J. At. Mol. Sci.
doi: 10.4208/jams.070514.091614a

Vol. **5**, No. 4, pp. 331-337
November 2014

Decoherence induced by the environment for dimerized anisotropic XY spin chain

Zhong-Jie Wang*, Xu Fang, and Ren-Gui Zhu

College of Physics and electronic information, Anhui Normal University, Wuhu, 241000, China

Received 5 July 2014; Accepted (in revised version) 16 September 2014
Published Online 29 October 2014

Abstract. The Loschmidt echo of single qubit coupling to the environment for dimerized anisotropic XY spin chain in transverse field have been analyzed. We have obtained analytical expression for the Loschmidt echo. By numerically computing the Loschmidt echo, we find that the decay of coherence will be enhanced with the increasing of the staggered parameter.

PACS: 75.10.Jm

Key words: dimerized anisotropic XY spin chain, decoherence, Loschmidt echo

1 Introduction

Decoherence is considered to be the main obstacle in quantum information processing [1]. The uncontrolled interactions of the system with the environment will result in a suppession of cohrence or loss of entanglement within system, namely, so called local decoherence or nonlocal decoherence [2, 3]. In study of decoherence, the environment is usually modelled as a collection of harmonic oscillators or spin-1/2 particles. In recent years, the study of spin-bath environment has been attracted special attentions [4-7]. For instance, it is found that quantum phase transitions in the spin-bath environment enhance the decay of coherence of single qubit interacting homogeneously with .all spins in the environment [8]. This phenomenon has also been observed in the experiment [9]. The decay of coherence is characterized by the Loschmidt echo (LE) which first appears in the context of quantum chaos. The properties of the Loschmidt echo have been extensively investgated in various spin-bath environments such as XY spin chain [8, 10], XY spin chain with three-site interaction [11], many-body system with two XY spin chain [12], time-dependent spin chain [13], etc. These reseaches reveal some connections between

*Corresponding author. *Email address:* `wuliwzj@mail.ahnu.edu.cn` (Z. J. Wang).

the enhanced decoherence of the Loschmidt echo and quantum phase transitions. In the other hand, one has studied the properties of entanglement of two quabits coupling to the spin-bath environment in vicinity of quantum phase transition points [14].

In this paper, we investigate decoherence induced by the environment for dimerized anisotropic XY spin chain. The statistical properties of the dimerized XY spin chain have been extensively studied such as spin-Peierls transition [15, 16], correlation function [17]. Here we compute the Loschmidt echo of single qubit coupling to the dimerized anisotropic XY spin chain and analyze the properties of LE. We find that the decay of coherence will be enhanced with the increasing of the staggered parameter.

2 The Model

Let us study the decoherence of a spin-1/2 particle coupling to the environment formed by a chain of N spin-1/2 particles. We consider the qubit interacts equally all the spins in the chain and neglect the self-Hamiltonian of the system. The Hamiltonian of the total system is given by

$$H = H_B + H_{SB} \tag{1a}$$

$$H_B = -\sum_{k=1}^{N}\left\{\frac{1}{2}[1+(-1)^k\delta](S_k^x S_{k+1}^x - S_k^y S_{k+1}^y) + \lambda S_k^z\right\} \tag{1b}$$

$$H_{SB} = -g|1><1| \oplus \sum_{k=1}^{N} S_k^z \tag{1c}$$

where periodic boubdary conditions are imposed, i.e. $S_{k+N}^a = S_k^a(\alpha = x, y, z)$, δ is referred to as staggered parameter, S_k^a are the Pauli operators for the kth site of the chain, λ and g are coupling parameters, two eigenstates of Pauli operator. S^z for the qubit are represented by $|1>$ and $|0>$). It is noted that the envirinment (to see Eq. (1b)) is described by the dimerized anisotropic XY spin chain in transverse magnetic field. If the parameter $\delta = 0$, then the envirinment will be degenerated to the anisotropic XY spin chain in which quantum phase transition was in detail investiged in ref. [18]. In the following study, we set $0<\delta<1$. As noted later, the decoherence of the qubit is dependent of the evolution of the environment with different effective Hamitonian

$$H^{(a)} = H_B - ag|1><1| \otimes \sum_{k=1}^{N} S_k^z \tag{2}$$

where a = 0, 1. In the following, we diagonalize the Hamitonian $H^{(a)}$. Making Jordan-Wigner transformation

$$S_k^x = \exp\{i\pi \sum_{j=1}^{k-1} a_j^+ a_j\}(a_k - a_k^+) \tag{3a}$$

$$S_k^y = i\exp\{i\pi \sum_{j=1}^{k-1} a_j^+ a_j\}(a_k - a_k^+) \tag{3b}$$

$$S_k^z = 2a_k^+ a_k - 1 \tag{3c}$$

and followed by Fourier transformation

$$a_k = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \exp\left\{ i\frac{2\pi jk}{N} f_j \right\} \tag{4a}$$

$$a_k^+ = \frac{1}{\sqrt{N}} \sum_{j=1}^{N} \exp\left\{ -i\frac{2\pi jk}{N} f_j^+ \right\} \tag{4b}$$

we can obtain the transformed Hamitonian $H^{(a)}$ as

$$\begin{aligned} H^{(a)} = & -i2 \sum_{k=1}^{N/2} \left\{ \sin(\frac{2\pi k}{N})(f_k^+ f_{-k}^+ - f_{-k} f_k) \right\} \\ & -2\delta \sum_{k=1}^{N/2} \left\{ \cos(\frac{2\pi k}{N})(f_{N/2-k}^+ f_k^+ + f_k f_{N/2-k}) \right\} \\ & +2\delta \sum_{k=1}^{N/2} \left\{ \cos(\frac{2\pi k}{N})(f_{-k}^+ f_{N/2+k}^+ + f_{N/2+k} f_{-k}) \right\} \\ & -2\lambda^{(a)} \sum_{k=1}^{N/2} \left\{ f_k^+ f_k + f_{N/2+k}^+ f_{N/2+k} \right\} \end{aligned} \tag{5}$$

where $\lambda^{(a)} = \lambda + ag$, f_j^+ and f_j are fermion operators, the constant term is omitted. Next we make Bogoliubov tranformation

$$f_k = \alpha_k \cos(\theta_k) + i\beta_k \sin(\theta_k) \tag{6a}$$

$$f_{N/2+k} = \beta_k \cos(\theta_k) + i\alpha_k \sin(\theta_k) \tag{6b}$$

where $\tan(2\theta_k) = -2\delta\cot(2\pi k/N)$, α_k and β_k are fermion operators, we can obtain the transformed Hamitonian $H^{(a)}$ with the form of

$$\begin{aligned} H^{(a)} = & -i2 \sum_{k=1}^{N/2} \left\{ \sin(\frac{2\pi k}{N})[(\alpha_k^+ \alpha_{-k}^+ - \alpha_{-k}\alpha_k)\cos^2(\theta_k) + (\beta_k^+ \beta_{-k}^+ - \beta_{-k}\beta_k)\sin^2(\theta_k)] \right\} \\ & +i4\delta \sum_{k=1}^{N/2} \left\{ \cos(\frac{2\pi k}{N})[(\alpha_k^+ \alpha_{-k}^+ - \alpha_{-k}\alpha_k) + \beta_{-k}^+ \beta_k^+ - \beta_k \beta_{-k})\cos(\theta_k)\sin(\theta_k)] \right\} \\ & -2\lambda^{(a)} \sum_{k=1}^{N/2} \left\{ \alpha_k^+ \alpha_k + \beta_k^+ \beta_k \right\} \end{aligned} \tag{7}$$

In order to eliminate off-diagonal terms for α_k and β_k in Eq. (7), we further make Bogoliubov tranformation with the form of

$$\alpha_k = c_k^{(a)}\cos(\varphi_k^{(a)}) + ic_{-k}^{(a)}\sin(\varphi_k^{(a)}) \tag{8a}$$

$$\beta_k = d_k^{(a)}\cos(\phi_k^{(a)}) + id_{-k}^{(a)}\sin(\phi_k^{(a)}) \tag{8b}$$

where

$$\tan(2\varphi_k^{(a)}) = \frac{2}{\lambda^{(a)}}[\delta\cos(2\pi k/N)\sin(2\theta_k) - \sin(2\pi k/N)\cos^2(\theta_k)] \tag{9a}$$

$$\tan(2\phi_k^{(a)}) = \frac{2}{\lambda^{(a)}}[\delta\cos(2\pi k/N)\sin(2\theta_k) - \sin(2\pi k/N)\sin^2(\theta_k)] \tag{9b}$$

and c_k and d_k are fermion operators. Thus we obtain the diagonalized Hamitonian $H^{(a)}$ as

$$H^{(a)} = \sum_{k=1}^{N/2}\left\{\Lambda_k^{+(a)} d_k^{+(a)} d_k^{(a)} + \Lambda_k^{-(a)} c_k^{+(a)} c_k^{(a)}\right\} \tag{10a}$$

where

$$\Lambda_k^{+(a)} = 4[\sin(2\pi k/N) + 2\delta\cos(2\pi k/N)]\sin(2\phi_k^{(a)}) - 2\lambda^{(a)}\cos(2\phi_k^{(a)}) \tag{10b}$$

$$\Lambda_k^{-(a)} = 4[\sin(2\pi k/N) - 2\delta\cos(2\pi k/N)]\sin(2\varphi_k^{(a)}) - 2\lambda^{(a)}\cos(2\varphi_k^{(a)}) \tag{10c}$$

We can see that the Hamitonian $H^{(a)}$ has the gapped energy spectrum with acoustic and optical branches. At zero temperature and in the presence of $\delta = 0$, the model undergoes two distinct second order quantum phase transitions [18]. One is the Ising transition from aparamagnetic to a ferromagnetic phase when $\lambda = 0$, another is the anisotropic transition from a ferromagnet with magnetization in the direction to one with magnetization in the y direction when $\lambda < 0$. In this paper we investigate the features of the Loschmidt echo.

3 The Loschmidt echo

In order to analyze decoherence of the qubit induced by the spin chain enrionment, we will consider the Loschmidt echo which is defined as [6]

$$L(t) = |<G|e^{itH^{(0)}}e^{-itH^{(0)}}|G>|^2 \tag{11}$$

where $|G>$ is the initial state of the environment. Now we assume that the ground states for the Hamiltonian $H^{(a)}$ is $|G^{(a)}>$, i.e., $c_k^{(a)}|G^{(a)}>=0$, $d_k^{(a)}|G^{(a)}>=0$. In the following, for simplicity, we select $|G>=|G^{(0)}>$. From Eq. (6) and Eq. (8), we can find relation between the two ground states as follows

$$|G^{(0)}>=[\cos(\varphi_k^{(0)}-\varphi_k^{(1)})+i\sin(\varphi_k^{(0)}-\varphi_k^{(1)})c_k^{(1)}c_{-k}^{+(1)}] \\ \otimes[\cos(\phi_k^{(0)}-\phi_k^{(1)})+i\sin(\phi_k^{(0)}-\phi_k^{(1)})d_k^{(1)}d_{-k}^{+(1)}]|G^{(1)}> \tag{12}$$

Inserting Eq. (12) into Eq. (11) , we can obtain

$$L(\lambda,\delta)=\prod_{k=1}^{N/2}\left[1-\sin^2(\varphi_k^{(0)}-\varphi_k^{(1)})\sin^2(\Lambda_k^{-(1)}t)\right] \\ \left[1-\sin^2(\phi_k^{(0)}-\phi_k^{(1)})\sin^2(\Lambda_k^{+(1)}t)\right] \tag{13}$$

In what follows we shall investigate in detail the properties of the Loschmidt echo in different coupling regimes. In the weak coupling regime, namely, the case of the parameter $g<1$, we compute numerically the changing of $L(\lambda,\delta)$ with the parameter λ for various values δ and fixed time t, as shown in Fig. 1. It is observed that the evolution of $L(\lambda,\delta,t)$ with the parameter λ displays the characteristic of single flat peak shape in the vicinity of the critical point $\lambda=0$. With the increasing of the value of δ, the width of the peak will be slightly enhanced and the decay of $L(\lambda,\delta,t)$ will be strenghten. If we increase coupling strengh $g=0.4$, from Fig. 2 we can observe that the width of the peak will be significantly increased. In the strong coupling regime, namely, the case of the parameter $g>1$, we draw a diagram of the change of $L(\lambda,\delta,t)$ with the parameter λ, shown as in Fig. 3. we can observe that the shapes of the peaks are acuminate rather than flat in the vicinity of the critical point $\lambda=0$ for the parameter $g=10$ and fixed time $t=0.001$. Unlike the case of the weak coupling, the decay of $L(\lambda,\delta,t)$ does not reach zero in the vicinity of the critical point.

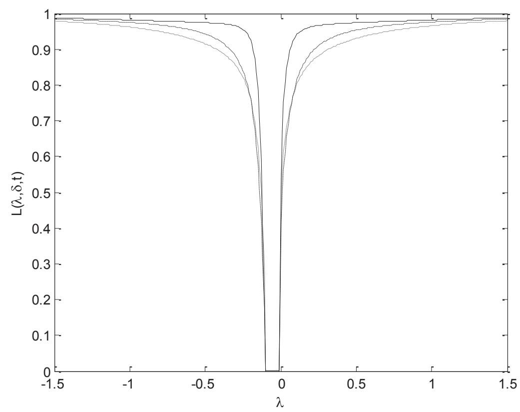

Figure 1: The Loschmidt echo $L(\lambda,\delta,t)$ as a function of magnetic intensity λ with different staggered parameter δ for fixed time (we take $N=500$, $g=0.1$, $t=0.1$), solid line: $\delta=0.1$, dashed line: $\delta=0.3$,: dotted line $\delta=0.5$.

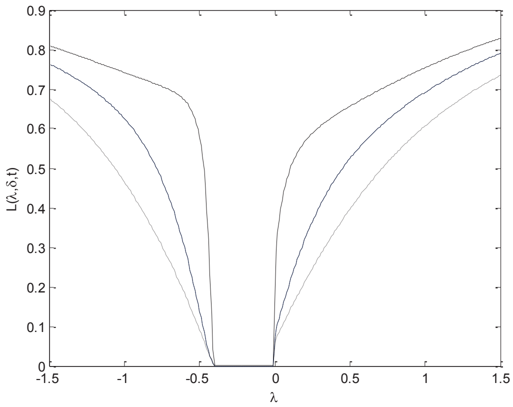

Figure 2: The Loschmidt echo $L(\lambda,\delta,t)$ as a function of magnetic intensity λ with different staggered parameter δ for fixed time (we take $N=500$, $g=0.4$, $t=0.1$), solid line: $\delta=0.1$, dashed line: $\delta=0.3$,: dotted line $\delta=0.5$.

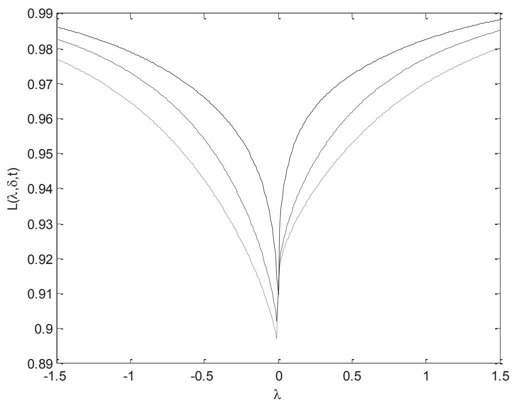

Figure 3: The Loschmidt echo $L(\lambda,\delta,t)$ as a function of magnetic intensity λ with different staggered parameter δ for fixed time (we take $N=500$, $g=10$, $t=0.001$), solid line: $\delta=0.1$, dashed line: $\delta=0.3$,: dotted line $\delta=0.5$.

Finally, it is interesting that we compare the properties of the Loschmidt echo in the system with that in the other systems, for instance, XY spin chain system investigated in ref. [8]. For the XY spin chain system, the Loschmidt echo $L(\lambda,\delta,t)$ shows maximum decaying in the critical point, and shows slow decaying away from the critical point. However, in our system, the Loschmidt echo $L(\lambda,\delta,t)$ shows stronger decaying away from the critical point. We guess that the decaying of the Loschmidt echo $L(\lambda,\delta,t)$ is related to the propertyies of quantum phase transitions.

4 Conclusion

In conclusion, we have analyzed the Loschmidt echo of single qubit induced by the environment for dimerized anisotropic XY spin chain. We have obtained analytical expression for the Loschmidt echo and computed numerically the Loschmidt echo. We find that the decay of coherence will be enhanced with the increasing of the staggered parameter δ.

References

[1] W. H. Zurek, Rev. Mod. Phys. 75 (2003) 715.
[2] T. Yu and J. H. Eberly, Phys. Rev. Lett. 93 (2004) 140404.
[3] C. Pineda and T. H. Seligman, Phys. Rev. A73 (2006) 012305.
[4] F. M. Cucchietti, J. P. Paz , and W. H. Zurek, Phys. Rev. A72 (2005) 052113.
[5] D. D. Bhaktavatsala Rao, Phys. Rev. A76 (2007) 042312.
[6] C. Cormick and J. P. Paz, Phys. Rev. A77 (2008) 022317.
[7] N. V. Prokof'ev and P. C. E. Stamp, Rep. Prog. Phys. 63 (2000) 669.
[8] H. T. Quan, *et al.*, Phys. Rev. Lett. 96 (2006) 140604.
[9] Jingfu Zhang, Phys. Rev. Lett. 100 (2008) 100501.
[10] L. C. Venuti, *et al.*, Phys. Rev. Lett. 107 (2011) 010403.
[11] H. L. Lian, Chin. Phys. C 36 (2012) 479.
[12] P. R. Zangara, *et al.*, Phys. Rev. A86 (2012)012322.
[13] F. M. Cucchietti, C. H. Lewenkopf, and H. M. Pastawski, Phys. Rev. E74 (2006) 026207.
[14] F. Verstract, M. A. Martin-Delgado,and J. I. Cirac, Phys. Rev. Lett. 92 (2004) 087201.
[15] J. Riera and A. dobry, Phys. Rev. A51 (1995) 16095.
[16] J. Sirker, *et al.*, Phys. Rev. Lett. 101 (2008) 157204.
[17] A. Herzog, *et al.*, Phys. Rev. B 84 (2011) 134428.
[18] J. E. Bunder and Ross H. McKenzie, Phys. Rev. B 60 (1999) 344.