J. At. Mol. Sci.
doi: 10.4208/jams.041711.051011a

Vol. 3, No. 1, pp. 73-77
February 2012

A simple scheme for realizing six-photon entangled state based on cavity quantum electrodynamics

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Received 17 April 2011; Accepted (in revised version) 10 May 2011
Published Online 28 September 2011

Abstract. A simple scheme is presented for generating six-photon entangled state with resonant interaction between a cascade type four-level atom and two three-mode cavities. In the proposed protocol, the quantum information is encoded on Fock states of the cavity fields. We solve Schrödinger equation and obtain quantum states of interaction system.The detection of the atom can collapse the cavity to the desired six-photon entangled state.

PACS: 03.67.-a, 03.67.Lx, 42.50.-p

Key words: quantum information, cascade four-level atom, three-mode cavity, entangled state

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1 Introduction

Cavity quantum electrodynamics (QED) is an ideal candidate for implementing quantum information processing [1]. The reason is based on the following two points. (i) Photons are ideal carriers for fast and reliable communication over long distances, and the atoms are good memorizers for storing and processing quantum information. Thus the combination of atoms and photons can be useful in quantum computation. (ii) The atoms trapped in a high-Q cavity have long decoherence time [2]. Entanglement of two or more particles is the most intriguing characteristic of quantum mechanics. In recent years, several physical system have been suggested to generate quantum entanglement [3–5], among which cavity quantum electrodynamics (QED) system is viewed as a promising tool in that recent development in cavity QED techniques have made us to produce quantum entanglement between cavity fields [6] and between atoms [7]. Entangled states not only are recognized as an essential ingredient for testing the foundation of quantum mechanics, but also have many significant applications in quantum-information processing (QIP) [8]. Generally, the more particles that can be entangled, the more clearly nonclassical effects are exhibited, and the more useful the states

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are for quantum applications [9]. Thus generation and manipulation of multipartite entangled states are very important tasks in QIP and have been attracting much attention. Dür *et al.* have shown that there are two inequivalent classes of tripartite entanglement states, i.e., the GHZ class and the W class, under stochastic local operations and classical communications [10]. GHZ type of entangled state has many interesting properties. For example, the three-particle GHZ state is maximally stable against noise, maximally violates Bell inequalities, and can be used to implement perfect teleportation [11]. Meng et al have proposed a scheme for preparing an N atoms GHZ entangled state through the interaction between N atoms and a cavity [12]. Although many schemes for tripartite entangled states have been studied [13, 14], the report about preparing multi-photon ($N>3$) entangled state is very few.

2 Realization of six-photon entangled state

We consider the resonant interaction of a cascade type four-level atom with a three-mode cavity field is shown in Fig. 1. The interaction Hamitonian for such a system can be described as ($\hbar=1$) [15]

$$H_1=g_1\Big(a^+|g\rangle\langle i|+a|i\rangle\langle g|\Big)+g_2\Big(b^+|i\rangle\langle j|+b|j\rangle\langle i|\Big)+g_3\Big(c^+|j\rangle\langle e|+c|e\rangle\langle j|\Big), \qquad (1)$$

where $a^+(b^+,c^+)$ and $a(b,c)$ are the creation and annihilation operators for the cavity fields, respectively, $g=g_1=g_2=g_3$ is the coupling constant of the interaction of atom with the cavity mode.

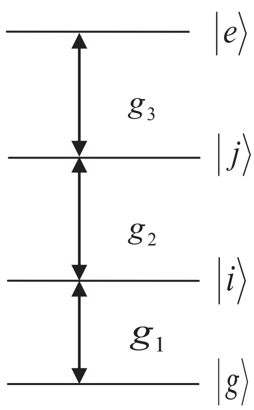

Figure 1: Atomic level structure of cascade type four-level atom coupling to a three-mode cavity.

For simplicity, assume that the cavities are initially prepared in $|0,0,0\rangle_1,|0,0,0\rangle_2$, the atom is initially in $|e\rangle$, we obtain the system initially prepared in the state

$$\Big|\phi(t=0)\Big\rangle=|e\rangle|0,0,0\rangle_1|0,0,0\rangle_2. \qquad (2)$$

We consider a general state of

$$\left|\phi(t)\right\rangle = C_1(t)|e,0,0,0\rangle + C_2(t)|j,0,0,1\rangle + C_3(t)|i,0,1,1\rangle + C_4(t)|g,1,1,1\rangle. \quad (3)$$

Using the Schrodinger equation, we can get a group of first-order time differential equations of $C_1(t), C_2(t), C_3(t), C_4(t)$

$$i\frac{\partial C_1}{\partial t} = gC_2, \quad (4a)$$

$$i\frac{\partial C_2}{\partial t} = g(C_1 + C_3), \quad (4b)$$

$$i\frac{\partial C_3}{\partial t} = g(C_2 + C_4), \quad (4c)$$

$$i\frac{\partial C_4}{\partial t} = gC_3. \quad (4d)$$

On eliminating $C_1(t), C_2(t), C_3(t)$, we obtain the following fourth-order time differential equation for C_4

$$\frac{\partial^4 C_4}{\partial t^4} + 3g^2\frac{\partial^2 C_4}{\partial t^2} + g^4 C_4 = 0. \quad (5)$$

The initial condition for C_4 at $t = 0$ are

$$C_4(0) = 0, \quad \frac{\partial C_4(0)}{\partial t^2} = 0, \quad \frac{\partial^2 C_4(0)}{\partial t^2} = 0, \quad \frac{\partial^3 C_4(0)}{\partial t^3} = ig^3. \quad (6)$$

We consider a solution of the form

$$C_4(t) = Ae^{\omega_1 t} + Be^{\omega_2 t} + Ce^{\omega_3 t} + De^{\omega_4 t}, \quad (7)$$

where $\omega_1, \omega_2, \omega_3$ and ω_4 are the roots the fourth-order polynomial equation

$$z^4 + 3g^2 z^2 + g^4 = 0. \quad (8)$$

The A, B, C, D are as follows

$$A = \frac{ig^3}{(\omega_1 - \omega_2)(\omega_1 - \omega_3)(\omega_1 - \omega_4)}, \quad (9a)$$

$$B = \frac{ig^3}{(\omega_1 - \omega_2)(\omega_2 - \omega_3)(\omega_3 - \omega_4)}, \quad (9b)$$

$$C = \frac{ig^3}{(\omega_1 - \omega_3)(\omega_2 - \omega_3)(\omega_3 - \omega_4)}, \quad (9c)$$

$$D = \frac{ig^3}{(\omega_1 - \omega_4)(\omega_2 - \omega_4)(\omega_3 - \omega_4)}. \quad (9d)$$

From C_4, we can get C_1,C_2,C_3. C_1 as follows

$$C_1=1+\frac{i}{g}\left(\frac{A(\omega_1^2+g^2)}{\omega_1}(e^{\omega_1 t}-1)+\frac{B(\omega_2^2+g^2)}{\omega_2}(e^{\omega_2 t}-1)\right.$$
$$\left.+\frac{C(\omega_3^2+g^2)}{\omega_3}(e^{\omega_3 t}-1)+\frac{D(\omega_4^2+g^2)}{\omega_4}(e^{\omega_4 t}-1)\right). \tag{10}$$

According to the effective Hamiltonian (1), we solve corresponding Schrödinger equation and obtain expression of the state. We send the atom through the first cavity, after an interaction time t_1, the state evolves into

$$|\phi(t)\rangle=\Big(C_1(t_1)|e\rangle|0,0,0\rangle_1+C_2(t_1)|j\rangle|0,1,1\rangle_1$$
$$+C_3(t_1)|i\rangle|0,1,1\rangle+1+C_4(t_1)|g\rangle|1,1,1\rangle_1\Big)|0,0,0\rangle_2. \tag{11}$$

Then we let the atom pass through the second cavity, and after an interaction time t_2,the atom-cavity system evolves into the state

$$|\phi(t_1+t_2)\rangle=\frac{1}{2}\Big(C_1(t_1)|0,0,0\rangle_1\big(C_1(t_2)|e\rangle|0,0,0\rangle_2+C_2(t_2)|j\rangle|0,0,1\rangle_2+C_3(t_2)|i\rangle|0,1,1\rangle_2$$
$$+C_4(t_2)|g\rangle|1,1,1\rangle_2\big)+C_2(t_1)|0,0,1\rangle_1\big(\cos(\sqrt{2}gt_2)+1\big)|j\rangle|0,0,0\rangle_2$$
$$-\sqrt{2}i\sin(\sqrt{2}gt_2)|i\rangle|0,1,0\rangle_2+\big(\cos(\sqrt{2}gt_2)-1\big)|g\rangle|1,1,0\rangle_2\Big)$$
$$+C_3(t_1)|0,1,1\rangle\big(\cos(gt_2)|i\rangle|0,0,0\rangle_2-i\sin(gt_2)|g\rangle|1,0,0\rangle_2\big)$$
$$+C_4(t_1)|g\rangle|1,1,1\rangle_1|0,0,0\rangle_2. \tag{12}$$

By choosing $C_3(t_1)=0$, $\sqrt{2}gt_2=2n\pi$ (n is an integer), we can obtain state

$$\Big|\phi(t_1+t_2)\Big\rangle=C_1(t_1)|0,0,0\rangle_1\big(C_1(t_2)|e\rangle|0,0,0\rangle_2+C_2(t_2)|j\rangle|0,0,1\rangle_2+C_3(t_2)|i\rangle|0,1,1\rangle_2$$
$$+C_4(t_2)|g\rangle|1,1,1\rangle_2\big)+C_2(t_1)|0,0,1\rangle\Big(\big(\cos(\sqrt{2}gt_2)+1\big)|j\rangle|0,0,0\rangle_2$$
$$-\sqrt{2}i\sin(\sqrt{2}gt_2)|i\rangle|0,1,0\rangle_2\Big)+C_4(t_1)|g\rangle|1,1,1\rangle_1|0,0,0\rangle_2. \tag{13}$$

We can perform a measurement on the atom, if the atom is detected in the state $|g\rangle$ the cavity field collapses into the following state

$$\Big|\phi(t_1+t_2)\Big\rangle_g\to\Big(C_1(t_1)C_4(t_2)|0,0,0\rangle_1|1,1,1\rangle_2+C_4(t_1)|1,1,1\rangle_1|0,0,0\rangle_2\Big). \tag{14}$$

The states $|\phi(t_1+t_2)\rangle_g$ is six-photon entangled state, when $C_1(t_1)C_4(t_2)=C_4(t_1)$, $|\phi(t_1+t_2)\rangle_g$ is six-photon GHZ state.

3 Conclusions

The implementation of the quantum entangled state requires the passage of a cascade type four-level atom through two three-mode cavity. As with any proposal for quantum computing implementation, its success ultimately depends on being able to complete many coherent dynamics during the decoherence time, the atomic and cavity lifetimes should being the larger than the interaction time of the atoms with the cavity fields. The photon number in the two cavity modes should also remain unaltered during interaction with the atom. In summary, we have proposed a simple method for generation of six-photon entangled state via cavity QED. Comparing with previous schemes, our proposal is more simple and feasible since the detection of atom can collapse the cavity to the desired six-photon entangled state.

Acknowledgments. The project is supported by the National Natural Science Foundation of China under Grant No. 11004050, the Key Scientific Research Fund of Hunan Provincial Education Department under Grant No. 09A013, and the Science and Technology Research Foundation of Hunan Province under Grant No. 2010FJ4120.

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