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Generation of three-atom W state via nonresonant Jaynes-Cummings model

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Abstract. A simple scheme is presented for generating three-atom W state via nonresonant Jaynes-Cummings model. In the proposed protocol, it is injected two two-level atom initially prepared in the excited state and ground state through the cavity prepared in the vacuum state. The third two-level atom is prepared in the ground state and three two-level atom are injected into the second cavity prepared in the vacuum state. The experiment of the proposed scheme is feasibile.

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Key words: two-level atom, cavity field, nonresonant interaction, W state

1 Introduction

Entanglement is not only the most striking feature of quantum mechanics, but also an important physical resource in quantum information processing (QIP), such as quantum teleportation, quantum dense coding, and quantum cryptography. Recently, multiparticle entanglement has attracted much interest due to its potential applications in QIP. In general, the more particles that can be entangled, the more clearly nonclassical effects are exhibited, and the more useful the states are for quantum applications [1-8]. Thus generation and manipulation of multipartite entangled states are very important tasks in QIP and have been attracting much attention. Dür et al have shown that there are two inequivalent classes of tripartite entanglement states, i.e., the W class and the GHZ class, under stochastic local operations and classical communications [9]. Zheng presented a scheme for the generation of the W state for trapped ions [10]. Zhong proposed an alternative scheme to generate four-photon W state via resonant cavity QED [11]. W type of entangled states have many interesting properties. Hence, the preparation of W state

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become a critical technique in QIP. In this paper, we propose a scheme for generation of three-atom W state via cavity QED. It is based on the nonresonnant Jaynes-Cummings model of two and three identical atoms simultaneously interacting with single-mode cavity field in the vacuum state. The proposed scheme is simple and feasible because we only employ three two-level atoms and two single-mode cavities, and is easier to be realized in experiment.

2 Generation of W state via nonresonant Jaynes-Cummings model

We consider that N identical two-level atoms are simultaneously coupled to the same mode of cavity field, in which we are considering dipole-dipole coupling between atoms. The Hamiltonian of the system can be written as ($\hbar=1$)

$$H=\omega_0 a^+a+\sum_{i=1}^{N}\omega_i S_i^Z+\sum_{i=1}^{N}g_i(a^+S_i^-+aS_i^+)+\sum_{i\neq j}\Omega_{ij}S_i^+S_j^-, \tag{1}$$

where a and a^+ is the annihilation and creation operators for the cavity field, S_i^Z, S_i^- and S_i^+ are the inverssion, lowering and raising operators for the i-th atom,respectively, ω_i is the i-th atomic transition frequenccy and ω_0 is the field frequency, g_i is the i-th atom-field coupling constant and Ω_{ij} is the strength of the dipole-dipole interaction between atom i and j. We assume that each atomic transition frequency is highly detuned from cavity mode, i.e., the detuning $|\Delta_i|=|\omega_i-\omega_0|\gg g_i$. For simplicity, N two-level atoms are assumed to be identical so that $g_1=g_2=\cdots=g$, $\omega_1=\omega_2=\cdots=\omega$, $\Delta_1=\Delta_2=\cdots=\Delta$, and $\Omega_{ij}=\Omega$. The effective Hamiltonian for such a system in the interaction picture is given by [12, 13]

$$H_{eff}=\sum_{i=1}^{N}\frac{2g^2}{\Delta}a^+aS_i^Z+\Omega\sum_{i\neq j}S_i^+S_j^- \tag{2}$$

Assume that atoms is initially in $|\psi(0)\rangle_a$ and the cavity is initially in $|\psi(0)\rangle_f$, then the initial state of the atoms-cavity field system is

$$|\psi(0)\rangle_s=|\psi(0)\rangle_a\otimes|\psi(0)\rangle_f \tag{3}$$

The evolution of the whole system is

$$|\psi(t)\rangle=\exp(-iH_{eff}t)|\psi(0)\rangle_s \tag{4}$$

For $N=3$ in Eq. (2), we assume that two identical single-mode cavity fields are initially prepared in the vacuum state $|0\rangle$, atom 3 is initially in $|g\rangle_3$. Atoms 1 and 2 are prepared in the state $|eg\rangle_{12}$, which are sent through the first cavity simultaneously. After interaction time τ_1, the state of system becomes [13]

$$|\psi(\tau_1)\rangle_{12s}=[\cos(\Omega\tau_1)|eg\rangle_{12}-i\sin(\Omega\tau_1)|ge\rangle_{12}]\otimes|0\rangle_f \tag{5}$$

We choose $\Omega\tau_1=\frac{\pi}{4}$ and a classical field is applied to rotate atom 1 along the Z-axis by the angle $\frac{\pi}{4}$, such a tranformation is

$$U=\exp[-i\frac{\pi}{4}(|e\rangle\langle e|-|g\rangle\langle g|)] \tag{6}$$

In Eq. (5), the state of atoms becomes

$$|\psi(\tau_1)\rangle'_{12s}=\frac{1}{\sqrt{2}}(|eg\rangle_{12}+|ge\rangle_{12}) \tag{7}$$

Now, we simultaneously send atoms 1, 2 and 3 into the second cavity field initially in the vacuum state $|0\rangle$, the state of atoms is $|\psi(\tau_1)\rangle'_{12s}|g\rangle_3$. After interaction time τ_2, the exits the cavity and thus the state of system becomes

$$|\Psi\rangle_{123s}\rightarrow\frac{1}{\sqrt{2}}(\cos\theta_2-\frac{i}{3}\sin\theta_2)(|eg\rangle_{12}+|ge\rangle_{12})|g\rangle_3-\frac{2\sqrt{2}}{3}\sin\theta_2|gg\rangle_{12}|e\rangle_3 \tag{8}$$

where $\theta_2=\frac{3\Omega\tau_2}{2}$, we set $\frac{1}{2}(\cos^2\theta_2+\frac{1}{9}\sin^2\theta_2)=\frac{8}{9}\sin^2\theta_2$, i.e., $ctg\theta_2=\frac{\sqrt{15}}{3}$, we can obtain following W state

$$|\Psi\rangle'_{123s}\rightarrow\frac{1}{\sqrt{3}}[\exp(i\varphi)(|geg\rangle_{123}+|gge\rangle_{123})+|egg\rangle_{123}] \tag{9}$$

where $ctg\varphi=-\frac{3\sqrt{2}}{2}ctg\theta_2$.

3 Conclusions

In summary, we have proposed a simple method for generation of 3-atom W state via the nonresonant Jaynes-Cummings model. It is worthwhile to note that this interaction does not give rise to any quantum noise due to the fact that the cavity field is still in the vacuum state $|0\rangle$. Our scheme is easier to realize in experiment, we can choose Rtheydberg atoms with principal quantum number 50 and 51 [14]. Our scheme only requires two single-mode cavities in the vacuum state and a classical field.

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