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Scheme for generating W states via distant cavities

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Abstract. A scheme for generation of W state via distant cavities is presented. Employing resonant interactions between atoms and cavities, choosing different initial states, we can obtain non-maximally and maximally entangled states. In addition, our scheme could be easily generalized to generate N-atom W state. In contrast to the original scheme, our scheme is insensitive to the atomic spontaneous emission and cavity decay, it made the schemes more easily realize on experiments.

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Key words: W state, distant cavity, beam-splitter

1 Introduction

Quantum entanglement, one of the most fascinating features of quantum mechanics, not only provides an important tool for distinguishing the quantum mechanics from the classical physics, but also gives the possibility to test quantum mechanics against a local hidden variable theory [1-3]. In addition, quantum entanglement plays a key role in quantum information processing. For example, control a small amount qubits has been achieved in cavity QED, ion traps, etc. In order to achieve large-scale quantum information processing, we must find good methods for expand simple physical system. At the same time, for communication purposes to bring about the communication. Quantum information has to be shared among separated quantum nodes. To bring about the communication, stationary qubits are entangled by using photons is our best bet [4-7]. There are many protocols for remote entangling operations and probabilistic two-qubits gates were realized [8-16] The Barrett-Kok and zheng shi-biao scheme is particularly promising, since it is fully scalable and robust against experimental imperfections [12,13]. C. W. Chou and D. L. Moehring has proposed an experimental scheme of the remote entangling operations (or probabilistic two-qubit gates) between separated qubits have also

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been done in both atomic ensembles [17] and trapped single atoms [18-20]. They are important ingredients for fault-tolerant distributed quantum computation [12, 21-23]. There are a lot of classes of multipartite entanglement, for example, GHZ (Greenberger-Horne-Zeilinger) states [24], cluster states [25], and W states [26]. The W state has a strong property that even any particle of the state has been discarded the other particles also be in entangled state [27-29]. The W state can be used in quantum key distribution, teleportation, leader election, and information splitting [30-32]. The decoherence existed in W states can be counteracted in purification scheme [33]. The preparation of the W states by using optics has been discussed so far extensively both theoretically and experimentally [34-41]. In addition, the W state has been prepared in other systems, such as cavity QED or ion traps [42-46].

In this paper, we present an alternative scheme for generating a special W state in cavity QED. In contrast to the original scheme, our scheme is insensitive to the atomic spontaneous emission and cavity decay. In addition, the special W state could be used for perfect teleportation and dense coding with a probability of 100% in Ref. [47]. Furthermore, our scheme could generate maximally three-atom W state, and it can be scalable to N-atom W state.

The paper is organized as follows. In Section 2, we propose a method for generating a special W state via three distant cavities. Meanwhile, the N-atom W state can also be realized. In Section 3, we propose a scheme for generating maximally entangled state. Finally, we give a summary in Section 4.

2 Generation of W state through distant cavities

The atoms have one excited state $|e\rangle$ and two ground states $|g\rangle$ and $|f\rangle$, as shown in Fig. 1. The transition $|e\rangle \to |g\rangle$ is resonantly coupled to the cavity mode. The transition $|e\rangle \to |f\rangle$ is dipole forbidden. The setup is shown in Fig. 2. Three distant atoms are trapped in three separate single-mode optical cavities, respectively. Photons leaking out of the cavities are mixed on two beam splitters, which destroy which-path information. Then the photons are detected by two photon detectors. We assume here that the cavities are one sided so that the only photon leakage occurs through the sides of the cavities facing the beam splitter like in Ref. [13].

In our scheme each atom is first entangled with the corresponding cavity mode via resonant interaction. The detection of one photon leaking out of the cavities and passing through two beam splitters corresponds to the measurement of the joint state of the three cavities; it collapses the three distant atoms to an entangled state.

Assume that the atom is initially in the state

$$|\phi_j\rangle = \frac{1}{\sqrt{2}}(|g_j\rangle + |f_j\rangle) \tag{1}$$

The three cavities are initially in the vacuum state $|0\rangle$. The first step is the transfer of one photon to the cavity through a half-cycle of the vacuum Rabi oscillation of the

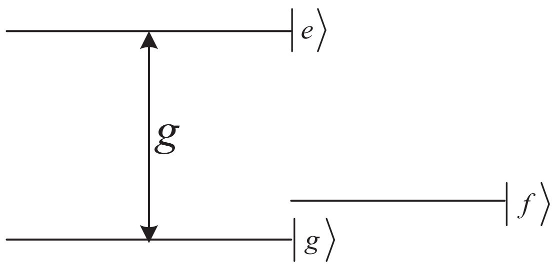

Figure 1: The level configuration of the atoms. The transition $|g\rangle \to |e\rangle$ is resonantly coupled to the cavity mode and the additional ground state $|f\rangle$ is not coupled to the cavity mode.

atom-cavity system. The vacuum Rabi half-cycle is initiated by exciting the state $|g_j\rangle$ to $|e_j\rangle$. This leads to

$$|\phi_j\rangle = \frac{1}{\sqrt{2}}(|e_j\rangle + |f_j\rangle) \tag{2}$$

The emission or nonemission of a photon depends on whether the initial state is $|g_j\rangle$ or $|f_j\rangle$, providing the essential tool for generating entanglement between the atom and cavity field. Atom j is initially in a ground state and the entanglement between atom j and cavity j is obtained after a Rabi quater-cycle. The aim of using the initial state $|\phi_j\rangle'$ is to let the basis states $|f_1\rangle|g_2\rangle|g_3\rangle$, $|g_1\rangle|f_2\rangle|g_3\rangle$ and $|g_1\rangle|g_2\rangle|f_3\rangle$ be equally damped, as shown below.

In the interaction picture, the Hamiltonian in each cavity is

$$H_j = g(a_j s_j^+ + a_j^+ s_j^-). \tag{3}$$

Whereas $s_j^+ = |e_j\rangle\langle g_j|$ and $s_j^- = |g_j\rangle\langle e_j|$ are the raising and lowering operators of the j-th $(j=1,2,3)$ atom, a_j^+ and a_j are the creation and annihilation operators of the jth cavity mode, and g is the atom-cavity coupling strength. The Hamiltonian of Eq. (3) does not include the effects of the atomic spontaneous emission and cavity decay. Under the condition that no photon is detected either by the atomic spontaneous emission or by the leakage through the cavity mirror, the evolution of the system is governed by the conditional Hamiltonian

$$H_{con,j} = H_j - \frac{i\kappa}{2} a_j^+ a_j - \frac{i\Gamma}{2} |e_j\rangle\langle_j e| \tag{4}$$

where κ is the cavity decay rate and Γ is the atomic spontaneous emission rate. The time evolution for the state $|e_j\rangle|0_j\rangle$ is

$$|e_j\rangle|0_j\rangle \to e^{-(\kappa+\Gamma)t/4}\Big\{\Big[\cos(\beta t) + \frac{\kappa-\Gamma}{4}\sin(\beta t)\Big]|e_j\rangle|0_j\rangle - i\frac{g}{\beta}\sin(\beta t)|g_j\rangle|1_j\rangle\Big\} \tag{5}$$

whereas $\beta = \sqrt{g^2 - (\kappa-\Gamma)^2/16}$.

After an interaction time t_1 given by $\tan(\beta t_1) = 4\beta/(\Gamma - \kappa)$, the whole system evolves to

$$
\begin{aligned}
|\Psi_1\rangle = \frac{1}{2\sqrt{2}} & \Big\{ |f_1\rangle|0_1\rangle - i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|g_1\rangle|1_1\rangle \Big\} \\
& \otimes \Big\{ |f_2\rangle|0_2\rangle - i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|g_2\rangle|1_2\rangle \Big\} \\
& \otimes \Big\{ |f_3\rangle|0_3\rangle - i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|g_3\rangle|1_3\rangle \Big\}.
\end{aligned} \tag{6}
$$

Now we perform the transformation

$$|f_j\rangle \to |g_j\rangle, \quad |g_j\rangle \to -|f_j\rangle.$$

This leads to

$$
\begin{aligned}
|\Psi_2\rangle = \frac{1}{2\sqrt{2}} & \Big\{ |g_1\rangle|0_1\rangle + i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|f_1\rangle|1_1\rangle \Big\} \\
& \otimes \Big\{ |g_2\rangle|0_2\rangle + i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|f_2\rangle|1_2\rangle \Big\} \\
& \otimes \Big\{ |g_3\rangle|0_3\rangle + i\frac{g}{\beta} e^{-(\kappa+\Gamma)t_1/4} \sin(\beta t_1)|f_3\rangle|1_3\rangle \Big\}.
\end{aligned} \tag{7}
$$

After the transformation the atom-cavity interaction is frozen since $H_j|\Psi_2\rangle = 0$. Now we wait for the photon detectors to click. The registering of a click at one of the photon detectors corresponds to the action of the jump operators $\frac{1}{2}[\sqrt{2}a_3 \pm (a_1 + a_2)]$ on the state $|\Psi_2\rangle$, where "+" corresponds to the detection of photon at the photon detector D_+, while

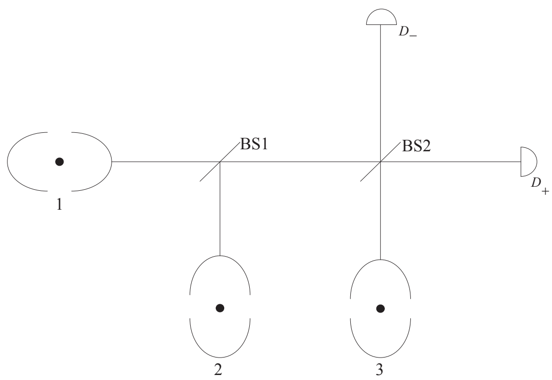

Figure 2: The experimental setup. Three distant atoms ate trapped in separate cavities. Photos leak through the sides of the cavities facing the beam-splitter (50:50, BS) and then are detected by the photon detectors D_+ and D_-.

"-" corresponds to the detection of photon at the photon detector D_-. The system is then projected to

$$\begin{aligned}|\Psi_3\rangle =& i\frac{g}{4\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle\\ &\pm i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &\pm i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &-\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1}\sin^2(\beta t_1)|g_1\rangle|f_2\rangle|f_3\rangle|0_1\rangle(\sqrt{2}|1_2\rangle|0_3\rangle\pm|0_2\rangle|1_3\rangle)\\ &-\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1}\sin^2(\beta t_1)|f_1\rangle|g_2\rangle|f_3\rangle|0_2\rangle(\sqrt{2}|1_1\rangle|0_3\rangle\pm|0_1\rangle|1_3\rangle)\\ &\mp\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1}\sin^2(\beta t_1)|f_1\rangle|f_2\rangle|g_3\rangle|0_1\rangle(|0_1\rangle|1_2\rangle+|1_1\rangle|0_2\rangle)\\ &+i\frac{g^3}{4\sqrt{2}\beta^3}e^{-3(\kappa+\Gamma)t_1/4-3\kappa\tau_1/2}\sin^3(\beta t_1)|f_1\rangle|f_2\rangle|f_3\rangle[\sqrt{2}|1_1\rangle|1_2\rangle|0_3\rangle\\ &\quad\pm(|0_1\rangle|1_2\rangle|1_3\rangle+|1_1\rangle|0_2\rangle|1_3\rangle)].\end{aligned}\tag{8}$$

Here τ_1 is the waiting time. In comparison with the scheme of Ref. [5], after the detection of the photon atom 1 is entangled with atom 2, 3and the cavity modes, and the three basis states $|g_1\rangle|g_2\rangle|f_3\rangle$, $|g_1\rangle|f_2\rangle|g_3\rangle$ and $|f_1\rangle|g_2\rangle|g_3\rangle$ are equally damped. Then we wait for another time τ_2. Suppose that no photon is detected during this period. Due to the cavity decay the system evolves to

$$\begin{aligned}|\Psi_4\rangle =& i\frac{g}{4\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle\\ &\pm i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &\pm i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &-\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|g_1\rangle|f_2\rangle|f_3\rangle|0_1\rangle(\sqrt{2}|1_2\rangle|0_3\rangle\pm|0_2\rangle|1_3\rangle)\\ &-\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|f_1\rangle|g_2\rangle|f_3\rangle|0_2\rangle(\sqrt{2}|1_1\rangle|0_3\rangle\pm|0_1\rangle|1_3\rangle)\\ &\mp\frac{g^2}{4\sqrt{2}\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|f_1\rangle|f_2\rangle|g_3\rangle|0_1\rangle(|0_1\rangle|1_2\rangle+|1_1\rangle|0_2\rangle)\end{aligned}$$

$$+i\frac{g^3}{4\sqrt{2}\beta^3}e^{-3(\kappa+\Gamma)t_1/4-3\kappa\tau_1/2-\kappa\tau_2}\sin^3(\beta t_1)|f_1\rangle|f_2\rangle|f_3\rangle[\sqrt{2}|1_1\rangle|1_2\rangle|0_3\rangle \pm(|0_1\rangle|1_2\rangle|1_3\rangle+|1_1\rangle|0_2\rangle|1_3\rangle)]. \tag{9}$$

If τ_2 is long enough so that $e^{-\kappa\tau_2/2}\ll 1$ the last four terms of $|\Psi_4\rangle$ can be discarded. This leads to

$$|\Psi_5\rangle=i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)[\sqrt{2}|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle \pm(|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle+|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle)]. \tag{10}$$

Three cavity modes left in the vacuum state $|0_1\rangle|0_2\rangle|0_3\rangle$. We obtain

$$|\Psi_6\rangle=N[\sqrt{2}|g_1\rangle|g_2\rangle|f_3\rangle\pm(|g_1\rangle|f_2\rangle|g_3\rangle+|f_1\rangle|g_2\rangle|g_3\rangle)], \tag{11}$$

where $N=i\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)$. We sequentially perform the following transformations on atom j, $|f_j\rangle\to|g_j\rangle$, $|g_j\rangle\to|e_j\rangle$ after atoms were taken out from cavity.

This leads to

$$|\Psi_7\rangle=N[\sqrt{2}|e_1\rangle|e_2\rangle|g_3\rangle\pm(|e_1\rangle|g_2\rangle|e_3\rangle+|g_1\rangle|e_2\rangle|e_3\rangle)]. \tag{12}$$

In the following, we discuss that the deviation from the ideal case shall be considered with the time difference Δt_1, Δt_2. Then the quantum state

$$|\Psi_5\rangle'=i\frac{g}{4\sqrt{2}\beta}\Big[e^{-(\kappa+\Gamma)(t_1+\Delta t_2+\tau_1)/4-\kappa\tau_1/2}\sin(\beta t_1+\beta\Delta t_2+\beta\tau_1)\sqrt{2}|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle \pm(e^{-(\kappa+\Gamma)(t_1+\Delta t_2+\tau_1)/4-\kappa\tau_1/2}\sin(\beta t_1+\beta\Delta t_1+\beta\tau_1)|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle +e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\Big]$$

will be generated. The difference between state $|\Psi_5\rangle$ and $|\Psi_5\rangle'$ state can be characterized in terms of fidelity $F=|\langle\Psi_5||\Psi_5\rangle'|^2$.

$$F=\frac{g^2}{32\beta^2}e^{-(\kappa+\Gamma)t_1-2\kappa\tau_1}\sin^2(\beta t_1)\Big[2e^{-(\kappa+\Gamma)(\Delta t_2+\tau_1)/4}\sin(\beta t_1+\beta\Delta t_2+\beta\tau_1) +e^{-(\kappa+\Gamma)(\Delta t_2+\tau_1)/4}\sin(\beta t_1+\beta\Delta t_2+\beta\tau_1)+\sin(\beta t_1)\Big]^2.$$

If $\Delta t_1\ll 1$, $\Delta t_2\ll 1$ holds, we have $F\approx 1$. In this case the operation is only slightly affected.

The success probability is

$$P=\frac{A}{2(1+B)}e^{\frac{-(\kappa+\Gamma)\arcsin(\frac{B}{1+B})^{\frac{1}{2}}}{\beta}},$$

where $A = g^2(\Gamma-\kappa)^2$, $B = [16g^2-(\kappa-\Gamma)^2](\Gamma-\kappa)^2$, $\beta = [g^2-((\kappa-\Gamma)^2)/16]^{\frac{1}{2}}$.

Through a unitary transformation operator (corresponding to the detection of photon at the photon detector D_-), we obtain

$$|\Psi_8\rangle = N(|e_1\rangle|g_2\rangle|e_3\rangle - i|g_1\rangle|e_2\rangle|e_3\rangle - i\sqrt{2}|e_1\rangle|e_2\rangle|g_3\rangle. \tag{13}$$

An experimental feasible protocol for realizing dense coding and teleportation by use $|\Psi_8\rangle$ has been implemented in Ref. [47]. What is more, the probability of implementing the quantum dense coding using the state is 1.

Next we describe how to generating an N-atom entangled state. Assume that each atom is initially in the state

$$|\phi_j\rangle'' = \frac{1}{\sqrt{2}}(|g_j\rangle + |f_j\rangle). \tag{14}$$

The N cavities are initially in the vacuum state $|0\rangle$.

Reiterate the above process from the Eq. (1) to the Eq. (5) operations. After an interaction time t_1 given by $\tan(\beta t_1) = 4\beta/(\Gamma-\kappa)$ the whole system evolves to

$$\begin{aligned}
|\Psi_1\rangle' = &\frac{1}{2\sqrt{2}}\Big\{|f_1\rangle|0_1\rangle - i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_1\rangle|1_1\rangle\Big\} \\
&\otimes\Big\{|f_2\rangle|0_2\rangle - i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_2\rangle|1_2\rangle\Big\} \\
&\otimes\Big\{|f_3\rangle|0_3\rangle - i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_3\rangle|1_3\rangle\Big\} \\
&\vdots \\
&\otimes\Big\{|f_n\rangle|0_n\rangle - i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_n\rangle|1_n\rangle\Big\}.
\end{aligned} \tag{15}$$

Now we perform the transformation

$$|f_j\rangle \to |g_j\rangle, \; |g_j\rangle \to -|f_j\rangle.$$

This leads to

$$\begin{aligned}
|\Psi_2\rangle' = &\frac{1}{2\sqrt{2}}\Big\{|g_1\rangle|0_1\rangle + i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|f_1\rangle|1_1\rangle\Big\} \\
&\otimes\Big\{|g_2\rangle|0_2\rangle + i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|f_2\rangle|1_2\rangle\Big\} \\
&\otimes\Big\{|g_3\rangle|0_3\rangle + i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|f_3\rangle|1_3\rangle\Big\} \\
&\vdots \\
&\otimes\Big\{|g_n\rangle|0_n\rangle + i\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|f_n\rangle|1_n\rangle\Big\}.
\end{aligned} \tag{16}$$

After the transformation the atom-cavity interaction is frozen since $H_j|\Psi_2\rangle' = 0$. Now we wait for the photon detectors to click. The registering of a click at one of the photon detectors corresponds to the action of the jump operators $\frac{1}{2}[\sum_{i\geq 3}^{n} 2^{i-2/2}a_i \pm (a_1+a_2)]$ on the state $|\Psi_2\rangle'$, where "+" corresponds to the detection of photon at the photon detector D_+, while "-" corresponds to the detection of photon at the photon detector D_-. The system is then projected to Reiterate these operations from Eq. (8) to Eq. (10) we can obtain N-atoms W state.

$$|\Psi_7\rangle' = N\Big[\sum_{i\geq 3}^{n} 2^{i-2/2}|e_1\rangle\cdots|e_{i-1}\rangle|g_i\rangle|e_{i+1}\rangle\cdots|e_n\rangle \pm (|e_1\rangle|g_2\rangle|e_3\rangle\cdots|e_n\rangle + |g_1\rangle|e_2\rangle|e_3\rangle\cdots|e_n\rangle)\Big]. \tag{17}$$

The success probability is

$$P = \frac{A'}{2(1+B)} e^{\frac{-(\kappa+\Gamma)\arcsin(\frac{B}{1+B})^{\frac{1}{2}}}{\beta}},$$

where $N \geq 3$, $A' = g^2(\Gamma-\kappa)^2/2^{\frac{N-3}{2}}$, $B = [16g^2-(\kappa-\Gamma)^2](\Gamma-\kappa)^2$, $\beta = [g^2-((\kappa-\Gamma)^2)/16]^{\frac{1}{2}}$.

3 Generating a maximally entangled W state from non-maximally entangled states

We assume in Sec. II $|\phi_j\rangle \to |\varphi_j\rangle$ $(j=1,2)$ and $|\phi_j\rangle \to |\varphi_3\rangle$,

$$|\varphi_j\rangle = a|g_j\rangle + b|f_j\rangle \tag{18}$$

where $j=1,2$, with a,b being unknown coefficients, $a^2+b^2=1$,

$$|\varphi_3\rangle = c|g_3\rangle + d|f_3\rangle \tag{19}$$

with c,d being unknown coefficients, $c^2+b^2=1$.

Reiterate the operations from the Eq. (1) to Eq. (5). After an interaction time t_1 the whole system evolves to

$$\begin{aligned}|\Psi_1\rangle'' = &\Big\{a|f_1\rangle|0_1\rangle - ib\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_1\rangle|1_1\rangle\Big\}\\ &\otimes\Big\{a|f_2\rangle|0_2\rangle - ib\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_2\rangle|1_2\rangle\Big\}\\ &\otimes\Big\{c|f_3\rangle|0_3\rangle - id\frac{g}{\beta}e^{-(\kappa+\Gamma)t_1/4}\sin(\beta t_1)|g_3\rangle|1_3\rangle\Big\}.\end{aligned} \tag{20}$$

Reiterate the operations from the Eq. (6) to Eq. (8), this leads to

$$\begin{aligned}|\Psi_4\rangle'' =& ia^2d\frac{g}{\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle\\ &\pm iabc\frac{g}{2\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &\pm iabc\frac{g}{2\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &-abd\frac{g^2}{2\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|g_1\rangle|f_2\rangle|f_3\rangle|0_1\rangle(\sqrt{2}|1_2\rangle|0_3\rangle\pm 0_2\rangle|1_3\rangle)\\ &-abd\frac{g^2}{2\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|f_1\rangle|g_2\rangle|f_3\rangle|0_2\rangle(\sqrt{2}|1_1\rangle|0_3\rangle\pm 0_1\rangle|1_3\rangle)\\ &\mp b^2c\frac{g^2}{2\beta^2}e^{-(\kappa+\Gamma)t_1/2-\kappa\tau_1-\kappa\tau_2/2}\sin^2(\beta t_1)|f_1\rangle|f_2\rangle|g_3\rangle|0_1\rangle(|0_1\rangle|1_2\rangle+1_1\rangle|0_2\rangle)\\ &+ib^2d\frac{g^3}{2\beta^3}e^{-3(\kappa+\Gamma)t_1/4-3\kappa\tau_1/2-\kappa\tau_2}\sin^3(\beta t_1)|f_1\rangle|f_2\rangle|f_3\rangle\Big[\sqrt{2}|1_1\rangle|1_2\rangle|0_3\rangle\\ &\quad\pm(|0_1\rangle|1_2\rangle|1_3\rangle+|1_1\rangle|0_2\rangle|1_3\rangle)\Big]\end{aligned} \tag{21}$$

If τ_2 is long enough so that $e^{-\kappa\tau_2/2}\ll 1$ the last four terms of $|\Psi_4\rangle''$ can be discarded. This leads to

$$\begin{aligned}|\Psi_5\rangle'' =& ia^2d\frac{g}{\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|f_3\rangle|0_3\rangle\\ &\pm iabc\frac{g}{2\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|g_1\rangle|0_1\rangle|f_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\\ &\pm iabc\frac{g}{2\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)|f_1\rangle|0_1\rangle|g_2\rangle|0_2\rangle|g_3\rangle|0_3\rangle\end{aligned} \tag{22}$$

Three cavity modes left in the vacuum state $|0_1\rangle|0_2\rangle|0_3\rangle$. If unknown coefficients a, b, c, d satisfy $ad=\frac{1}{\sqrt{2}}bc$, this leads to $|\Psi_5\rangle''\rightarrow|\Psi_6\rangle''$.

$$|\Psi_6\rangle''=N'\Big[|g_1\rangle|g_2\rangle|f_3\rangle\pm(|g_1\rangle|f_2\rangle|g_3\rangle+|f_1\rangle|g_2\rangle|g_3\rangle)\Big], \tag{23}$$

where $N'=iabc\frac{g}{4\sqrt{2}\beta}e^{-(\kappa+\Gamma)t_1/4-\kappa\tau_1/2}\sin(\beta t_1)$.

The success probability is

$$P=\frac{A''}{2(1+B)}e^{\frac{-(\kappa+\Gamma)\arcsin(\frac{B}{1+B})^{\frac{1}{2}}}{\beta}},$$

where $A''=a^2b^2c^2g^2(\Gamma-\kappa)^2$, $B=[16g^2-(\kappa-\Gamma)^2](\Gamma-\kappa)^2$, $\beta=[g^2-((\kappa-\Gamma)^2)/16]^{\frac{1}{2}}$.

Then we sequentially perform the following transformations on atom j, $|f_j\rangle \to |g_j\rangle$, $|g_j\rangle \to |e_j\rangle$. We can obtain a maximally W state,

$$|\Psi_7\rangle'' = N'\Big[|e_1\rangle|e_2\rangle|g_3\rangle \pm (|e_1\rangle|g_2\rangle|e_3\rangle + |g_1\rangle|e_2\rangle|e_3\rangle)\Big] \tag{24}$$

Now we discuss the feasibility of the present scheme. The success probability of our scheme increases as the needed interaction time t_1 decreases. Due to the imperfection of the photon detectors, there is a probability that two photons or three photons have leaked out of the cavities but only one photon is detected during the interaction detection, which leads to the state $|f_1\rangle|f_2\rangle|g_3\rangle|0_1\rangle|0_2\rangle|0_3\rangle$, $|f_1\rangle|g_2\rangle|f_3\rangle|0_1\rangle|0_2\rangle|0_3\rangle$, $|g_1\rangle|f_2\rangle|f_3\rangle|0_1\rangle|0_2\rangle|0_3\rangle$ or $|f_1\rangle|f_2\rangle|f_3\rangle|0_1\rangle|0_2\rangle|0_3\rangle$. The scheme is conditional upon the detection of emitted photons. If one of the emissions is not detected, the scheme fails and the procedure restarts. Set the detection efficiency to be η. Then the success probability is $P' = \eta^2 P$.

In order to perform the transformation in Eq. (8) we use a pair of off-resonant classical fields with the same Rabi frequency Ω to drive the transitions $|g_j\rangle \to |h_j\rangle$ and $|f_j\rangle \to |h_j\rangle$, where $|h_j\rangle$ is an auxiliary excited state. The two classical fields are detuned from the respective transitions by the same amount δ. In the case that the detuning δ is much larger than the Rabi frequency Ω the upper level $|h_j\rangle$ can be adiabatically eliminated and the two classical fields just induce the Raman transition between the states $|g_j\rangle$ and $|f_j\rangle$ [48]. The Raman coupling strength is $\lambda = \Omega^2/\delta$. The time needed to perform the required transformation is $\pi/2\lambda$. Under the condition $\lambda \gg g$, the atom-cavity interaction can be neglected during this transformation. Set $\Omega = 3\times 10^2 g$ and $\delta = 10\Omega$. During this transformation the probability that each atom exchanges an excitation with the cavity mode is on the order of $(g\pi/2\lambda)^2 \approx 2.7\times 10^{-3}$.

The required atomic level configuration can be achieved in Cs. The hyperfine levels $|F=4,\ m=-1\rangle$ and $|F=4,\ m=0\rangle$ of $6S_{1/2}$ can act as the ground states $|g\rangle$ and $|f\rangle$, respectively, while the hyperfine levels $|F'=5,\ m'=0\rangle$ and $|F'=5,\ m'=-1\rangle$ of $5^2P_{3/2}$ can act as the excited states $|e\rangle$ and $|h\rangle$, respectively. In a recent cavity QED experiment with Cs atoms trapped in an optical cavity, the corresponding atom-cavity coupling strength is $g = 2\pi\times 34$ MHz [49]. The decay rates for the atomic excited states and the cavity mode are $\Gamma = 2\pi\times 2.6$ MHz and $\kappa = 2\pi\times 4.1$ MHz, respectively. The required interaction time t_1 is about $7.4\times 10^{-3}\mu s$. The waiting times τ_1 and τ_2 are on the order of $2/\kappa \approx 7.8\times 10^{-2}\mu s$ and $20/\kappa \approx 7.8\times 10^{-1}\mu s$, respectively. The total time needed to complete the entanglement state is on the order of $0.86\mu s$. Set $\delta t_1 = 0.05 t_1$ and $\eta = 0.6$. The present scheme works in the Lamb-Dicke regime, i.e., the spatial extension of the atomic wave function should be much smaller than the wavelength of the light fields. In a recent experiment [50] the localization to the Lamb-Dicke limit of the axial motion was demonstrated for a single atom trapped in an optical cavity.

4 Conclusions

We have proposed a simple scheme for the generation of three-atom maximally W state, N-atom W state and a special three-atom W state in the paper. Meanwhile, in Ref. [47], Agrawal and Pati proved that $|\Psi_8\rangle$ could be used for perfect teleportation and dense coding with a probability of 100%. The scheme involves uses resonant atoms with an additional ground state not coupled to the cavity field. In addition, our scheme is insensitive to the atomic spontaneous emission and cavity decay, it made the schemes more easily realize on experiments.

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