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Teleportation of a product state of an arbitrary single-particle state

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Abstract. This paper proposes two schemes for teleporting a product state of an arbitrary single-particle state from a sender to a receiver via a four-particle entangled cluster state. The two different quantum channels are used, while the successful probabilities of these two schemes are different. In the first proposal, the successful probability is 1.0 and in the second proposal, the successful probability is $4q^2$ if the receiver performs an appropriate unitary operation.

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Key words: quantum teleportation, probabilistic teleportation, Bell state measurement (BSM), unitary transformation

1 Introduction

Recently, much attention has been paid to quantum information. Entanglement is considered as the fundamental resource of quantum information processing such as quantum teleportation, quantum dense coding, and quantum secret sharing and so on. Quantum teleportation, first proposed by Bennett *et al.* [1] in 1993, can transmit an unknown quantum state from a sender to a receiver at a distant location via a quantum channel with the help of some classical information. As quantum teleportation is one of the basic methods of quantum communication [2] and may be useful in quantum computation [3], it has attracted much attention, and some experimental work has been reported [4, 5], much theoretical work has been reported over the past decade [6–23].

However, all of the aforementioned schemes are focused on some entangled states. In their schemes, the unknown quantum state, which is transmitted between two parties, usually is a single-particle or two-particles entangled state or three-particles entangled state, even four-particles entangled state, little attention has been devoted to teleporting a product state. In

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this paper, we present two schemes for teleportating a product state via a four-particle cluster state. In the first scheme, quantum channel is a maximally cluster state; while in the second scheme, quantum channel is a non-maximally cluster state. The successful possibility of our first scheme is reach 1.0, and the successful probability of our second scheme is $4q^2$.

The rest of this paper is organized as follows. The Section 2 presents the first scheme for teleportating via a maximally cluster state. Probabilistic teleportation via a non-maximally cluster state is described in Section 3. Finally, a short conclusion is given in Section 4.

2 Teleportation of a product state of arbitrary single-particle via a four-particle cluster state

In our scheme, the two parties, a sender (namely, Alice) and a receiver (namely, Bob), Alice has a product state of arbitrary single-particle (i.e. $|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b$), which she wants to send to Bob

$$|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b, \tag{1}$$

where $|\phi\rangle_a=\alpha_a|0\rangle_a+\beta_a|1\rangle_a$, $|\phi\rangle_b=\alpha_b|0\rangle_b+\beta_b|1\rangle_b$. α_a, α_b, β_a and β_b are any set of complex numbers and need satisfying the following conditions: $|\alpha_a|^2+|\beta_a|^2=1$, $|\alpha_a|>|\beta_a|$, $|\alpha_b|^2+|\beta_b|^2=1$ and $|\alpha_b|>|\beta_b|$.

A cluster state is used as quantum channel between Alice and Bob, which is in the following state

$$|\varphi\rangle_{1234}=\frac{1}{2}\Big(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle\Big)_{1234}. \tag{2}$$

Particles a, b, 2 and 3 belong to Alice; particles 1 and 4 belong to Bob. Initially, the joint system before Alice's measurement can be written as:

$$\begin{aligned}
|\psi\rangle_{ab1234}=&|\phi\rangle_a\otimes|\phi\rangle_b\otimes|\varphi\rangle_{1234}\\
=&\frac{1}{2}\Big(\alpha_a|0\rangle_a+\beta_a|1\rangle_a\Big)\Big(\alpha_b|0\rangle_b+\beta_b|1\rangle_1\Big)\Big(|0000\rangle+|0011\rangle+|1100\rangle-|1111\rangle\Big)_{1234}\\
=&\frac{1}{2}\Big(\alpha_a\alpha_b|000000\rangle+\alpha_a\alpha_b|000011\rangle+\alpha_a\alpha_b|001100\rangle-\alpha_a\alpha_b|001111\rangle\\
&+\alpha_a\beta_b|010000\rangle+\alpha_a\beta_b|010011\rangle+\alpha_a\beta_b|011100\rangle-\alpha_a\beta_b|011111\rangle\\
&+\beta_a\alpha_a|100000\rangle+\beta_a\alpha_b|100011\rangle+\beta_a\alpha_b|101100\rangle-\beta_a\alpha_b|101111\rangle\\
&+\beta_a\beta_b|110000\rangle+\beta_a\beta_b|110011\rangle+\beta_a\beta_b|111100\rangle-\beta_a\beta_b|111111\rangle\Big)_{ab1234}.
\end{aligned} \tag{3}$$

In order to realize the teleportation, twice Bell-state measurements on particles (a, 2) and particles (b, 3) are made by Alice, respectively, which will cause particles (1, 4) collapses into one of the following state

$${}_{b3}\Big\langle\phi^{\pm}\Big|_{a2}\Big\langle\phi^{+}\Big||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|00\rangle\pm\alpha_a\beta_b|01\rangle+\beta_a\alpha_b|10\rangle\mp\beta_a\beta_b|11\rangle\Big)_{14}, \tag{4}$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|00\rangle\pm\alpha_a\beta_b|01\rangle-\beta_a\alpha_b|10\rangle\pm\beta_a\beta_b|11\rangle\Big)_{14}, \tag{5}$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|01\rangle\pm\alpha_a\beta_b|00\rangle-\beta_a\alpha_b|11\rangle\pm\beta_a\beta_b|10\rangle\Big)_{14}, \tag{6}$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|01\rangle\pm\alpha_a\beta_b|00\rangle+\beta_a\alpha_b|11\rangle\mp\beta_a\beta_b|10\rangle\Big)_{14}, \tag{7}$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|10\rangle\mp\alpha_a\beta_b|11\rangle+\beta_a\alpha_b|00\rangle\pm\beta_a\beta_b|01\rangle\Big)_{14}, \tag{8}$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|10\rangle\mp\alpha_a\beta_b|11\rangle-\beta_a\alpha_b|00\rangle\mp\beta_a\beta_b|01\rangle\Big)_{14}, \tag{9}$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(-\alpha_a\alpha_b|11\rangle\pm\alpha_a\beta_b|10\rangle+\beta_a\alpha_b|01\rangle\pm\beta_a\beta_b|00\rangle\Big)_{14}, \tag{10}$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(-\alpha_a\alpha_b|11\rangle\pm\alpha_a\beta_b|10\rangle-\beta_a\alpha_b|01\rangle\mp\beta_a\beta_b|00\rangle\Big)_{14}, \tag{11}$$

where $|\phi^{\pm}\rangle$ and $|\psi^{\pm}\rangle$ are four Bell states. $|\phi^{\pm}\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$, $|\psi^{\pm}\rangle=(|00\rangle\pm|11\rangle)/\sqrt{2}$.

After doing that, Alice tells Bob her measurement results via a classical channel. Finally, Bob can obtain the unknown state on particles 1 and 4 by performing appropriate unitary transformations. We discuss the operations in detail below.

First Bob performs a quantum controlled phase gate operation on the particles 1 and 4, where the particle 1 is the control bit and the particle 4 is the target bit, i.e., if and only if particle 1 is in the state $|1\rangle$, particle 4 is performed an operation of Pauli operator (σ_z).Thus the Eqs. (4)-(11) become

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|00\rangle\pm\alpha_a\beta_b|01\rangle+\beta_a\alpha_b|10\rangle\pm\beta_a\beta_b|11\rangle\Big)_{14},$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|00\rangle\pm\alpha_a\beta_b|01\rangle-\beta_a\alpha_b|10\rangle\mp\beta_a\beta_b|11\rangle\Big)_{14},$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|01\rangle\pm\alpha_a\beta_b|00\rangle+\beta_a\alpha_b|11\rangle\pm\beta_a\beta_b|10\rangle\Big)_{14},$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|01\rangle\pm\alpha_a\beta_b|00\rangle-\beta_a\alpha_b|11\rangle\mp\beta_a\beta_b|10\rangle\Big)_{14},$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|10\rangle\pm\alpha_a\beta_b|11\rangle+\beta_a\alpha_b|00\rangle\pm\beta_a\beta_b|01\rangle\Big)_{14},$$

$$_{b3}\left\langle\phi^{\pm}\right|_{a2}\left\langle\psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|10\rangle\pm\alpha_a\beta_b|11\rangle-\beta_a\alpha_b|00\rangle\mp\beta_a\beta_b|01\rangle\Big)_{14},$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|11\rangle\pm\alpha_a\beta_b|10\rangle+\beta_a\alpha_b|01\rangle\pm\beta_a\beta_b|00\rangle\Big)_{14},$$

$$_{b3}\left\langle\psi^{\pm}\right|_{a2}\left\langle\psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{4}\Big(\alpha_a\alpha_b|11\rangle\pm\alpha_a\beta_b|10\rangle-\beta_a\alpha_b|01\rangle\mp\beta_a\beta_b|00\rangle\Big)_{14},$$

Without loss of generality, if Alice's measurement results are $|\phi^{+}\rangle_{a2}$ and $|\psi^{-}\rangle_{b3}$ respectively, then the particle pair (1, 4) is collapsed into the state

$$\Big(\alpha_a\alpha_b|01\rangle-\alpha_a\beta_b|00\rangle+\beta_a\alpha_b|11\rangle-\beta_a\beta_b|10\rangle\Big)_{14}=\Big(\alpha_a|0\rangle_1+\beta_a|1\rangle_1\Big)\Big(\alpha_b|1\rangle_4-\beta_b|0\rangle_4\Big).$$

Bob need to perform $U_{14}=I^1\otimes\sigma_x^4\otimes\sigma_z^4$ (or $I^1\otimes\sigma_y^4$) on the particle pair (1, 4) to reconstruct the original state. The others possible cases are described in Table 1. Thus the product state of arbitrary single-particle can be simultaneously reproduced on Bob's side successfully, and the total successful probability of the teleportation is $(1/4)^2\times 16=1$.

Table 1: The unitary transformations corresponding to Alice's measurement results.

Alice's result	Bob's operations	Alice's result	Bob's operations
$\lvert\phi^+\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$I^1\otimes I^4$	$\lvert\psi^+\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_x^1\otimes I^4$
$\lvert\phi^+\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$I^1\otimes\sigma_z^4$	$\lvert\psi^+\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_x^1\otimes\sigma_z^4$
$\lvert\phi^-\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_z^1\otimes I^4$	$\lvert\psi^-\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_y^1\otimes I^4$
$\lvert\phi^-\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_z^1\otimes\sigma_z^4$	$\lvert\psi^-\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_y^1\otimes\sigma_z^4$
$\lvert\phi^+\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$I^1\otimes\sigma_x^4$	$\lvert\psi^+\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_x^1\otimes\sigma_x^4$
$\lvert\phi^+\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$I^1\otimes\sigma_y^4$	$\lvert\psi^+\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_x^1\otimes\sigma_y^4$
$\lvert\phi^-\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_z^1\otimes\sigma_x^4$	$\lvert\psi^-\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_y^1\otimes\sigma_x^4$
$\phi^-\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_z^1\otimes\sigma_y^4$	$\lvert\psi^-\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_y^1\otimes\sigma_y^4$

3 Probabilistic teleportation of a product state of arbitrary single-particle via a non-maximally four-particle cluster state

In this section, we will also teleport the product state $|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b$, and we will take a non-maximally four-particle cluster state as the quantum channel

$$|\varphi\rangle_{1234}=\Big(m|0000\rangle+n|0011\rangle+p|1100\rangle-q|1111\rangle\Big)_{1234}, \tag{12}$$

where m, n, p and q are any set of complex numberes and need satisfying the following conditions: $|m|^2+|n|^2+|p|^2+|q|^2=1$ and $|m|>|n|>|p|>|q|$. Particles a, b, 2 and 3 belong to Alice; particles 1 and 4 belong to Bob. Initially, the total state of the system can be expressed as

$$\begin{aligned}
&|\phi\rangle_{ab1234}=|\phi\rangle_a\otimes|\phi\rangle_b\otimes|\varphi\rangle_{1234}\\
=&\Big(\alpha_a|0\rangle_a+\beta_a|1\rangle_a\Big)\Big(\alpha_b|0\rangle_b+\beta_b|1\rangle_b\Big)\Big(m|0000\rangle+n|0011\rangle+p|1100\rangle-q|1111\rangle\Big)_{1234}\\
=&\Big(\alpha_a\alpha_b m|000000\rangle+\alpha_a\alpha_b n|000011\rangle+\alpha_a\alpha_b p|001100\rangle-\alpha_a\alpha_b q|001111\rangle\\
&\quad+\alpha_a\beta_b m|010000\rangle+\alpha_a\beta_b n|010011\rangle+\alpha_a\beta_b p|011100\rangle-\alpha_a\beta_b q|011111\rangle\\
&\quad+\beta_a\alpha_b m|100000\rangle+\beta_a\alpha_b n|100011\rangle+\beta_a\alpha_b p|101100\rangle-\beta_a\alpha_b q|101111\rangle\\
&\quad+\beta_a\beta_b m|110000\rangle+\beta_a\beta_b n|110011\rangle+\beta_a\beta_b p|111100\rangle-\beta_a\beta_b q|111111\rangle\Big)_{ab1234}.
\end{aligned} \tag{13}$$

Similarly, at the first step, Alice need to make twice Bell-state measurements on particles (a, 2) and particles (b, 3) to realize the teleportation, then all the 16 possible collapsed states of particles (1, 4) are

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(m\alpha_a\alpha_b|00\rangle\pm n\alpha_a\beta_b|01\rangle+p\beta_a\alpha_b|10\rangle\mp q\beta_a\beta_b|11\rangle\right)_{14}, \quad (14)$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(m\alpha_a\alpha_b|00\rangle\pm n\alpha_a\beta_b|01\rangle-p\beta_a\alpha_b|10\rangle\pm q\beta_a\beta_b|11\rangle\right)_{14}, \quad (15)$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(n\alpha_a\alpha_b|01\rangle\pm m\alpha_a\beta_b|00\rangle-q\beta_a\alpha_b|11\rangle\pm p\beta_a\beta_b|10\rangle\right)_{14}, \quad (16)$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(n\alpha_a\alpha_b|01\rangle\pm m\alpha_a\beta_b|00\rangle+q\beta_a\alpha_b|11\rangle\mp p\beta_a\beta_b|10\rangle\right)_{14}, \quad (17)$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(p\alpha_a\alpha_b|10\rangle\mp q\alpha_a\beta_b|11\rangle+m\beta_a\alpha_b|00\rangle\pm n\beta_a\beta_b|01\rangle\right)_{14}, \quad (18)$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(p\alpha_a\alpha_b|10\rangle\mp q\alpha_a\beta_b|11\rangle-m\beta_a\alpha_b|00\rangle\mp n\beta_a\beta_b|01\rangle\right)_{14}, \quad (19)$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(-q\alpha_a\alpha_b|11\rangle\pm p\alpha_a\beta_b|10\rangle+n\beta_a\alpha_b|01\rangle\pm m\beta_a\beta_b|00\rangle\right)_{14}, \quad (20)$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(-q\alpha_a\alpha_b|11\rangle\pm p\alpha_a\beta_b|10\rangle-n\beta_a\alpha_b|01\rangle\mp m\beta_a\beta_b|00\rangle\right)_{14}. \quad (21)$$

At the second step, Bob performs a quantum controlled phase gate operation on the particles 1 and 4, where the particle 1 is the control bit and the particle 4 is the target bit, i.e., if and only if particle 1 is in the state $|1\rangle$, particle 4 is performed an operation of Pauli operator (σ_z). Thus the Eqs. (14)–(21) become

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(m\alpha_a\alpha_b|00\rangle\pm n\alpha_a\beta_b|01\rangle+p\beta_a\alpha_b|10\rangle\pm q\beta_a\beta_b|11\rangle\right)_{14},$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(m\alpha_a\alpha_b|00\rangle\pm n\alpha_a\beta_b|01\rangle-p\beta_a\alpha_b|10\rangle\mp q\beta_a\beta_b|11\rangle\right)_{14},$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \phi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(n\alpha_a\alpha_b|01\rangle\pm m\alpha_a\beta_b|00\rangle+q\beta_a\alpha_b|11\rangle\pm p\beta_a\beta_b|10\rangle\right)_{14},$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \phi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(n\alpha_a\alpha_b|01\rangle\pm m\alpha_a\beta_b|00\rangle-q\beta_a\alpha_b|11\rangle\mp p\beta_a\beta_b|10\rangle\right)_{14},$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(p\alpha_a\alpha_b|10\rangle\pm q\alpha_a\beta_b|11\rangle+m\beta_a\alpha_b|00\rangle\pm n\beta_a\beta_b|01\rangle\right)_{14},$$

$$_{b3}\left\langle \phi^{\pm}\right|_{a2}\left\langle \psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(p\alpha_a\alpha_b|10\rangle\pm q\alpha_a\beta_b|11\rangle-m\beta_a\alpha_b|00\rangle\mp n\beta_a\beta_b|01\rangle\right)_{14},$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \psi^{+}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(q\alpha_a\alpha_b|11\rangle\pm p\alpha_a\beta_b|10\rangle+n\beta_a\alpha_b|01\rangle\pm m\beta_a\beta_b|00\rangle\right)_{14},$$

$$_{b3}\left\langle \psi^{\pm}\right|_{a2}\left\langle \psi^{-}\right||\Psi\rangle_{ab1234}=\frac{1}{2}\left(q\alpha_a\alpha_b|11\rangle\pm p\alpha_a\beta_b|10\rangle-n\beta_a\alpha_b|01\rangle\mp m\beta_a\beta_b|00\rangle\right)_{14}.$$

At the third step, after knowing the results from Alice, Bob will perform relevant unitary transformation to reproduce the unknown state on particles 1 and 4. For an example, if the measurement results are $|\phi^+\rangle_{a2}$ and $|\psi^+\rangle_{b3}$ respectively, then the particle pair (1, 4) is collapsed into the state $(n\alpha_a\alpha_b|01\rangle+m\alpha_a\beta_b|00\rangle+q\beta_a\alpha_b|11\rangle+p\beta_a\beta_b|10\rangle)_{14}$. Bob need to perform $U_{14}=I^1\otimes\sigma_x^4$ on the particle pair (1, 4) to reconstruct the original state, respectively. Then the particle pair (1, 4) is changed into the state

$$|\varphi\rangle_{14}^2=\frac{1}{2}(n\alpha_a\alpha_b|00\rangle_{14}+m\alpha_a\beta_b|01\rangle_{14}+q\beta_a\alpha_b|10\rangle_{14}+p\beta_a\beta_b|11\rangle_{14}).$$

The others possible cases are described in Table 2.

Table 2: The unitary transformations corresponding to Alice's measurement results and the states of $|\varphi\rangle_{14}$ under the unitary transformation U_1 operation.

Alice's results	Bob's operations	The states of $\lvert\varphi\rangle_{14}$ under the unitary transformation U_1 operation
$\lvert\phi^+\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$I^1\otimes I^4$	$\lvert\varphi\rangle_{14}^1=\frac{1}{2}\big(m\alpha_a\alpha_b\lvert 00\rangle+n\alpha_a\beta_b\lvert 01\rangle+p\beta_a\alpha_b\lvert 10\rangle+q\beta_a\beta_b\lvert 11\rangle\big)_{14}$
$\lvert\phi^+\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$I^1\otimes\sigma_z^4$	
$\lvert\phi^-\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_z^1\otimes I^4$	
$\lvert\phi^-\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_z^1\otimes\sigma_z^4$	
$\lvert\phi^+\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$I^1\otimes\sigma_x^4$	$\lvert\varphi\rangle_{14}^2=\frac{1}{2}\big(n\alpha_a\alpha_b\lvert 00\rangle+m\alpha_a\beta_b\lvert 01\rangle+q\beta_a\alpha_b\lvert 10\rangle+p\beta_a\beta_b\lvert 11\rangle\big)_{14}$
$\lvert\phi^+\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$I^1\otimes\sigma_y^4$	
$\lvert\phi^-\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_z^1\otimes\sigma_x^4$	
$\lvert\phi^-\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_z^1\otimes\sigma_y^4$	
$\lvert\psi^+\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_x^1\otimes I^4$	$\lvert\varphi\rangle_{14}^3=\frac{1}{2}\big(p\alpha_a\alpha_b\lvert 00\rangle+q\alpha_a\beta_b\lvert 01\rangle+m\beta_a\alpha_b\lvert 10\rangle+n\beta_a\beta_b\lvert 11\rangle\big)_{14}$
$\lvert\psi^+\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_x^1\otimes\sigma_z^4$	
$\lvert\psi^-\rangle_{a2}\lvert\phi^+\rangle_{b3}$	$\sigma_y^1\otimes I^4$	
$\lvert\psi^-\rangle_{a2}\lvert\phi^-\rangle_{b3}$	$\sigma_y^1\otimes\sigma_z^4$	
$\lvert\psi^+\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_x^1\otimes\sigma_x^4$	$\lvert\varphi\rangle_{14}^4=\frac{1}{2}\big(q\alpha_a\alpha_b\lvert 00\rangle+p\alpha_a\beta_b\lvert 01\rangle+n\beta_a\alpha_b\lvert 10\rangle+m\beta_a\beta_b\lvert 11\rangle\big)_{14}$
$\lvert\psi^+\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_x^1\otimes\sigma_y^4$	
$\lvert\psi^-\rangle_{a2}\lvert\psi^+\rangle_{b3}$	$\sigma_y^1\otimes\sigma_x^4$	
$\lvert\psi^-\rangle_{a2}\lvert\psi^-\rangle_{b3}$	$\sigma_y^1\otimes\sigma_y^4$	

If Bob gets the state

$$|\varphi\rangle_{14}^1=(m\alpha_a\alpha_b|00\rangle+n\alpha_a\beta_b|01\rangle+p\beta_a\alpha_b|10\rangle+q\beta_a\beta_b|11\rangle)_{14},$$

at the fourth step, to carry out this evolution, Bob need to introduce an auxiliary qubit with the original state $|0\rangle_A$, under the basis

$$\left\{|000\rangle_{45A},|010\rangle_{45A},|100\rangle_{45A},|110\rangle_{45A},|001\rangle_{45A},|011\rangle_{45A},|101\rangle_{45A},|111\rangle_{45A}\right\},$$

a collective unitary transformation U_1 is made, where

$$U_1=\begin{pmatrix} A_1 & A_2 \\ A_2 & -A_1 \end{pmatrix},$$
$$A_1=\mathrm{diag}(a_0,a_1,a_2,a_3),$$

$$A_2=\mathrm{diag}\left(\sqrt{1-a_0{}^2},\sqrt{1-a_1{}^2},\sqrt{1-a_2{}^2},\sqrt{1-a_3{}^2}\right),$$
$$(a_0,a_1,a_2,a_3)=\left(\frac{q}{m},\frac{q}{n},\frac{q}{p},1\right),$$
$$U_1=\begin{pmatrix} \frac{q}{m} & 0 & 0 & 0 & \sqrt{1-\left(\frac{q}{m}\right)^2} & 0 & 0 & 0 \\ 0 & \frac{q}{n} & 0 & 0 & 0 & \sqrt{1-\left(\frac{q}{n}\right)^2} & 0 & 0 \\ 0 & 0 & \frac{q}{p} & 0 & 0 & 0 & \sqrt{1-\left(\frac{q}{p}\right)^2} & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \sqrt{1-\left(\frac{q}{m}\right)^2} & 0 & 0 & 0 & -\frac{q}{m} & 0 & 0 & 0 \\ 0 & \sqrt{1-\left(\frac{q}{n}\right)^2} & 0 & 0 & 0 & -\frac{q}{n} & 0 & 0 \\ 0 & 0 & \sqrt{1-\left(\frac{q}{p}\right)^2} & 0 & 0 & 0 & -\frac{q}{p} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Then with the unitary transformation U_1 operation, the unnormalized product state $|\varphi\rangle_{14}^1\otimes|0\rangle_A$ will be changed as follows

$$U_1|\varphi\rangle_{45}^1\otimes|0\rangle_A=\frac{q}{2}\Big(\alpha_1\alpha_2|00\rangle_{45}+\alpha_1\beta_2|01\rangle_{45}+\beta_1\alpha_2|10\rangle_{45}+\beta_1\beta_2|11\rangle_{45}\Big)\otimes|0\rangle_A$$
$$+\frac{1}{2}\Big(\sqrt{m^2-q^2}\alpha_1\alpha_2|00\rangle_{45}+\sqrt{n^2-q^2}\alpha_1\beta_2|01\rangle_{45}+\sqrt{p^2-q^2}\beta_1\alpha_2|10\rangle_{45}\Big)\otimes|1\rangle_A. \quad (22)$$

Eq. (22) is also unnormalized; a measurement on auxiliary particle follows. The result $|1\rangle_A$

Table 3: The value of a_i $(i=0,1,2,3)$ in the unitary transformation U_1.

The state of the particle pair (1,4)	a_0	a_1	a_2	a_3
$\vert\varphi\rangle_{14}^1$	$\frac{q}{m}$	$\frac{q}{n}$	$\frac{q}{p}$	1
$\vert\varphi\rangle_{14}^2$	$\frac{q}{n}$	$\frac{q}{m}$	1	$\frac{q}{p}$
$\vert\varphi\rangle_{14}^3$	$\frac{q}{p}$	1	$\frac{q}{m}$	$\frac{q}{n}$
$\vert\varphi\rangle_{14}^4$	1	$\frac{q}{p}$	$\frac{q}{n}$	$\frac{q}{m}$

means the failed teleportation; while if the result is $|0\rangle_A$, Bob will obtain the state $(\alpha_a\alpha_b|00\rangle+\alpha_a\beta_b|01\rangle+\beta_a\alpha_b|10\rangle+\beta_a\beta_b|11\rangle)_{14}$, which is the original state $|\phi\rangle_{ab}$,

$$|\phi\rangle_{ab}=|\phi\rangle_a\otimes|\phi\rangle_b=\left(\alpha_a|0\rangle_a+\beta_a|1\rangle_a\right)\left(\alpha_b|0\rangle_b+\beta_b|1\rangle_b\right).$$

The whole optimal probability, which Bob can obtain $|\phi\rangle_{ab}$, is obtained as $q^2/4$.

Similarly, the others possible states can be discussed in the same way and are described in Table 2. While the value of a_i $(i=0,1,2,3)$ in the unitary transformation U_1 is different and is described in Table 3. Synthesizing all cases (16 kinds in all), the total optimal probabilities of successful teleportation is $(q^2/4)\times 16=4q^2$.

4 Conclusion

In this paper, two different schemes for teleporting a product state of arbitrary single-particle are proposed. In the first scheme, we use a four-particle cluster state as the quantum channel to teleport the product state, the receiver Bob can simultaneously reconstruct the original state according to Alice's measurement results, and the successful possibility is 1.0. In the latter scheme we teleport this product state via a non-maximally four-particle cluster state. In two different schemes, Bob should perform a quantum controlled phase gate operation besides the unitary transformation. Contrast to the first scheme, in order to realize the teleportation, Bob should introduce an auxiliary qubit. The advantage of the latter scheme is that the quantum channel is a more general state. The successful possibility and fidelity of this scheme can be $4q^2$.

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