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Muon's anomalous magnetic moment effects on laser assisted Coulomb scattering process

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Abstract. Laser assisted Coulomb scattering by relativistic electron and heavy electron (muon) is studied by using Salamin waves [3] in the Weak Field Approximation (WFA). Both electron and muon are described by the Dirac equation, with the anomalous magnetic moment effects fully included. The generalization of this paper to heavy electron (muon) gives interesting insights as to how the mass affects the magnitude of the differential cross sections. No significant difference in the muon's DCS with and without AMM effects was detected.

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1 Introduction

The muon anomalous magnetic moment is one of the most precisely measured quantities in particle physics. Recent high precision measurements at Brookhaven reveal a discrepancy by 3.2 standard deviations from the electroweak Standard Model which could be a hint for an unknown contribution from physics beyond the Standard Model. A muon looks like a copy of an electron, which at first sight is just much heavier $m_\mu/m_e \sim$ 206.7682838. However, unlike the electron, it is unstable and its lifetime is actually rather short. The first measurement of $(g_\mu - 2)/2$ was performed at Columbia in 1960 [4] with a result $a_\mu = 0.00122$ at a precision of about 5%. Soon later in 1961, at the CERN cyclotron

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(1958-1962), the first precision determination became available [5]. Surprisingly, nothing special was observed within the 0.4% level of accuracy of the experiment. It was the first real evidence that the muon was just a heavy electron. In particular, this meant that the muon is point-like and no extra short distance effects could be seen. This latter point of course is a matter of accuracy and the challenge to go further was evident.

The $E821$ experiment at Brookhaven National Laboratory (BNL) [7] studied the precession of muon and anti-muon in a constant external magnetic field as they circulated in a confining storage ring. The $E821$ experiment reported the following average value (from the July 2007 review by Particle Data Group)

$$a_\mu = \frac{g-2}{2} = 0.00116592080$$

Our aim in this paper is to shed some light on a difficult and recently addressed description of laser-assisted processes that incorporate the muon's anomaly. The process under study is the laser-assisted coulomb scattering collision of a Dirac-Volkov muon. We focus on the relativistic muonic dressing with the addition of the muon's anomaly. Some results are rather surprising, bearing in mind the small value of a_μ. In Sec. 2, we present the formalism as well as the coefficients that intervene in the expression of the DCS. In Sec. 3, we discuss the results we have obtained. Throughout this work, we use atomic units $\hbar = m_e = e$ and work with the metric tensor $g^{\mu\nu} = g_{\mu\nu} = diag(1,-1,-1,-1)$. In many equations of this paper, the Feynman 'slash notation' is used. For any $4-$vector A, $\not{A} = A^\mu \gamma_\mu = A^0\gamma_0 - \mathbf{A}.\gamma$ where the matrices γ are the well known Dirac matrices.

2 Theory

The second-order Dirac equation for a muon in the presence of an external electromagnetic field is given by

$$\left[\left(p - \frac{1}{c}A\right)^2 - m_\mu^2 c^2 - \frac{i}{2c}F_{\mu\nu}\sigma^{\mu\nu}\right]\psi(x) = 0 \tag{1}$$

where m_μ represents the mass of the muon, $\sigma^{\mu\nu} = \frac{1}{2}[\gamma^\mu, \gamma^\nu]$, γ^μ are the Dirac matrices and $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field tensor. A^μ is the four-vector potential. The plane wave solution of the second-order equation is known as the Volkov state [2]

$$\psi(x) = \left(1 + \frac{\not{k}\not{A}}{2c(kp)}\right)\frac{u(p,s)}{\sqrt{2VQ_0}}\exp\left[-i(qx) - i\int_0^{kx}\frac{(Ap)}{c(kp)}d\phi\right] \tag{2}$$

The second-order Dirac equation for a muon with anomalous magnetic moment (AMM) effects in the presence of an external electromagnetic field is given by

$$\left[\left(p - \frac{1}{c}A\right)^2 - m_\mu^2 c^2 - \frac{i}{2c}F_{\mu\nu}\sigma^{\mu\nu} + ia_\mu\left(\not{p} - \frac{\not{A}}{c} + m_\mu c\right)F_{\mu\nu}\sigma^{\mu\nu}\right]\psi(x) = 0 \tag{3}$$

where $a_\mu = \kappa_\mu/(4m_\mu)$, with κ_μ is the muon's anomaly. The term $F_{\mu\nu}\sigma^{\mu\nu}$ stems from the fact that the muon has a spin-one-half, and the term multiplying a_μ is due to their AMM. It is possible to rewrite the exact solution of Eq. (3) found by Salamin [3] as

$$\psi(x) = \exp[-(\alpha \not{k}\not{A} + \beta\not{k} + \delta\not{p}\not{k}\not{A})]\frac{u(p,s)}{\sqrt{2VQ_0}}e^{-i(qx)-i\int_0^{kx}\frac{(Ap)}{c(kp)}d\phi} \tag{4}$$

with

$$\alpha = \left(\frac{\kappa_\mu c}{2} - \frac{1}{c}\right)/2(k.p) \quad ; \quad \beta = \frac{\kappa_\mu A^2}{4m_\mu c(k.p)} \quad ; \quad \delta = \frac{\kappa_\mu}{4m_\mu(k.p)} \tag{5}$$

Finally, we take up the weak-field approximation (WFA). This is the case when the intensity of the laser field is small such that the resulting quiver energy E_q of the particle, defined as its average classical energy in an oscillating electric field, is comparable to its rest energy E_0. Retaining only terms of order one in the expansion of the first exponential in (4) leaves one with

$$\psi(x) = [1-(\alpha \not{k}\not{A} + \beta\not{k} + \delta\not{p}\not{k}\not{A})]\frac{u(p,s)}{\sqrt{2VQ_0}}\exp\left[-i(qx)-i\int_0^{kx}\frac{(Ap)}{c(kp)}d\phi\right] \tag{6}$$

Handling the interaction of a particle with an external electromagnetic field within the context of the minimal coupling scheme ignores altogether the fact that the particle possesses an anomalous magnetic moment. In other words, the Volkov state of a particle does not take into account the small, but maybe important, contribution to the particle's dynamics coming from its interaction with the field through this part of its total magnetic moment. For our general exact solution (4) of the non-minimally coupled Dim equation to be correct, it should necessarily yield the Volkov state when κ, the particle's anomaly, is set equal to zero. When this is done, equation (5) gives $\beta = \delta = 0$ and $\alpha = -1/2c(k.p)$. With this at hand, equation (6) reduces to equation (2).

We turn now to the calculation of the transition amplitude with the AMM. The interaction of the dressed electrons with the central Coulomb field

$$A^\mu = \left(-\frac{Z}{|x|}, 0, 0, 0\right) \tag{7}$$

is considered as a first-order perturbation. This is well justified if $Z\alpha \ll 1$, where Z is the nuclear charge of the nucleus considered. We evaluate the S-matrix element for the transition $(i \longrightarrow f)$

$$S_{fi} = \frac{iZ}{c}\int d^4x\,\overline{\psi}_{q_f}(x)\frac{\gamma^0}{|x|}\psi_{q_i}(x) \tag{8}$$

We first consider the quantity

$$\overline{\psi}_{q_f}(x)\frac{\gamma^0}{|x|}\psi_{q_i}(x) = \frac{1}{\sqrt{2Q_iV}}\frac{1}{\sqrt{2Q_fV}}\overline{u}(p_f,s_f)\overline{R}(p_f)\frac{\gamma^0}{|x|}R(p_i)u(p_i,s_i) \times\exp\{-i[S(q_f,x)-S(q_i,x)]\} \tag{9}$$

We have

$$\exp\{-i[S(q_f,x)-S(q_i,x)]\}=\exp\{i(q_f-q_i).x-iz\sin(\phi-\phi_0)\} \tag{10}$$

where z is such that

$$z=\sqrt{\alpha_1^2+\alpha_2^2} \tag{11}$$

whereas the quantities α_1 and α_2 are given by

$$\alpha_1=\frac{(a_1.p_i)}{c(k.p_i)}-\frac{(a_1.p_f)}{c(k.p_f)},\quad \alpha_2=\frac{(a_2.p_i)}{c(k.p_i)}-\frac{(a_2.p_f)}{c(k.p_f)} \tag{12}$$

and the phase ϕ_0 is such that $\phi_0=\arccos(\alpha_1/z)=\arcsin(\alpha_2/z)=\arctan(\alpha_2/\alpha_1)$. After some algebraic manipulation, one gets

$$\begin{aligned}\overline{u}(p_f,s_f)\overline{R}(p_f)\gamma^0R(p_i)u(p_i,s_i)=\overline{u}(p_f,s_f)[&C_0+C_1\cos\phi+C_2\sin\phi\\&+C_3\cos2\phi+C_4\sin2\phi]u(p_i,s_i)\end{aligned} \tag{13}$$

The five coefficients C_0, C_1, C_2, C_3 and C_4 can be obtained using REDUCE [8] and are explicitly given in our previous work [9]. The coefficients C_i contain all the information about the muon's AMM effects since they depend on $\kappa_\mu=4a_\mu m_\mu$.

We now introduce the well-known relations involving ordinary Bessel functions

$$\left\{\begin{array}{c}1\\ \cos(\phi)\\ \sin(\phi)\\ \cos(2\phi)\\ \sin(2\phi)\end{array}\right\}e^{-iz\sin(\phi-\phi_0)}=\sum_{n=-\infty}^{+\infty}\left\{\begin{array}{c}B_{0n}\\ B_{1n}\\ B_{2n}\\ B_{3n}\\ B_{4n}\end{array}\right\}e^{-in\phi} \tag{14}$$

with

$$\left\{\begin{array}{c}B_{0n}\\ B_{1n}\\ B_{2n}\\ B_{3n}\\ B_{4n}\end{array}\right\}=\left\{\begin{array}{c}J_n(z)e^{in\phi_0}\\ \left(J_{n+1}(z)e^{i(n+1)\phi_0}+J_{n-1}(z)e^{i(n-1)\phi_0}\right)/2\\ \left(J_{n+1}(z)e^{i(n+1)\phi_0}-J_{n-1}(z)e^{i(n-1)\phi_0}\right)/2i\\ \left(J_{n+2}(z)e^{i(n+2)\phi_0}+J_{n-2}(z)e^{i(n-2)\phi_0}\right)/2\\ \left(J_{n+2}(z)e^{i(n+2)\phi_0}-J_{n-2}(z)e^{i(n-2)\phi_0}\right)/2i\end{array}\right\} \tag{15}$$

Therefore,

$$\overline{R}(p_f)\gamma^0R(p_i)=\sum_{n=-\infty}^{+\infty}\Lambda_ne^{-in\phi} \tag{16}$$

where

$$\Lambda_n = C_0 B_{0n}(z) + C_1 B_{1n} + C_2 B_{2n} + C_3 B_{3n} + C_4 B_{4n} \tag{17}$$

Proceeding along the lines of standard QED calculations, for the formal differential cross section (DCS) expression in the presence of a circularly polarized laser field and taking into account the AMM effects of the muon, the calculation is now reduced to the computation of traces of γ matrices. This is routinely done using Reduce [8]. We consider the unpolarized DCS. Therefore, the various polarization states have the same probability and the actual calculated DCS is given by summing over the final polarization s_f and averaging over the initial polarization s_i. Then, the unpolarized DCS is formally given by

$$\left.\frac{d\overline{\sigma}}{d\Omega_f}\right|_{Q_f=Q_i+n\omega} = \frac{Z^2}{c^4}\frac{|\mathbf{q}_f|}{|\mathbf{q}_i|}\sum_{n=-\infty}^{+\infty}\frac{1}{|\mathbf{q}_f-\mathbf{q}_i-n\mathbf{k}|^4}\frac{1}{2}\sum_{s_i}\sum_{s_f}|M^n_{fi}|^2\Big|_{Q_f=Q_i+n\omega} \tag{18}$$

However, the novelty in the various stages of the calculations is contained in the term that contains all the information about the muon's AMM effects

$$\frac{1}{2}\sum_{s_i}\sum_{s_f}|M^n_{fi}|^2 \quad = \quad \frac{1}{2}\mathrm{Tr}[(\not{p}_f c + c^2)\Lambda_n(\not{p}_i c + c^2)\overline{\Lambda}_n] \tag{19}$$

$$\overline{\Lambda}_n = \gamma^0 \Lambda_n^{\dagger} \gamma^0 \tag{20}$$

We would like to emphasize that the REDUCE code we have written gave very long analytical expressions for the spinorial part $\frac{1}{2}\sum_{s_i}\sum_{s_f}|M^n_{fi}|^2$, which were difficult to incorporate in the corresponding latex manuscript. So we prefer to give below, as an example, just the coefficient multiplying the bessel function $j_n(z)^2$ which is given by

$$\mathcal{A} = \frac{1}{16(k.p_f)(k.p_i)m_\mu^4 c^4}(\mathcal{A}_1 + \mathcal{A}_2) \tag{21}$$

with

$$\begin{aligned}
\mathcal{A}_1 =\, & 2(a_1.p_f)^2(a_2.p_i)^2\kappa^4c^4\omega^2 - (a_1.p_f)^2\kappa^4(k.p_i)^2|\mathbf{a}|^2c^6 + 2(a_1.p_f)^2\kappa^4(k.p_i)|\mathbf{a}|^2c^4E_i\omega - 8(a_1.p_f)^2 \\
& \kappa^3 m_\mu^2|\mathbf{a}|^2c^4\omega^2 + 8(a_1.p_f)^2\kappa^2(k.p_i)^2m_\mu^2c^6 - 16(a_1.p_f)^2\kappa^2(k.p_i)m_\mu^2c^4E_i\omega + 16(a_1.p_f)^2\kappa^2m_\mu^4c^6\omega^2 \\
& + 12(a_1.p_f)^2\kappa^2m_\mu^2|\mathbf{a}|^2c^2\omega^2 - 16(a_1.p_f)^2\kappa m_\mu^4c^4\omega^2 - 4(a_1.p_f)(a_1.p_i)(a_2,p_f)(a_2.p_i)\kappa^4c^4\omega^2 + 2 \\
& \times(a_1.p_f)(a_1.p_i)\kappa^4(k.p_f)(k.p_i)|\mathbf{a}|^2c^6 - 2(a_1.p_f)(a_1.p_i)\kappa^4(k.p_f)|\mathbf{a}|^2c^4E_i\omega - 2(a_1.p_f)(a_1.p_i)\kappa^4(k.p_i) \\
& \times|\mathbf{a}|^2c^4E_f\omega - 2(a_1.p_f)(a_1.p_i)\kappa^4m_\mu^2|\mathbf{a}|^2c^6\omega^2 - 2(a_1.p_f)(a_1.p_i)\kappa^4|\mathbf{a}|^2c^4\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i| + 2 \\
& \times(a_1.p_f)(a_1.p_i)\kappa^4|\mathbf{a}|^2c^2E_fE_i\omega^2 + 16(a_1.p_f)(a_1.p_i)\kappa^3m_\mu^2|\mathbf{a}|^2c^4\omega^2 - 16(a_1.p_f)(a_1.p_i)\kappa^2(k.p_f)(k.p_i) \\
& \times m_\mu^2c^6 + 16(a_1.p_f)(a_1.p_i)\kappa^2(k.p_f)m_\mu^2c^4E_i\omega + 16(a_1.p_f)(a_1.p_i)\kappa^2(k.p_i)m_\mu^2c^4E_f\omega - 16(a_1.p_f) \\
& \times(a_1.p_i)\kappa^2m_\mu^4c^6\omega^2 - 24(a_1.p_f)(a_1.p_i)\kappa^2m_\mu^2|\mathbf{a}|^2c^2\omega^2 + 16(a_1.p_f)(a_1.p_i)\kappa^2m_\mu^2c^4\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f| \\
& \times|\mathbf{p}_i| - 16(a_1.p_f)(a_1.p_i)\kappa^2m_\mu^2c^2E_fE_i\omega^2 + 32(a_1.p_f)(a_1.p_i)\kappa m_\mu^4c^4\omega^2 + 2(a_1.p_i)^2(a_2,p_f)^2\kappa^4c^4\omega^2
\end{aligned}$$

$$-(a_1.p_i)^2\kappa^4(k.p_f)^2|\mathbf{a}|^2c^6+2(a_1.p_i)^2\kappa^4(k.p_f)|\mathbf{a}|^2c^4E_f\omega-8(a_1.p_i)^2\kappa^3m_\mu^2|\mathbf{a}|^2c^4\omega^2+8(a_1.p_i)^2\kappa^2$$
$$\times(k.p_f)^2m_\mu^2c^6-16(a_1.p_i)^2\kappa^2(k.p_f)m_\mu^2c^4E_f\omega+16(a_1.p_i)^2\kappa^2m_\mu^4c^6\omega^2+12(a_1.p_i)^2\kappa^2m_\mu^2|\mathbf{a}|^2c^2\omega^2$$
$$-16(a_1.p_i)^2\kappa m_\mu^4c^4\omega^2-(a_2,p_f)^2\kappa^4(k.p_i)^2|\mathbf{a}|^2c^6+2(a_2,p_f)^2\kappa^4(k.p_i)|\mathbf{a}|^2c^4E_i\omega-8(a_2,p_f)^2\kappa^3m_\mu^2$$
$$\times|\mathbf{a}|^2c^4\omega^2+8(a_2,p_f)^2\kappa^2(k.p_i)^2m_\mu^2c^6-16(a_2,p_f)^2\kappa^2(k.p_i)m_\mu^2c^4E_i\omega+16(a_2,p_f)^2\kappa^2m_\mu^4c^6\omega^2+12$$
$$\times(a_2,p_f)^2\kappa^2m_\mu^2|\mathbf{a}|^2c^2\omega^2-16(a_2,p_f)^2\kappa m_\mu^4c^4\omega^2+2(a_2,p_f)(a_2.p_i)\kappa^4(k.p_f)(k.p_i)|\mathbf{a}|^2c^6-2(a_2,p_f)$$
$$\times(a_2.p_i)\kappa^4(k.p_f)|\mathbf{a}|^2c^4E_i\omega-2(a_2,p_f)(a_2.p_i)\kappa^4(k.p_i)|\mathbf{a}|^2c^4E_f\omega-2(a_2,p_f)(a_2.p_i)\kappa^4m_\mu^2|\mathbf{a}|^2c^6\omega^2$$
$$\times-2(a_2,p_f)(a_2.p_i)\kappa^4|\mathbf{a}|^2c^4\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|+2(a_2,p_f)(a_2.p_i)\kappa^4|\mathbf{a}|^2c^2E_fE_i\omega^2+16(a_2,p_f)$$
$$\times(a_2.p_i)\kappa^3m_\mu^2|\mathbf{a}|^2c^4\omega^2-16(a_2,p_f)(a_2.p_i)\kappa^2(k.p_f)(k.p_i)m_\mu^2c^6+16(a_2,p_f)(a_2.p_i)\kappa^2(k.p_f)m_\mu^2c^4E_i\omega$$
$$\times+16(a_2,p_f)(a_2.p_i)\kappa^2(k.p_i)m_\mu^2c^4E_f\omega-16(a_2,p_f)(a_2.p_i)\kappa^2m_\mu^4c^6\omega^2-24(a_2,p_f)(a_2.p_i)\kappa^2m_\mu^2|\mathbf{a}|^2$$
$$\times c^2\omega^2+16(a_2,p_f)(a_2.p_i)\kappa^2m_\mu^2c^4\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|-16(a_2,p_f)(a_2.p_i)\kappa^2m_\mu^2c^2E_fE_i\omega^2+32$$
$$\times(a_2,p_f)(a_2,p_i)\kappa m_\mu^4c^4\omega^2-(a_2,p_i)^2\kappa^4(k.p_f)^2|\mathbf{a}|^2c^6+2(a_2,p_i)^2\kappa^4(k.p_f)|\mathbf{a}|^2c^4E_f\omega-8(a_2,p_i)^2\kappa^3m_\mu^2$$
$$\times|\mathbf{a}|^2c^4\omega^2+8(a_2,p_i)^2\kappa^2(k.p_f)^2m_\mu^2c^6-16(a_2,p_i)^2\kappa^2(k.p_f)m_\mu^2c^4E_f\omega+16(a_2,p_i)^2\kappa^2m_\mu^4c^6\omega^2+12$$
$$\times(a_2,p_i)^2\kappa^2m_\mu^2|\mathbf{a}|^2c^2\omega^2-16(a_2,p_i)^2\kappa m_\mu^4c^4\omega^2+2\kappa^4(k.p_f)(k.p_i)m_\mu^2|\mathbf{a}|^4c^8+2\kappa^4(k.p_f)(k.p_i)|\mathbf{a}|^4c^6$$
$$\times\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})|\mathbf{p}_f||\mathbf{p}_i|+2\kappa^4(k.p_f)(k.p_i)|\mathbf{a}|^4c^4E_fE_i-2\kappa^4(k.p_f)|\mathbf{a}|^6c^2E_i\omega-2\kappa^4(k.p_i)|\mathbf{a}|^6c^2E_f\omega-2$$
$$\times\kappa^4m_\mu^2|\mathbf{a}|^6c^4\omega^2+\kappa^4|\mathbf{a}|^8\omega^2-2\kappa^4|\mathbf{a}|^6c^2\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|+2\kappa^4|\mathbf{a}|^6E_fE_i\omega^2+8\kappa^3(k.p_f)^2m_\mu^2|\mathbf{a}|^4$$
$$\times c^6-16\kappa^3(k.p_f)(k.p_i)m_\mu^2|\mathbf{a}|^4c^6-16\kappa^3(k.p_f)m_\mu^2|\mathbf{a}|^4c^4E_f\omega+16\kappa^3(k.p_f)m_\mu^2|\mathbf{a}|^4c^4E_i\omega+8\kappa^3(k.p_i)^2$$
$$\times m_\mu^2|\mathbf{a}|^4c^6+16\kappa^3(k.p_i)m_\mu^2|\mathbf{a}|^4c^4E_f\omega-16\kappa^3(k.p_i)m_\mu^2|\mathbf{a}|^4c^4E_i\omega+16\kappa^3m_\mu^4|\mathbf{a}|^4c^6\omega^2+16\kappa^3m_\mu^2|\mathbf{a}|^4c^4$$
$$\times\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|-16\kappa^3m_\mu^2|\mathbf{a}|^4c^2E_fE_i\omega^2-16\kappa^2(k.p_f)^2m_\mu^4|\mathbf{a}|^2c^8-16\kappa^2(k.p_f)^2m_\mu^2|\mathbf{a}|^4c^4+16\kappa^2$$
$$\times(k.p_f)(k.p_i)m_\mu^4|\mathbf{a}|^2c^8+32\kappa^2(k.p_f)(k.p_i)m_\mu^2|\mathbf{a}|^4c^4-16\kappa^2(k.p_f)(k.p_i)m_\mu^2|\mathbf{a}|^2c^6\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})|\mathbf{p}_f||\mathbf{p}_i|$$

and,

$$\mathcal{A}_2=-16\kappa^2(k.p_f)(k.p_i)m_\mu^2|\mathbf{a}|^2c^4E_fE_i+32\kappa^2(k.p_f)m_\mu^4|\mathbf{a}|^2c^6E_f\omega-32\kappa^2(k.p_f)m_\mu^4|\mathbf{a}|^2c^6E_i\omega$$
$$+32\kappa^2(k.p_f)m_\mu^2|\mathbf{a}|^4c^2E_f\omega-16\kappa^2(k.p_f)m_\mu^2|\mathbf{a}|^4c^2E_i\omega-16\kappa^2(k.p_i)^2m_\mu^4|\mathbf{a}|^2c^8-16\kappa^2(k.p_i)^2$$
$$\times m_\mu^2|\mathbf{a}|^4c^4-32\kappa^2(k.p_i)m_\mu^4|\mathbf{a}|^2c^6E_f\omega+32\kappa^2(k.p_i)m_\mu^4|\mathbf{a}|^2c^6E_i\omega-16\kappa^2(k.p_i)m_\mu^2|\mathbf{a}|^4c^2E_f\omega$$
$$+32\kappa^2(k.p_i)m_\mu^2|\mathbf{a}|^4c^2E_i\omega-32\kappa^2m_\mu^6|\mathbf{a}|^2c^8\omega^2-16\kappa^2m_\mu^4|\mathbf{a}|^4c^4\omega^2-32\kappa^2m_\mu^4|\mathbf{a}|^2c^6\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})$$
$$\times\omega^2|\mathbf{p}_f||\mathbf{p}_i|+32\kappa^2m_\mu^4|\mathbf{a}|^2c^4E_fE_i\omega^2-8\kappa^2m_\mu^2|\mathbf{a}|^6\omega^2-16\kappa^2m_\mu^2|\mathbf{a}|^4c^2\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|$$
$$+16\kappa^2m_\mu^2|\mathbf{a}|^4E_fE_i\omega^2+32\kappa(k.p_f)^2m_\mu^4|\mathbf{a}|^2c^6-64\kappa(k.p_f)(k.p_i)m_\mu^4|\mathbf{a}|^2c^6-64\kappa(k.p_f)m_\mu^4|\mathbf{a}|^2$$
$$\times c^4E_f\omega+64\kappa(k.p_f)m_\mu^4|\mathbf{a}|^2c^4E_i\omega+32\kappa(k.p_i)^2m_\mu^4|\mathbf{a}|^2c^6+64\kappa(k.p_i)m_\mu^4|\mathbf{a}|^2c^4E_f\omega-64\kappa$$
$$\times(k.p_i)m_\mu^4|\mathbf{a}|^2c^4E_i\omega+64\kappa m_\mu^6|\mathbf{a}|^2c^6\omega^2+64\kappa m_\mu^4|\mathbf{a}|^2c^4\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|-64\kappa m_\mu^4|\mathbf{a}|^2c^2$$
$$\times E_fE_i\omega^2+32(k.p_f)(k.p_i)m_\mu^6c^8+32(k.p_f)(k.p_i)m_\mu^4c^6\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})|\mathbf{p}_f||\mathbf{p}_i|+32(k.p_f)(k.p_i)$$
$$\times m_\mu^4c^4E_fE_i-32(k.p_f)m_\mu^4|\mathbf{a}|^2c^2E_i\omega-32(k.p_i)m_\mu^4|\mathbf{a}|^2c^2E_f\omega-32m_\mu^6|\mathbf{a}|^2c^4\omega^2+16m_\mu^4|\mathbf{a}|^4$$
$$\times\omega^2-32m_\mu^4|\mathbf{a}|^2c^2\cos(\widehat{\mathbf{p}_i,\mathbf{p}_f})\omega^2|\mathbf{p}_f||\mathbf{p}_i|+32m_\mu^4|\mathbf{a}|^2E_fE_i\omega^2.$$

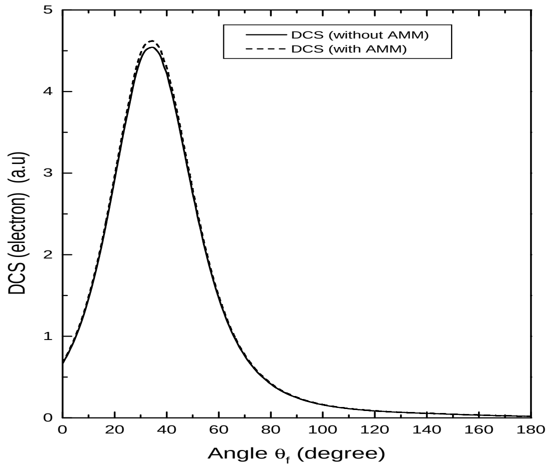

Figure 1: The DCSs of the electron (with and without AMM effects) scaled in 10^{-4} as a function of the angle θ_f in degrees for an electrical field strength of $\mathcal{E}=0.05$ a.u. and a relativistic parameter $\gamma=1.0053$. The corresponding number of photons exchanged is ± 100.

3 Result and discussion

The differential cross sections have been computed for the Coulomb scattering by electron and muon impact in a geometry ($\theta_i=45°$, $\phi_i=45°$, $\phi_f=90°$ and $0°\le\theta_f\le 180°$) for medium and intermediate energies (e.g., $\gamma=1.0053$ and 1.5) and for medium and intermediate intensities (e.g., $\mathcal{E}=0.05$ a.u. and 0.5 a.u.).

We assume the electromagnetic wave to be quasi-monochromatic and of circularly polarization with the vector potential

$$\mathbf{A}(\varphi)=|\mathbf{a}|(\hat{e}_1\cos(\varphi)+\hat{e}_2\sin(\varphi)) \tag{22}$$

with $|\mathbf{a}|$ is a slow varying amplitude of the vector $\mathbf{A}(t,\mathbf{r})$ with the phase $\varphi=kx$, $k=(\omega/c,\mathbf{k})$ is the four-wave vector of the laser field with frequency ω.

Three main conclusions can be drawn from Fig. 1. Firstly, in the non relativistic regime ($\gamma=1.0053$ and $\mathcal{E}=0.05$ a.u.), the different DCSs (with and without AMM) for the electron give almost identical curves. Secondly, the electron's AMM effects are not important in this regime. It does not mean that the electron's AMM effects are irrelevant but only that their contribution is too small to be noticeable. Thirdly, the numerical predictions appropriately show that the electron's DCS with AMM effects approaches the

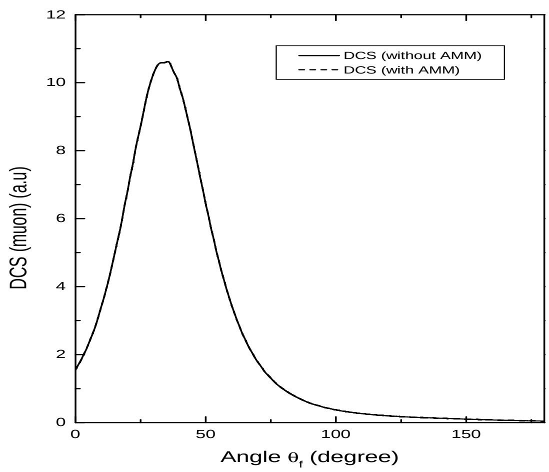

Figure 2: The DCSs of the muon (with and without AMM effects) scaled in 10^{-9} as a function of the angle θ_f in degrees for an electrical field strength of $\mathcal{E}=0.05$ a.u. and a relativistic parameter $\gamma=1.0053$. The corresponding number of photons exchanged is ± 100.

electron's DCS without AMM for the intensity $\mathcal{E}=0.05$ a.u. and for the incident electron's kinetic energy ($E \approx 2700$ eV) . Fig. 2 shows that the two DCSs (with and without AMM) for the muon give overlapping curves. The absolute differential cross sections show a decreasing behavior with increasing mass, showing an overall decrease of two orders of magnitude (from 10^{-4} to 10^{-8}). This result is rather reasonable since the DCSs decrease with the energy. Special relativity shows that relativistic mass m and relativistic parameter γ are connected to the relativistic total energy via the well-known relationship ($E=c^2\gamma m$). We notice in this equation that both relativistic parameter γ and relativistic mass m produce an increase of relativistic total energy. We present in Fig. 3 the results of our DCS calculations for Coulomb scattering by relativistic unpolarized incident electrons. We have used the formalism in which the incident electron is represented by Dirac-Volkov state [2] for comparison. We have also displayed the explicit contribution to the DCS due to the electron's AMM effects by using the Salamin waves [3].

By comparing the results with and without AMM effects in the regime in which we have ($\gamma=1.5$ and $\mathcal{E}=0.5$ a.u.), the DCS with AMM effects always overestimates the DCS without AMM effects by approximatively 3.2 orders of magnitude in the vicinity of $\theta_f=33°$. This figure shows major differences between DCS with AMM effects and AMM-free DCS when the electron's mass is used ($m_e=1$ in atomic units). Fig. 4 demonstrates the dependence on the angle θ_f of the muon's DCS (with and without AMM) for fixed various input parameters. One sees immediately that there are almost indistinguishable curves over the entire angular range which is the same situation as in Fig. 2.

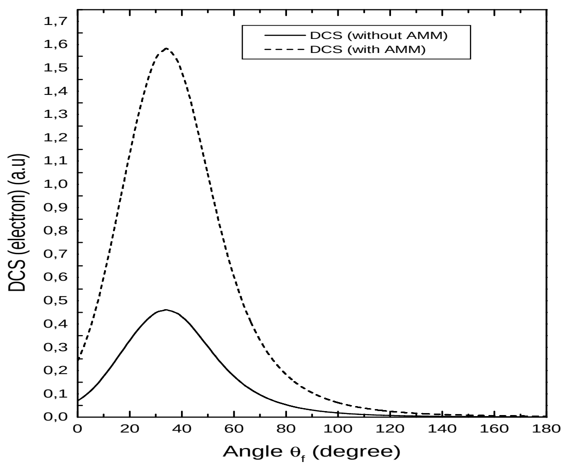

Figure 3: The DCSs of the electron (with and without AMM effects) scaled in 10^{-9} as a function of the angle θ_f in degrees for an electrical field strength of $\mathcal{E}=0.5$ a.u. and a relativistic parameter $\gamma=1.5$. The corresponding number of photons exchanged is ± 100.

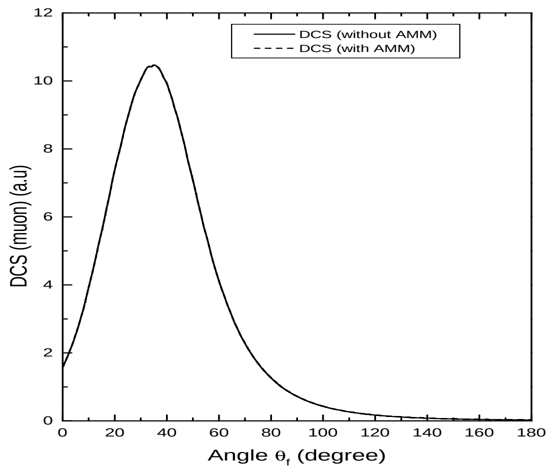

Figure 4: The DCSs of the muon (with and without AMM effects) scaled in 10^{-15} as a function of the angle θ_f in degrees for an electrical field strength of $\mathcal{E}=0.5$ a.u. and a relativistic parameter $\gamma=1.5$. The corresponding number of photons exchanged is ± 100.

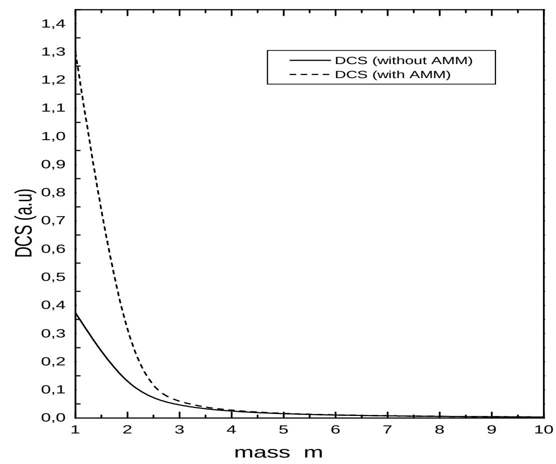

Figure 5: The DCSs (with and without AMM effects) scaled in 10^{-9} as a function of the mass m for an electrical field strength of $\mathcal{E}=0.5$ a.u., a relativistic parameter $\gamma=1.5$ and a fixed angle $\theta_f=45°$. The corresponding number of photons exchanged is ± 100.

It indicates that the contribution of the muon's anomalous magnetic moment effects to the differential cross section is not so important. In order to illustrate the mass effects on the DCSs, we have displayed in Fig. 5 the dependence of the calculated differential cross section on the projectile mass m at fixed impact energy $\gamma=1.5$. A comparison of our DCSs results (with and without AMM effects) for m varying from 1 to 10 reveals that they are graphically indistinguishable from the value of the mass 5, demonstrating that there is no effect of anomalous Magnetic moment from the value of mass 5. In order to clarify the situation in which we have overlapping curves for the two approaches in Fig. 5, we present in Fig. 6 the two approaches (DCS with and without AMM effects) that provide a rough decrease of the DCS magnitude versus the projectile mass ($150 \leq m \leq 250$). In general, DCSs decrease with the mass and give overlapping curves. Hence, from Figs. 5 and 6, we come to the conclusion that there is no effect of the muon's anomalous magnetic moment even at high energies and high intensities.

This theoretical study has been confirmed by the case of the Coulomb scattering of the electron when we take ($m=1$) in our general program. It should be emphasized that the above analysis can be generalized to the relativistic Coulomb scattering by incident particles with the same characterization (e^+, μ^+...). We finally note that the muon's AMM effects are not significant for the Coulomb scattering process in the first Born approximation. As we have seen above, within the range of validity of the WFA, the anomalous magnetic moment has an important effect on the differential cross section for the electron but not for the muon.

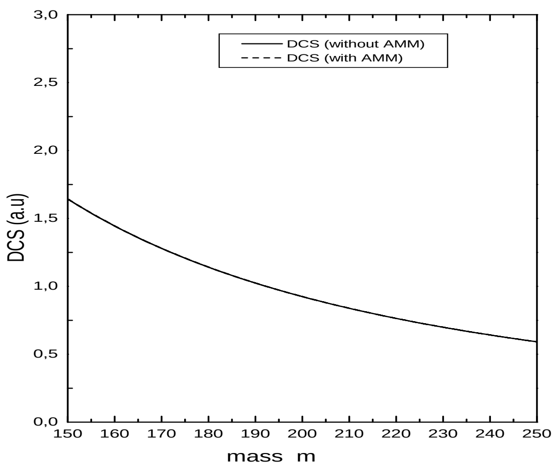

Figure 6: The DCSs (with and without AMM effects) scaled in 10^{-14} as a function of the mass m for an electrical field strength of $\mathcal{E}=0.5$ a.u., a relativistic parameter $\gamma=1.5$ and a fixed angle $\theta_f=45°$. The corresponding number of photons exchanged is ± 100.

4 Conclusion

In conclusion, let us enumerate shortly the main results of the present work. Extending the study of the Coulomb scattering process of an electron by a charged nucleus to the case of the muon showed that the muon's anomalous magnetic moment effects do not affect the behavior of the DCSs. In addition, we have demonstrated that the AMM effects vanish versus the mass projectile from the value 5. However, in order to check the accuracy of this physical result, we are currently in a point to investigate other approaches with other similar particles such as e^+, μ^+ and τ^- and prove that heavy particles with the same electron's characteristics give rise to the same results.

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