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Efficient scheme for implementing the Deutsch-Jozsa algorithm in cavity QED

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Abstract. A scheme is proposed for implementing a two-qubit quantum logic gate and realizing the Deutsch-Jozsa algorithm in cavity QED. In the scheme a three-level atom interacts with highly detuned cavity modes. The gate is not affected by the atomic decay rates because of the metastable lower levels are involved in the gate operations. The Deutsch-Jozsa algorithm is easily realized with current experimental techniques.

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Key words: cavity QED, quantum logic gate, Deutsch-Jozsa algorithm

1 Introduction

In the last decade, people have paid much attention to physical realization of quantum computer owing to that it can offer an enormously acceleration compared to the classical one, such as quantum search [1] and Shor factoring [2]. As we all know that a quantum computing network is made up of a series of one-qubit rotation and two-qubit gates [3]. Considering a simple example of quantum algorithm-the Deutsch-Jozsa algorithm, which combines quantum parallelism with quantum interference. Also, two-qubit gate is an important step in it. It aims to distinguish function $f(x)$ between constant and balanced on 2^n inputs [4,5]. The value of function $f(x)$ is 0 or 1 for each input. If $f(x)$ is the constant, for all the inputs, the function values will be constant. But the values of the balanced function are equal to 0 for half of all the inputs while 1 for the other half. So far, there are many proposals for realizing the Deutsch-Jozsa algorithm theoretically and experimentally in the NMR system [6,7], ion trap [8], homonuclear multispin systems [9] and cavity QED [10,11].

Cavity QED is regarded as an ideal system to realize quantum information processing [10-17]. Lately, Zheng *et al.* [10]. have proposed a scheme for implementing the

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Deutsch-Jozsa algorithm. Following this, Yang *et al.* [12] have come up with a scheme to implement the Deutsch-Jozsa algorithm in cavity QED by using Schrodinger cat states, the atomic spontaneous emission can be minimized and it doesn't need the Hadamard gates because of the use of Schrodinger cat states. Ma *et al.* [13] also have proposed an idea to realize the Deutsch-Jozsa algorithm which uses superconducting quantum interference devices. Recently , Vallone *et al.* [14] describe an experimental scheme of the Deutsch-Jozsa algorithm with a six-qubit cluster states in their scheme, and the basis of the original measurement model allowing the algorithm implementation is its biggest characteristics. In our work, we propose a way to implement two-qubit quantum gates and the Deutsch-Jozsa algorithm in cavity QED system, in which a three-level atom interacts with highly detuned cavity modes. And the gate error induced by atomic spontaneous emission is minimized during the gate operation. The required experimental techniques are easy obtainable. Therefore our scheme might be experimentally realizable by using present available techniques. Meanwhile, the experimental achievement of the Deutsch-Jozsa algorithm would give birth to more important role in quantum computation.

The paper is organized as follows. In Section 2, we propose a scheme to realize a two qubit C-NOT gate. In Section 3 is devoted to describe how to implement the Deutsch-Jozsa algorithm in detail. Discussion and conclusion are given in Section 4.

2 Two qubit C-NOT gate

Here we consider a three-level Λ-type atomic system, as shown in Fig. 1. The atom levels are devoted by $|g\rangle$, $|l\rangle$, and $|e\rangle$.

The cavity modes having the annihilation operators $\hat{a}$, $\hat{b}$ interact with the atomic transitions $|g\rangle$ to $|e\rangle$, $|l\rangle$ to $|e\rangle$, respectively. And the cavity-field mode frequencies and the corresponding atomic transition frequencies are w_1, w_2 and w_{eg}, w_{el}, respectively. The

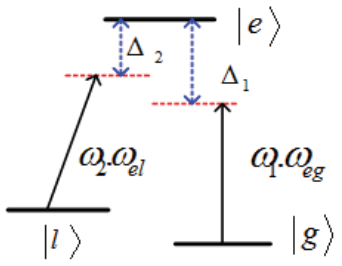

Figure 1: Schematic drawing of a three-lever atomic system in -type configuration. Here, ω_1, ω_2 are frequencies of the cavity-field mode. The parameters Δ_1 and Δ_2 are corresponding detuning of cavity-field mode from the respective atomic transition, respectively.

interaction Hamiltonian (assuming $\hbar=1$) in the interaction picture is

$$H=g_1\hat{a}e^{i\Delta_1 t}|e\rangle\langle g|+g_2\hat{b}e^{i\Delta_2 t}|e\rangle\langle l|+H.c. \tag{1}$$

where g_1, g_2 are the atom-field coupling strength for the two transitions. $\Delta_1=w_{eg}-w_1$, $\Delta_2=w_{el}-w_2$ are the one-photon detuning between the atomic transition frequencies and the cavity modes frequencies, respectively.

Assume the initial photon occupation numbers of the two cavity-mode to be n_1, n_2, respectively. Atom is initially in the state $|g\rangle$, then the state evolution of the system is

$$|\psi(t)\rangle=a_1(t)|g\rangle|n_1,n_2\rangle+a_2(t)|e\rangle|n_1-1,n_2\rangle+a_3(t)|n_1-1,n_2+1\rangle \tag{2}$$

where $a_i(t)$ represent occupation amplitude of the corresponding vectors, $a_i(t)$ are complex numbers. The time evolution of this system is decided by the Schrödinger equation

$$i\frac{\partial|\psi\rangle}{\partial t}=H|\psi\rangle \tag{3}$$

then we can obtain

$$\begin{aligned}
i\frac{\partial a_1}{\partial t}&=g_1\sqrt{n_1}e^{-i\Delta_1 t}a_2\\
i\frac{\partial a_2}{\partial t}&=g_1\sqrt{n_1}e^{-i\Delta_2 t}a_1+g_2\sqrt{n_2+1}e^{-i\Delta_2 t}a_3\\
i\frac{\partial a_3}{\partial t}&=g_2\sqrt{n_2+1}e^{-i\Delta_2 t}a_2
\end{aligned} \tag{4}$$

we can use the following transformation in order to simplify the above equation

$$\begin{aligned}
P_1&=a_1\\
P_2&=a_2e^{-i\Delta_1 t}\\
P_3&=a_3e^{-i(\Delta_1-\Delta_2)t}
\end{aligned} \tag{5}$$

Next, we assume that the coupling strength could satisfy $g_1=g_2=g$, and we work in the case $\Delta_i\geq g$, there is no energy exchange between the cavity and the atomic system. On this occasion, the excited state $|e\rangle$ can be adiabatically eliminated [18]. Hence we can neglect time derivative of p_2. Let $\frac{\partial p_2}{\partial t}=0$, the equation (4) can reduce to

$$\begin{aligned}
\frac{\partial p_1}{\partial t}&=\frac{ig^2}{\Delta_1}[n_1p_1+\sqrt{n_1(n_2+1)}p_3]\\
\frac{\partial p_3}{\partial t}&=-i(\Delta_1-\Delta_2)p_3+\frac{ig^2}{\Delta_1}[\sqrt{n_1(n_2+1)}p_1+(n_2+1)p_3]
\end{aligned} \tag{6}$$

Using initial conditions $n_1=1$ and $n_2=0$, we can obtain the solution of equation (6)

$$\begin{aligned}
p_1&=e^{ict}\left\{\left[\cos wt+\frac{i}{2w}(\Delta_1-\Delta_2)\sin wt\right]p_1(0)+\frac{ig^2}{\Delta_1 w}\sin wtp_3(0)\right\}\\
p_3&=e^{ict}\left\{\left[\cos wt-\frac{i}{2w}(\Delta_1-\Delta_2)\sin wt\right]p_3(0)+\frac{ig^2}{\Delta_1 w}\sin wtp_1(0)\right\}
\end{aligned} \tag{7}$$

where

$$c=\frac{1}{2}\left[\frac{2g^2}{\Delta_1}-(\Delta_1-\Delta_2)\right]$$
$$w=\frac{1}{2}\left[(\Delta_1-\Delta_2)^2+\frac{4g^4}{\Delta_1^4}\right] \tag{8}$$

If we set the case

$$\Delta_1-\Delta_2=\frac{2g^2}{\Delta_1} \tag{9}$$

and choose interaction time $wt=\pi$, assuming the initial condition $p_1(0)[p_3(0)=1]$, then $p_1(t)[p_3(t)=-1]$. So we get the two-qubit gate as follows

$$\begin{aligned}
&|0\rangle_a|0\rangle_b|g\rangle\rightarrow|0\rangle_a|0\rangle_b|g\rangle\\
&|0\rangle_a|0\rangle_b|l\rangle\rightarrow|0\rangle_a|0\rangle_b|l\rangle\\
&|0\rangle_a|1\rangle_b|g\rangle\rightarrow|0\rangle_a|1\rangle_b|g\rangle\\
&|0\rangle_a|1\rangle_b|l\rangle\rightarrow|0\rangle_a|1\rangle_b|l\rangle
\end{aligned} \tag{10}$$

Therefore we can get a C-NOT gate where cavity field mode $\hat{b}$ is control qubit and the atomic qubit is used as target qubit in computational subsystem from equation (10).

$$C_N=H^{\dagger}C_PH \tag{11}$$

in which H is the Hadamard gate on the atom with $|g\rangle\rightarrow(|g\rangle-|l\rangle)/\sqrt{2}$, $|e\rangle=(|g\rangle+|l\rangle)/\sqrt{2}$, C_P is the C-PHASE operation in equation (10).

3 Two-qubit Deutsch-Jozsa algorithm

The Deutsch-Jozsa algorithm is a simple quantum algorithm in the quantum computation, aimed to distinguish between constant and balanced function on 2^n inputs. The Deutsch-Jozsa algorithm needs only one query to decide whether function is constant or balanced, while a classical algorithm will require $2^n/2+1$ queries at worst. In theory, the Deutesch-Jozsa algorithm can be briefly represented as follows. First, we prepared the query qubit in the perposition state $(|0\rangle_a+|1\rangle_a)/\sqrt{2}$ and the auxiliary working qubit is prepared in the state $(|0\rangle_b-|1\rangle_b)/\sqrt{2}$. So the state of the whole system is

$$|\phi_1\rangle=\frac{1}{2}(|0\rangle_a+|1\rangle_a)(|0\rangle_b-|1\rangle_b) \tag{12}$$

while the function $f(x)$ is characterized by the unitary mapping transformation U_f, i.e., $U_f:|x\rangle|y\rangle\rightarrow|x\rangle|y\oplus f(x)\rangle$ in which $\oplus$ indicates addition modulo 2. The unitary transformation on the system leads to

$$|\phi_2\rangle=\frac{1}{2}\left[(-1)^{f(0)}|0\rangle_a+(-1)^{f(1)}|1\rangle_a\right](|0\rangle_b-|1\rangle_b) \tag{13}$$

For U_f, there are four possible transformations: for $U_{f,1}$, $f(0)=f(1)=0$; for $U_{f,2}$, $f(0)=f(1)=1$; for $U_{f,3}$, $f(0)=1$, $f(1)=0$; for $U_{f,4}$, $f(0)=0$, $f(1)=1$. Then a Hardmard gate is performed on the query qubit, after this transformation, the state of the query qubit becomes $|f(0)\oplus f(1)\rangle$. If the function is constant, the state of query qubit becomes $|0\rangle_a$, otherwise it becomes $|1\rangle_a$. Thus a measurement on the query qubit can tell us if the function is constant or balanced (as is demonstrated in Fig. 2).

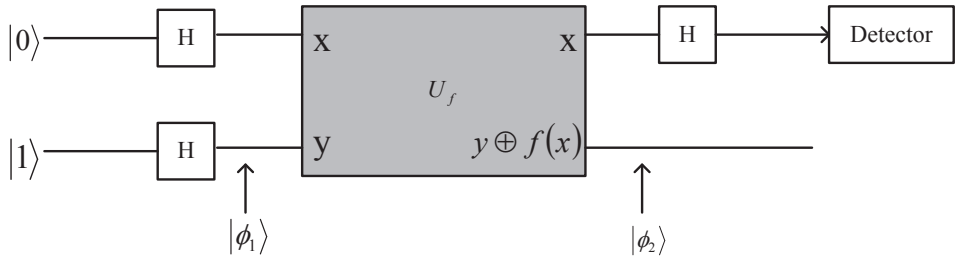

Figure 2: Quantum circuit for realizing the Deutsch-Jozsa algorithm.

In this part, we pay our attention on the experimental realization of the Deutsch-Jozsa algorithm in cavity QED. Now, we let the cavity mode serves as the query qubit and the level of atom plays as the auxiliary working qubit. Firstly we should prepare the cavity in the required state. Assume the cavity mode initilly is in the vacuum state and the first atom is in the state $(|g\rangle_1+|e\rangle_1)/\sqrt{2}$. In the interaction picture, the interaction Hamiltonian is

$$H_1 = ig(a\dagger S^- - aS^+) \tag{14}$$

in which S^+, S^- devote the raising and lowering operations for the atom, $a\dagger$, a are the creation and annihilation operation for the cavity mode, and g is coupling constant. After an interaction time t, the evolution of the state is given by

$$|e\rangle_1|0\rangle_c \rightarrow \cos gt|e\rangle_1|0\rangle_c + \sin gt|g\rangle_1|1\rangle_c \tag{15}$$

Hence, we choose $gt=\pi/2$, the cavity is in the state $(|0\rangle_c+|1\rangle_c)/\sqrt{2}$, with the atom left in the ground state. Then we send the second atom whose is in the state $(|g\rangle_2-|l\rangle_2)/\sqrt{2}$ pass the cavity. So the whole system consist of the second atom and the cavity and the state of this system is

$$|\varphi\rangle_{sys} = \frac{1}{2}(|0\rangle_c+|1\rangle_c)(|g\rangle_2-|l\rangle_2) \tag{16}$$

which can be rewritten as

$$|\varphi\rangle_1 = \frac{1}{2}(|0\rangle_c+|1\rangle_c)(|0\rangle_b-|1\rangle_b) \tag{17}$$

where $|0\rangle_b$ and $|1\rangle_b = |l\rangle$. In this condition , equation (17) has the same form with the equation (12). So we can make use of it to realize the Deutsch-Jozsa algorithm.

Secondly our goal is to reveal that the four transformations can be implemented in cavity QED.

In the case $f(0)=f(1)=0$, the atom is highly detuned with the cavity mode, which has no exchange between the atom and the cavity mode, and the state remains in the state $|\varphi\rangle_1$.

In the case $f(0)=f(1)=1$, the third atom is sended through the cavity. Choose the amplitudes and frequencies of the cavity field appropriately to make sure that the state of the second atom remains during the interaction between the cavity-field and the third atom. The third atom is initially in the ground state, the atom interacts with the cavity mode now. For a time t' later, we can gain

$$|g\rangle_3|1\rangle_c \rightarrow \cos gt'|g\rangle_3|1\rangle_c - \sin gt'|e\rangle_3|0\rangle_c \tag{18}$$

With the choice $gt'=\frac{\pi}{2}$, leading to

$$|g\rangle_3|1\rangle_c \rightarrow |e\rangle_3|0\rangle_c \tag{19}$$

Then we use a π-Ramsey pulse to achieve the single-qubit transformation on the atom, we have

$$|g\rangle_3 \rightarrow -|e\rangle_3, \quad |e\rangle_3 \rightarrow |g\rangle_3 \tag{20}$$

Next we perform the atom interacts with the cavity mode by the interaction time rightly, we gain $|e\rangle_3|0\rangle_c \rightarrow |g\rangle_3|1\rangle_c$. After these procedure above, resulting in

$$|\varphi\rangle_2=\frac{1}{2}(-|0\rangle_c-|1\rangle_c)(|0\rangle_b-|1\rangle_b) \tag{21}$$

In the case $f(0)=0$, $f(1)=0$, we perform the C-NOT gate of equation (11), we can easily obtain

$$|\varphi\rangle_3=\frac{1}{2}(|0\rangle_c-|1\rangle_c)(|0\rangle_b-|1\rangle_b) \tag{22}$$

In the case $f(0)=0$, $f(1)=1$, in order to realize it , we first perform equation (19), then we perform equation (20), finally, we can perform $|e\rangle_3|0\rangle \rightarrow |g\rangle_3|1\rangle_c$. These lead to

$$|\varphi\rangle_4=\frac{1}{2}(-|0\rangle_c+|1\rangle_c)(|0\rangle_b-|1\rangle_b) \tag{23}$$

Apparently we have achieved the unitary transformation via the process above. Thirdly, we send a fourth atom with the initial state $|g\rangle_4$ through the cavity for a interaction time t'', we have

$$\begin{aligned} &|g\rangle_4|0\rangle_c \rightarrow |g\rangle_4|0\rangle_c \\ &|g\rangle_4|1\rangle_c \rightarrow \cos gt''|g\rangle_4|1\rangle_c - \sin gt''|e\rangle_4|0\rangle_c \end{aligned} \tag{24}$$

Here we choose $gt''=\frac{3\pi}{2}$.

In the end, we perform the Hadamard operation on the fourth atom. If the function is constant, the state of the fourth atom becomes $|g\rangle_4$, and if the function is balanced, the state of the fourth atom is $|e\rangle_4$. In this way, a single measurement of the state of the atom is adequacy to decide the properties of the function.

4 Discussion and conclusion

It is indispensible for us to give a brief discussion on the experimental feasibility of the proposed scheme. For the Rydberg with principal quantum 50 and 51, the radiative time is about $T_r = 3\times10^{-2}s$, and the coupling constant is $g = 2\pi\times25kHz$ [19,20]. The required atom-cavity-field interaction time are $\Delta_1\pi/\sqrt{2}g^2 \approx 1.414\times10^{-4}s$ (with the choice $\Delta_1 = 10g$), $\pi/2g = 1.0\times10^{-5}s$, and $3\pi/2g = 1.5\times10^{-5}s$, respectively. Then the time required to complete the whole procedure is on the order of $T \approx 1.6\times10^{-3}s$, much shorter than T_r. Meanwhile the efficient decay time of the cavity is about $3.0\times10^{-2}s$, longer than the required time . Therefore, based on cavity QED techniques presently available, our scheme might be realizable.

In conclusion, we have described a concise scheme to realize the C-NOT gate and the Deutsch-Jozsa algorithm in cavity QED. Owing to the extraordinary role of the C-NOT gate in the process of the implementation of the Deutsch-Jozsa, we have realized it at first. In addition, the required experimental techniques of our scheme are easily available, so our scheme may be realized. The experimental achievement of the Deutsch-Jozsa algorithm would conceive more deeply effect on quantum computation, and it inspires trying to understand the law of computation.

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