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Effect of external field on the early stage decoherence

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Abstract. In this paper, the effects of an external field on the early stage decoherence are discussed both in the cavity QED and the spin chain channel. We calculate the evolution of the entanglement, find that the finite time disentanglement can be eliminated or induced by the present of an external field for some initial states.

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Key words: early stage decoherence, cavity QED, spin channel

1 Introduction

Entanglement and coherence are two basic conceptions in the quantum world, and are the foundation of the quantum information processing. In real systems, decoherence always take place because of the unavoidable interaction of these systems with their natural environment, leads to the decay of coherence and entanglement. Yu and Eberly [1] have studied the dynamics of bipartite entanglement between two atoms which coupling to their own dissipative environment respectively. They found that the entanglement can completely vanish in a finite time even with much slower local decoherence, termed entanglement sudden death (ESD). This surprising phenomenon which is contrary to our intuition on the decoherence is also appears in many other scenarios [2–6]. Yönaç *et al.* [2] study the disentanglement of two initially entangled Jaynes-Cummings atoms without any interaction between them; the effects of interaction between the particles and the couplings to the same environment have been discussed in Ref. [3,4]; and the ESD is demonstrated can also happen in closed systems [7]. The experiment evidence is recently pressed [8,9]. Although extensive works have been done in ESD, it is still unclear what causes it and what is the physics behind.

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These early stage decoherence may sometimes influence the quantum information process [10], e.g., in the entangled quantum information networks, the error correction technology [11,12] could make the very small degraded entanglement to be full usefulness, but incapacity in a totally disentanglement. An approach to quantum control is expected to avoid or unfold the entanglement sudden death as required. In this paper, we will demonstrate that the present of an appropriate external field will influence the decoherence process and to be a convenience approach to quantum control. Our paper is organized as follows, in Section 2, we introduce the measure of entanglement; the effects of external field in cavity QED and spin channel are discussed respectively in Section 3 and Section 4; concluding remarks are given in Section 5.

2 Measure of entanglement

To calculate the entanglement between two qubits, we choose the concurrence C defined by Wootters [13] as the convenient measure of entanglement. The concurrence varies from $C=0$ of a separable state to $C=1$ of a maximally entangled state. For a pure or mixed state of two qubits A and B, the concurrence may be calculated explicitly from the density matrix ρ as

$$C(\rho)=\max(0,\sqrt{\lambda_1}-\sqrt{\lambda_2}-\sqrt{\lambda_3}-\sqrt{\lambda_4}), \tag{1}$$

where the quantities λ_i are the eigenvalues in decreasing order of the matrix

$$\zeta=\rho(\sigma_y^A\otimes\sigma_y^B)\rho^*(\sigma_y^A\otimes\sigma_y^B), \tag{2}$$

where ρ^* denotes the complex conjugation of ρ in the standard basis and σ_y is the Pauli matrix expressed in the same basis as

$$\sigma_y=\begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}. \tag{3}$$

3 Atoms in driven cavities

In the original paper of Yu *et al.* [1,2], two Jaynes–Cummings atoms A and B are located inside two spatially separated cavities a and b. The two atoms are initially entangled but have no direct interaction afterwards. The Hamiltonian is

$$H=H_A+H_B=\omega a^\dagger a+g(a^\dagger\sigma_-^A+a\sigma_+^A)+\omega b^\dagger b+g(b^\dagger\sigma_-^B+b\sigma_+^B). \tag{4}$$

Closer to experimental reality is the case of an external (classical) field [14] with drive amplitude ε added as an additional item $\varepsilon(a^\dagger+a)$. Use the analogous scenario in Ref. [2] but instead of two driven cavities, the Hamiltonian can be rewritten as

$$H=\omega a^\dagger a+g(a^\dagger\sigma_-^A+\sigma_+^A a)+\varepsilon_A(a^\dagger+a)+\omega b^\dagger b+g(b^\dagger\sigma_-^B+\sigma_+^B b)+\varepsilon_B(b^\dagger+b). \tag{5}$$

We assume that the system is prepared firstly in a partially entangled initial state

$$|\Psi_0\rangle = |\psi\rangle_{AB} \otimes |00\rangle_{ab},$$

where

$$|\psi\rangle_{AB} = \cos\alpha|ee\rangle + \sin\alpha|gg\rangle,$$

and $|e\rangle(|g\rangle)$ denotes the up (low) level state of the atom. We also assume that $\omega = 1$, $g = 1$, and $\varepsilon_A = \varepsilon_B = \varepsilon$ in this paper. Then, we will study the time evolution of the entanglement between A and B when subjected to the Hamiltonian (5) and compare with the results of no-driven cavity [2].

Our calculation reveal that the external field will distinctly influence the evolution of entanglement. We plot the evolution curves of the entanglement between atoms AB, with an initial state $\alpha = \pi/6$ for different external fields $\varepsilon = 0$, $\varepsilon = 0.3$, and $\varepsilon = 1$ respectively in the Fig. 1. We can find that, for the no-driven cavity with the $\varepsilon = 0$, the entanglement evolute periodically and the ESD take place for each decline. With the present of the field $\varepsilon = 0.3$, the evolution become complicated and the death regions are reduced, and for the $\varepsilon = 1$ the ESD totally disappear.

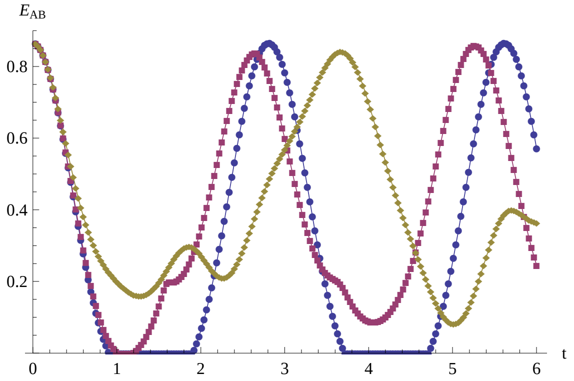


Figure 1: The evolution of entanglement E_{AB} for the initial state $\alpha = \pi/6$ in the situation that $\varepsilon = 0$(dot), $\varepsilon = 0.3$(square), and $\varepsilon = 1$(diamond).

We also plot the schematic curves and the contour plot of arbitrary α for different ε in the Fig. 2. We can find that, as the increasing of the external field, the regions of the disentanglement are obviously changed.

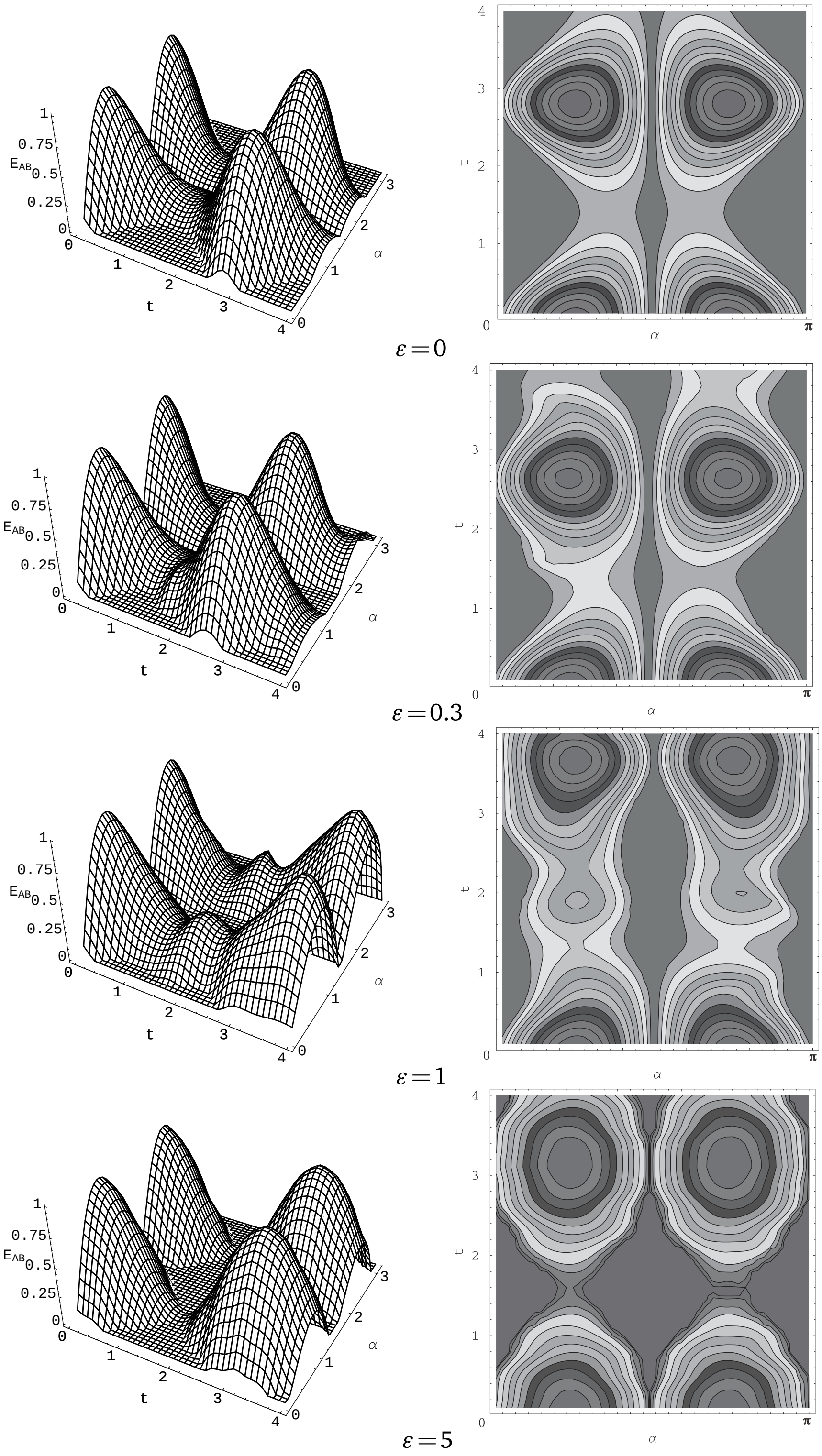
For another type initial entangled states

$$|\Phi_0\rangle = |\phi\rangle_{AB} \otimes |00\rangle_{ab}, \tag{6}$$

where

$$|\phi\rangle_{AB} = \cos\alpha|eg\rangle + \sin\alpha|ge\rangle. \tag{7}$$

There are no sudden death for these initial states in the no-driven situation [2]. As the present of the external field, the ESD appear and enlarge with the increasing of ε, seen in Fig. 3.



$\varepsilon=0$

$\varepsilon=0.3$

$\varepsilon=1$

$\varepsilon=5$

Figure 2: Schematic curves and contour plot for the initial states $|\Psi_0\rangle$ with arbitrary α for $\varepsilon=0$, $\varepsilon=0.3$, $\varepsilon=1$, and $\varepsilon=5$.

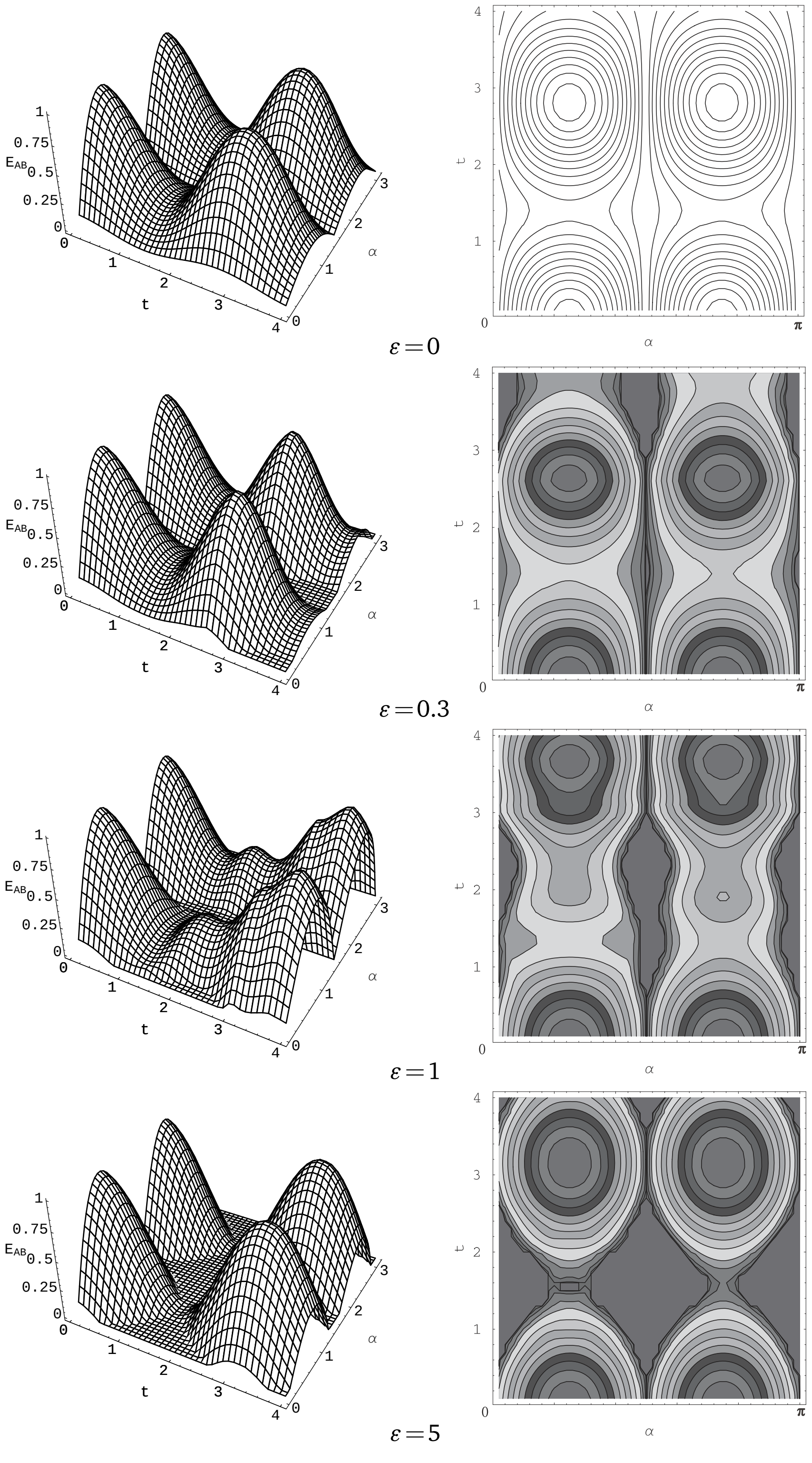


Figure 3: Schematic curves and contour plot for the initial states $|\Phi_0\rangle$ with arbitrary α for $\varepsilon=0$, $\varepsilon=0.3$, $\varepsilon=1$, and $\varepsilon=5$.

4 Spin chain channel

The spin systems have been proposed to be used in many quantum information processes [15]. The spin chains have also been considered as quantum "wires" for the quantum information transfer. S. Bose [16] have proposed a scheme to use a spin chain as a channel for short distance quantum communication. The communication is achieved by placing a spin state at one end of the chain *A* (Alice) and waiting for a specific amount of time to let this state propagate to the other end *B* (Bob) [16].

We then discuss the decoherence in a Heisenberg spin channel, the Hamiltonian can be written as

$$H=J\sum_{i}^{N-1}\vec{\sigma}_i\cdot\vec{\sigma}_{i+1}. \tag{8}$$

We assume that the two qubits *AB* belong to a chain with the number of sites $N=4$ are

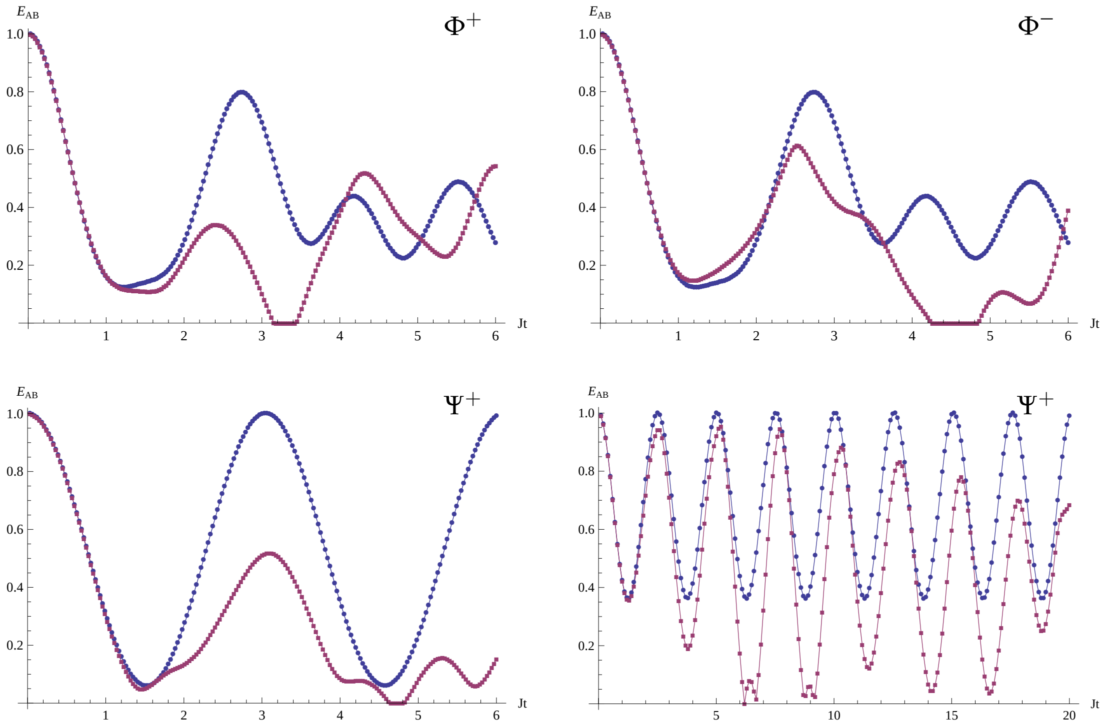


Figure 4: The entanglement evolution of four bell-type initial states with the transverse external field $B=1$(squared line) the dotted lines are the curves for $B=0$.

initially in one of the bell-type state

$$\begin{aligned}\Phi^+ &= \frac{1}{\sqrt{2}}(|11\rangle+|00\rangle)_{AB}\otimes|00\rangle,\\ \Phi^- &= \frac{1}{\sqrt{2}}(|11\rangle-|00\rangle)_{AB}\otimes|00\rangle,\\ \Psi^+ &= \frac{1}{\sqrt{2}}(|10\rangle+|01\rangle)_{AB}\otimes|00\rangle,\\ \Psi^- &= \frac{1}{\sqrt{2}}(|10\rangle-|01\rangle)_{AB}\otimes|00\rangle,\end{aligned} \tag{9}$$

where the $|1\rangle$ ($|0\rangle$) denotes the spin up (down) state. The evolution of the entanglement between *AB* for different initial states have been studied in Ref. [17], and there are no ESD for these four initial states.

Then, we add a transverse magnetic field on the spin channel, the Hamiltonian is written as

$$H = J\sum_{i}^{N-1}\vec{\sigma}_i\cdot\vec{\sigma}_{i+1}+\sum_{i}^{N}B\sigma_i^x, \tag{10}$$

where B is the intensity of the magnetic field. The evolution of entanglement will influenced by the additional field, the numerical results are plot in Fig. 4. We can find that, the ESD appears with the present of external field for the initial states Φ^+, Φ^-, Ψ^+, but not for the initial state Ψ^-.

5 Conclusion

The foundation of these amazing phenomenon of entanglement sudden death have not been revealed clearly and the influence on the quantum information process are not discussed sufficiently. But it should be considered in the quantum information process. We discussed the effect of an external field on these decoherence process both in cavity QED and spin chain channel. Our results show that the additional field will eliminate or induce the finite time disentanglement for some initial states, indicate that an external field can be used in the quantum control of the present of ESD.

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