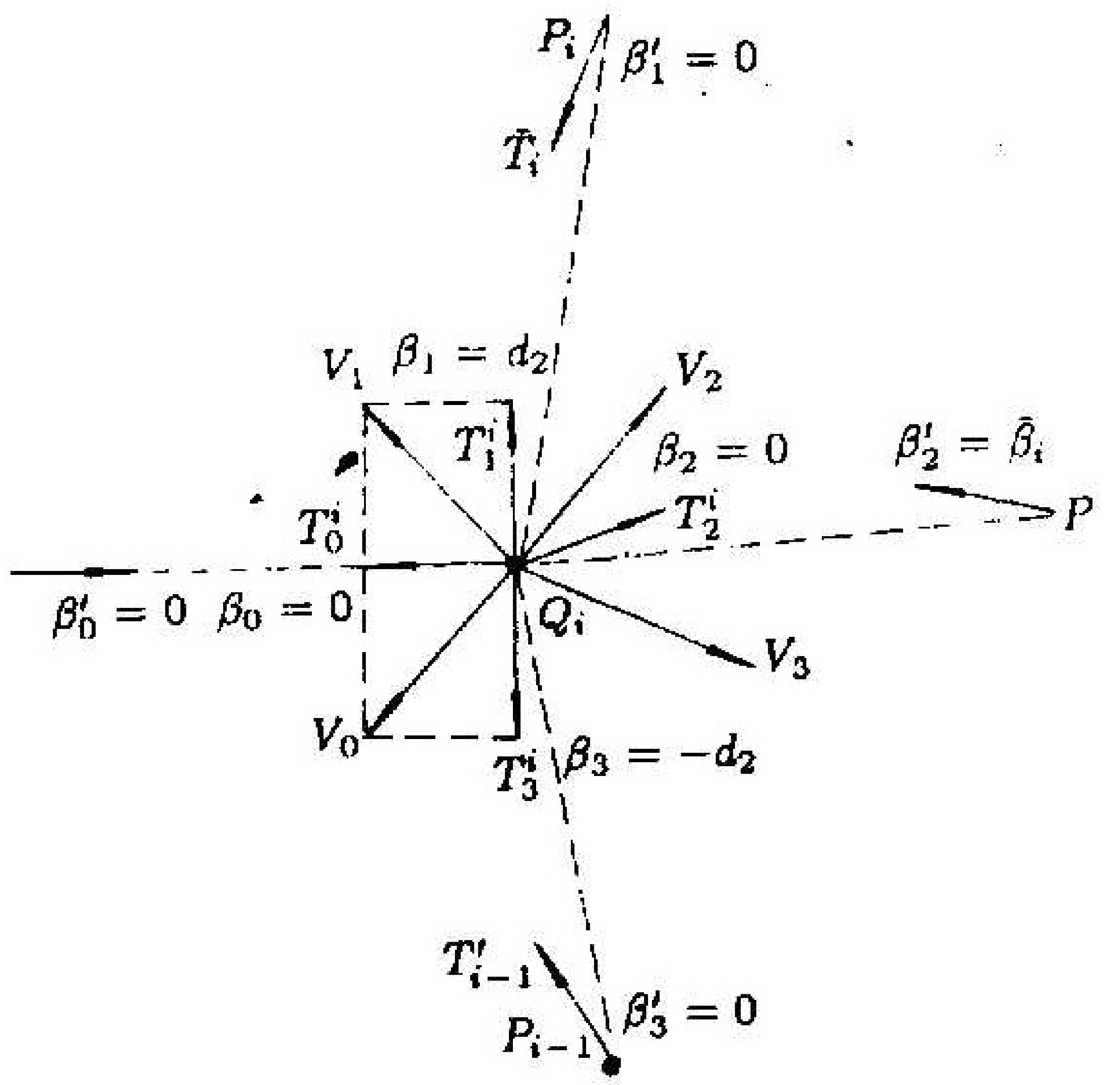


Fig.8. The geometric description of the tangent and twist vectors of Q_i

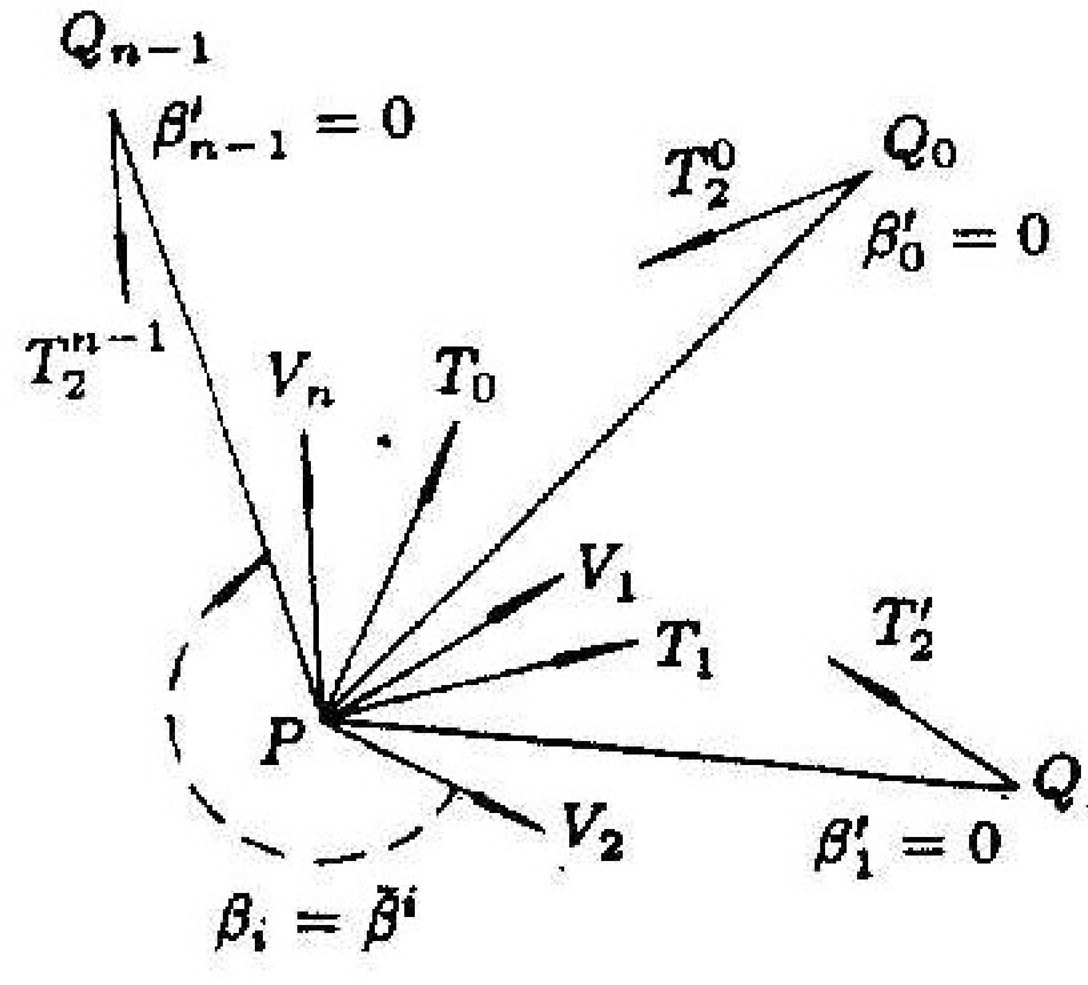


Fig.9. The geometric description of the tangent and twist vectors of center P

§5. Implementation

The algorithm for this method has been embedded in our newly developed solid modeling system MESSAGE which offers facilities of Euler operations, set operations and local shape modification for modeling objects bounded by planar, quadric surfaces and sculptured surfaces including bicubic rational B-spline surfaces and runs on Universe 68000. This new algorithm adds vertex and edge smoothing operations to MESSAGE such that the full use of these operations make this solid modeling system more user-friendly, easy to use for interactive design both in academic research and paractical use. The objects generated by using our smoothing operations from polyhedra in MESSAGE are still efficient in MESSAGE. Therefore, further operations such as set operations can be applied to the new objects. The shape parameters (d1, d2, d3 and d4) of the smoothing operations provide the user with more freedom and can be easily used to change the shape of the objects locally. The smoothing patches are described in an exact analytic form such that the algorithm is well designed.

The implementation and performance of the algorithm for this new method show that it is efficient, flexible and easy to manipulate.

Some examples of the results by using vertex smoothing operation to some solid objects in MESSAGE, including a car body and a telephone base, are shown in Fig.10 – Fig.15. The response time for generating them is quick and all within one minute. Thus, it can be used for interactive design.

§6. Conclusions

In this paper, a new concept and definition of G^1 smoothing surfaces used to smooth and modify polyhedra locally are given. Based on the common solution to geometric continuity of the first order, the necessary and sufficient conditions of G^1 continuity of an umbrella are obtained as the foundation of the new method for generating G^1 smoothing surfaces which are composed by planar and Bezier patches and expressed in explicit mathematical form such that the G^1 smoothing operations can be incorporated into the solid modeling system in common use. The algorithm is coded in C language and runs on Universe 68000. The computation of the program is simple, straightforward and reliable.

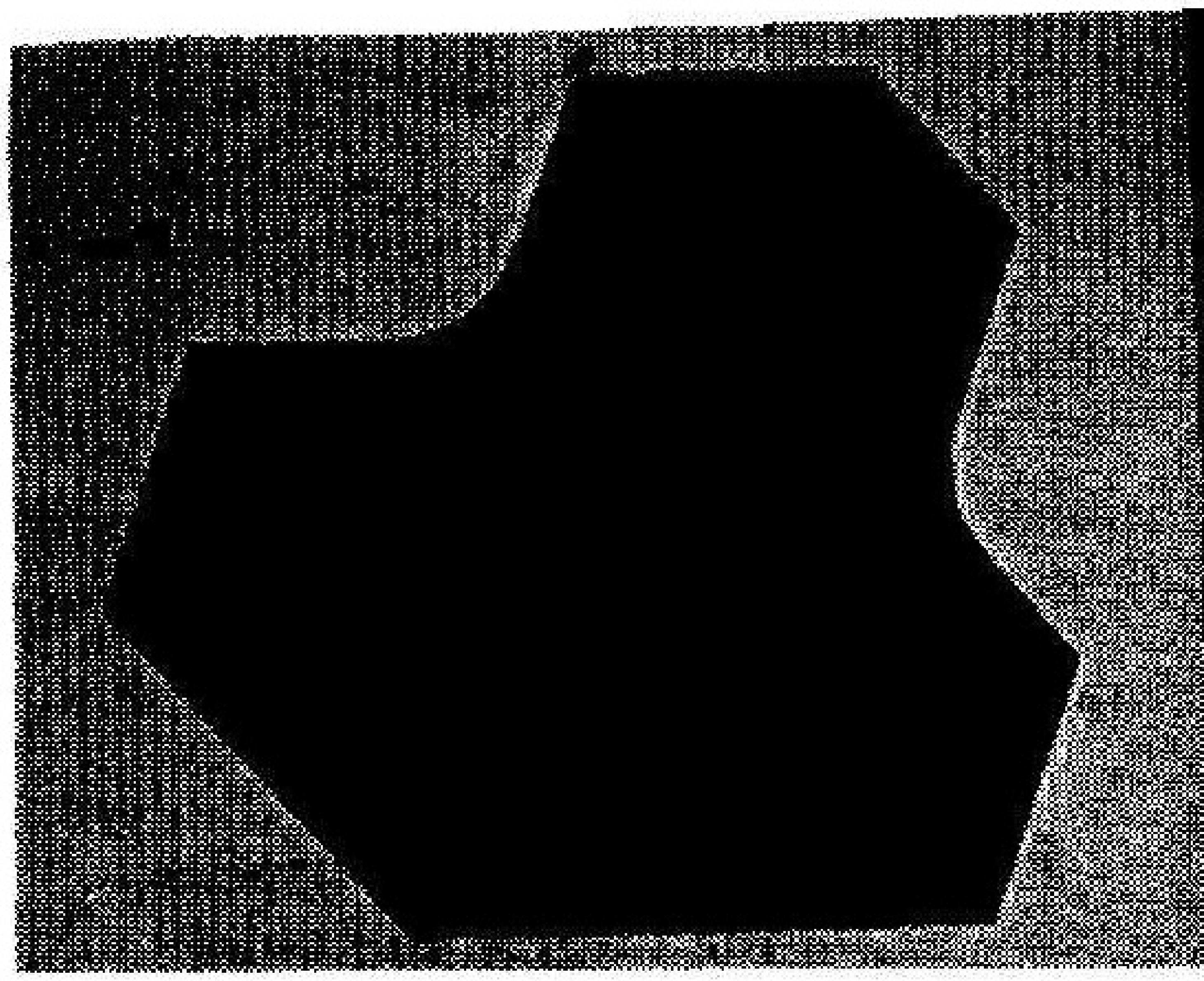


Fig. 10. Shaded picture of smoothing pentagon topology

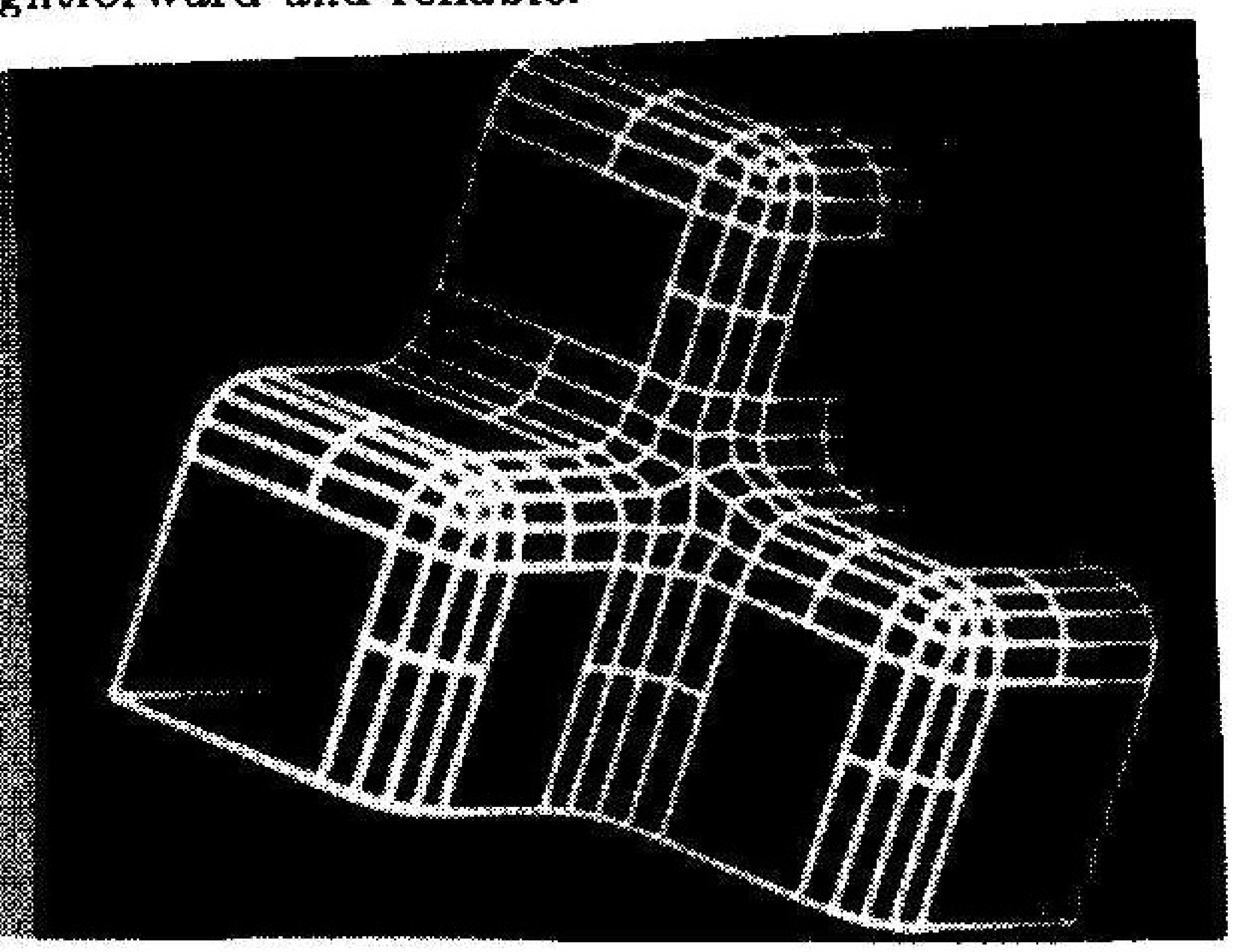


Fig. 11. Line drawing of smoothing hexagon topology

This algorithm can also be used as a powerful method represent a wide range of 3-D objects obtained from polyhedra by using smoothing surfaces to modify and change the shape. Adding these smoothing operations will expand the function of the existing modeling systems greatly and meet most needs of the interactive design and applications at the least cost.

The geometric continuity among the patches which form a closed surface is still a main difficulty in generating more general objects. Further study of this problem and the method to creat more general objects are still considerable problems in computer graphics and CAD/CAM fields. We have solved these problems in a special but very useful case. Currently, we are investigating the possibility of extending the concept and method to smooth and modify more general objects.

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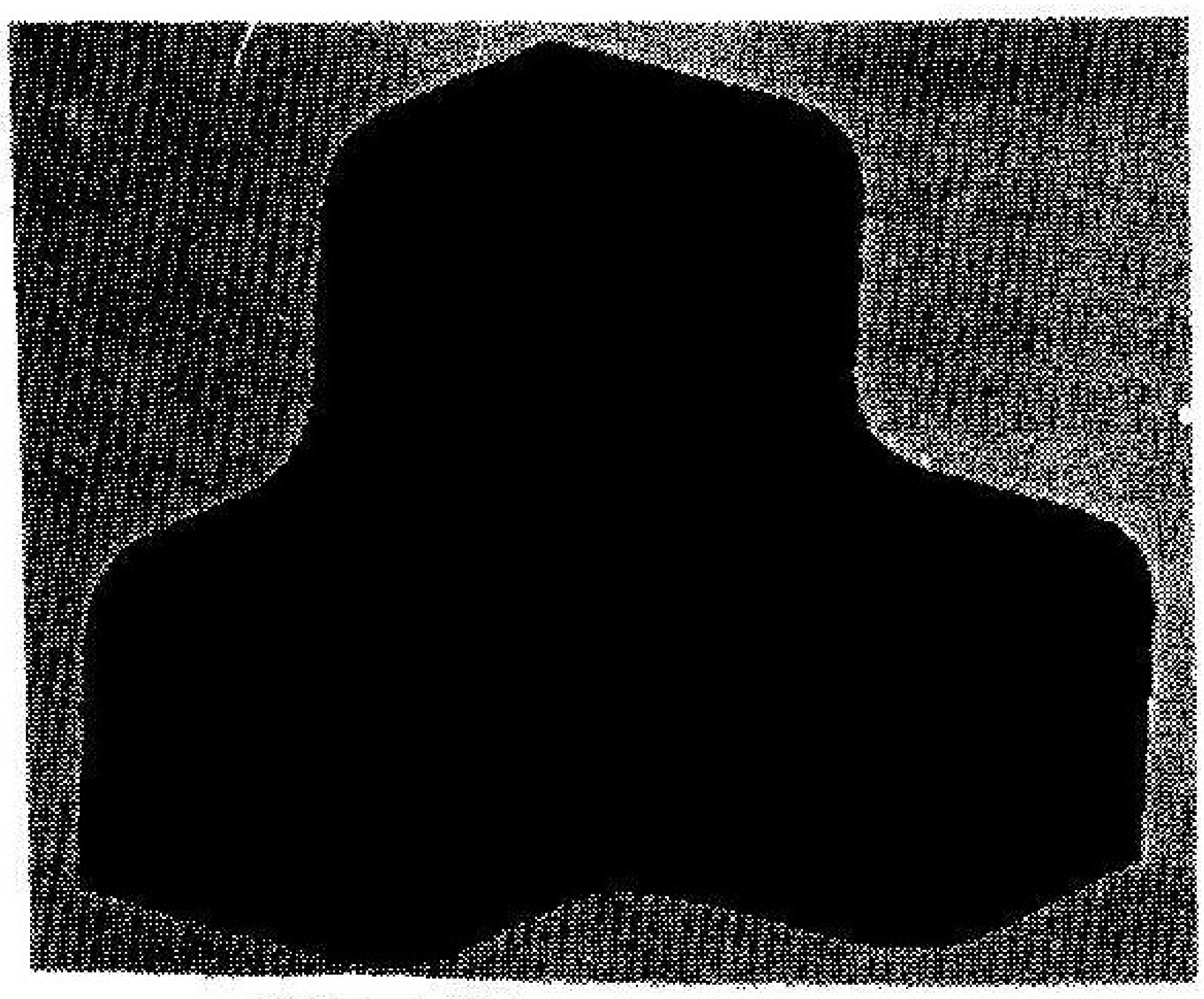

Fig. 12. Shaded picture of smoothing hexagon topology

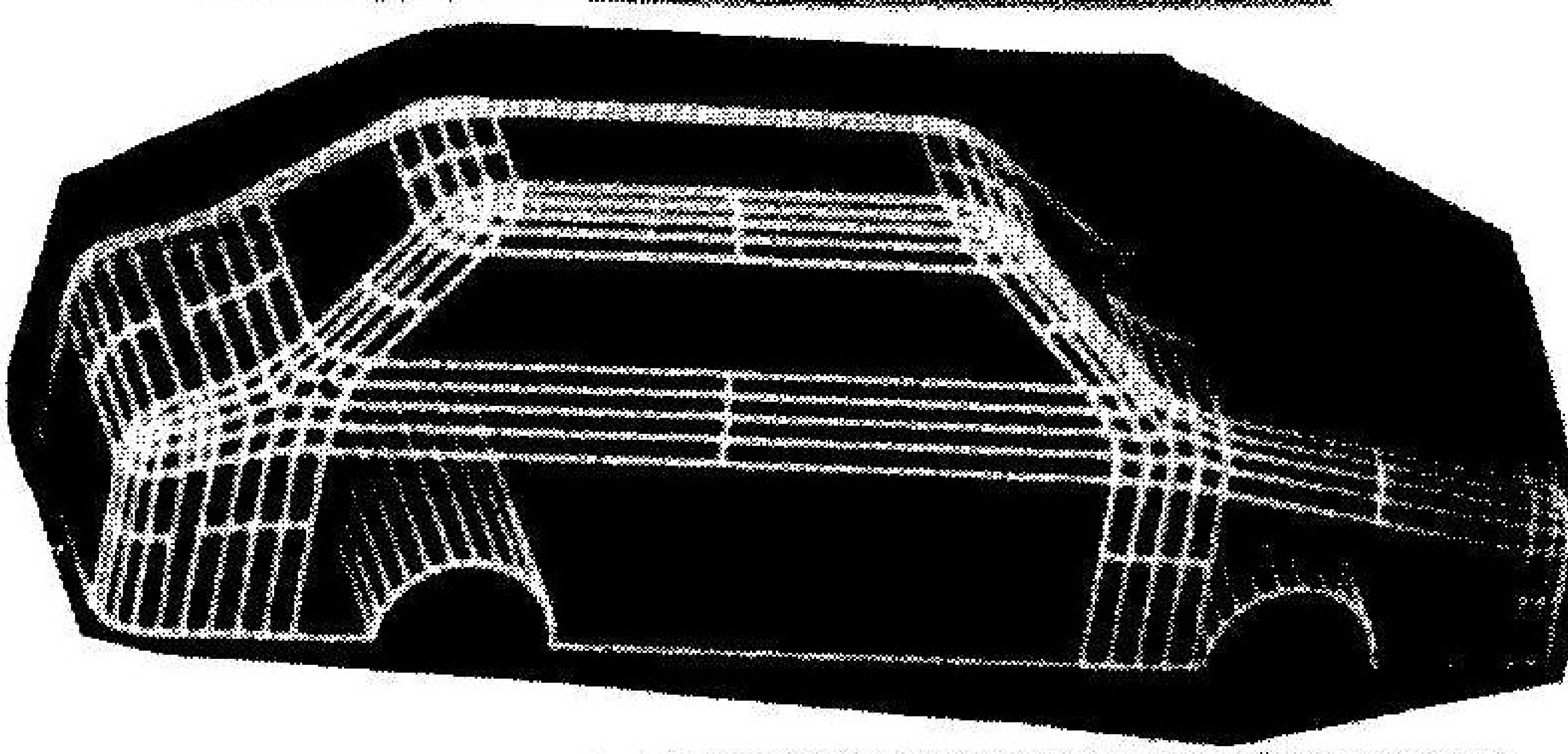

Fig. 13. Line drawing of a car body

Fig. 14. shaded picture of a car body

Fig. 15. Shaded picture of a telephon base

References

[1] A.A.G. Requicha and H.B. Voelcker, Solid Modeling: Current Status and Reserach Direction, *IEEE CG & A,* Vol. 3, No.10.

[2] A.R. Forrest, Freeform Surfaces and Solid Modelling, SIGGRAPH Free Form Curves and Surfaces, *Tutorial Notes.*

[3] E. Catmull and J. Clark, Recursively generated B-spline surface on arbitrary topological meshes, *CAD,* **10** : 6 (1978), 350–355.

[4] D. Doo, A subdivision algorithm for smoothing down irregular shaped polyhedra, Proc. Conf. Interactive Technique in CAD, 157–165, IEEE Computer Society 78CH1289-8C, 1978.

[5] Lu Wei, Jin Tong-guang and Liang You-dong, Surface generated by cutting and grinding polyhedron with arbitrary topological meshes: C–G surface, Proceedings of CADDM 87, Beijing, 1987.

[6] H. Chiyokura and F. Kimura, Design of solids with free-from surfaces, *Computer Graphics,* , **17** : 3 (1983), 289–298.

[7] M.A. Sabin, Non-rectangular Surface Patches Suitable for Inclusion in a B-Spline Surface, Eurographics 83, P.J.W ten Hagen (editor), Elsevier Science Publishers, North-Holland.

[8] J.R. Rossignac and A.A.G. Reguicha, Constant-radius blending in solid modeling, *Computer in Mechanical Engineering,* **3** : 1 (1984), 65–73.

[9] A.E. Middleditch and K.H. Sears, Blending surfaces for set theoretic volume modeling system, Proc. of SIGGRAPH 85, *Computer Graphics,* **19** : 3, 1985.

[10] B.A. Barsky and J.C. Betty, Local control of bias and tension, Proc. of SIGGRAPH 83, *Computer Graphics,* Vol.17, No.3, 1983.

[11] J.A. Gregory and J. Hahn, Geometric continuity and convex combination patches, *CAGD,* Vol.4, No.3, 1987.

[12] J.J. Van Wijk, Bicubic patches for approximating non-rectangular control-point meshes, *CAGD,* Vol.3, No.1, 1986.

[13] A.D. DeRose, Geometric Continuity: A Parametrization independent Measure of Continuity for Computer Aided Geometric Design, Technical Report No. 86-09-04, August, 1985.

[14] G. Farin, A construction for the visual C^1 continuity of polynomial surface patches, Computer Graphics and Image Processing 20, 272–282.

[15] Liang You-dong, Geometric continuity between two curves and two surfaces, National University Conference of Computational Mathematics, 1987.

[16] Ye Xiu-zi and Liang You-dong, Geometric continuity between surfaces and its applications, to appear.

[17] Liang You-dong and Ye Xiu-zi, Geometric continuity between curves and its applications, to appear.

[18] P.A. Firby and C.T. Gardener, Surface Topology.

[19] Liang You-dong and Yu Yi-yue, Mathematical principle of 3-D geometry modeling, *Applied Mathematics Journal of Chinese Universities,* **1** : 1 (1986), .