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Two kinds of entangled coherent states and their nonclassical effects

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Abstract. As for two kinds of entangled coherent states, we have studied the relationship between the entanglement and the nonclassical effects; we calculate their entanglement by the concurrence and their nonclassical effects, such as squeezing and antiquating. We find that the entanglement always corresponds with one of squeezing and antiquating and the larger a nonclassical effect is, the stronger entanglement is. The result shows the entanglement has a deep relationship with the nonclassical effects.

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Key words: entangled coherent states, nonclassical effects, concurrence

1 Introduction

Since 1935, entanglement has been recognized as one of the most puzzling features of quantum mechanics [1,2]. However, it is nowadays a widespread opinion that it also represents a fundamental resource for many quantum information protocols. Therefore, entanglement deserves to be analyzed in all respects. After the first experiment on quantum teleportation [3] and other quantum information processes using two-mode squeezing states [4,5], continuous variable systems have aroused great interest in the separable properties. So far, most theoretical and experimental work has focused on the entanglement properties of Gaussian states. For Gaussian states, the necessary and sufficient inseparability criterion has been fully developed [6, 7], Inseparability Criteria for Continuous Bipartite Quantum States has also been developed [8,9]. Using the total variance of a pair of Einstein-Podolsky-Roses type operators introduced by Duan *et al.* [6], generalized EPS entangled states (GEES) has been obtained and it has been proved that a state must be the two-mode squeezing state if the state is a GEES whether it is Gaussian or not and whether it is pure or not. However, there are some

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entangled coherent states which have entanglement, but do not have squeezing. Whether this kind of entanglement corresponds with some nonclassical effects of two-mode fields? [10]

In this paper, we analyze two kinds of entangled coherent states and calculate their entanglement by concurrence and their nonclassical effects, such as squeezing and antiquating, and find that the entanglement always follows one of squeezing and antiquating and the entanglement increases with the increase of a nonclassical effect. It shows the entanglement has a deep relationship with the nonclassical effects.

2 Squeezing and antiquating of two kinds entangled coherent states

Firstly, we consider the following bipartite entangled coherent states [11]

$$|\psi\rangle=\mu|\alpha,\alpha\rangle+\nu|-\alpha,-\alpha\rangle, \tag{1}$$

where $|\alpha\rangle,|-\alpha\rangle$ are normalized states of system 1 and similarly $|\alpha\rangle,|-\alpha\rangle$ are states of system 2 with complex μ and ν, after normalization, the bipartite states $|\psi\rangle$ are given by

$$|\psi\rangle=\frac{1}{N}\Big(\mu|\alpha,\alpha\rangle+\nu|-\alpha,-\alpha\rangle\Big), \tag{2}$$

where $N^2=|\mu|^2+|\nu|^2+(\mu^*\nu+\mu\nu^*)e^{-4R^2}$. The two no orthogonal states $|\alpha\rangle,|-\alpha\rangle$ are assumed to be linearly independent and span a two-dimensional subspace of the Hilbert space, and then we choose an orthogonal basis $\Big\{|0\rangle_i,|1\rangle_i\Big\}$ $(i=1,2)$ [12]

$$\begin{aligned}|0\rangle_1=|\alpha\rangle_1,|1\rangle_1=\frac{1}{\sqrt{1-P^2}}\Big(|-\alpha\rangle_1-P|\alpha\rangle_1\Big),|0\rangle_2\\ =|-\alpha\rangle_2,|1\rangle_2=\frac{1}{\sqrt{1-P^2}}\Big(|\alpha\rangle_2-P|-\alpha\rangle_2\Big),\end{aligned} \tag{3}$$

with $P=e^{-2R^2}$.

Under these bases, the entangled states $|\psi\rangle$ can be rewritten as

$$|\psi\rangle=\frac{1}{N}\Big((\mu P+\nu P)|00\rangle+\mu\sqrt{1-P^2}|01\rangle+\nu\sqrt{1-P^2}|10\rangle\Big), \tag{4}$$

which shows that the general entangled no orthogonal state is considered as a state of two logical quits, then it is straightforward to obtain the reduced density matrix ρ_1 and two eigenvalues of ρ_1 are given by [13]

$$\lambda_\pm=\frac{1}{2}\pm\frac{1}{2}\sqrt{1-\frac{4|\mu\nu|^2}{N^4}(1-P^2)^2}, \tag{5}$$

which are same with those of ρ_2. The corresponding eigenvectors of ρ_1 is denoted by $|\pm\rangle_1$, then the general theory of the Schmidt decomposition [14] implies that the normalized state $|\psi\rangle$ can be written as

$$|\psi\rangle = c_+|++\rangle + c_-|--\rangle, \tag{6}$$

with $c_\pm = \sqrt{\lambda_\pm}$. There are different measures of entanglement, one of measures is concurrence [15]. Since the state (1) is essentially two-state systems, we can characterize the entanglement of it by the concurrence. The concurrence for a pure state $|\psi\rangle$ is defined as

$$c = \left|\langle\psi|\sigma_y \bigotimes \sigma_y|\psi^*\rangle\right|. \tag{7}$$

Here σ_y is the vector of Pauli matrices, $|\psi^*\rangle$ is the complex conjugate of $|\psi\rangle$.
From Eqs. (6) and (7), we can obtain the entanglement of state (1)

$$c = 2c_+c_- = \frac{2|\mu||\nu|\left(1-e^{-4R^2}\right)}{|\mu|^2+|\nu|^2+\left(\mu\nu^*+\mu^*\nu\right)e^{-4R^2}}. \tag{8}$$

For simplicity, we consider μ, ν as real number, so

$$c = \frac{2|\mu||\nu|\left(1-e^{-4R^2}\right)}{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}. \tag{9}$$

Firstly, we consider the simplest situation $\theta=0$ in $\alpha=Re^{i\theta}$ and discuss the two-mode squeezing defined by Loudon and Knight [16], the field quadrature operator are given by

$$U_1 = \frac{a+b+a^+ + b^+}{2\sqrt{2}}, \qquad U_2 = \frac{a-a^+ + b - b^+}{2\sqrt{2}i}. \tag{10}$$

These operators satisfy the commutation relation $[U_1, u_2] = i/2$, which implies the uncertainly relation $\langle(\Delta U_1)^2(\Delta U_2)^2\rangle \geq 1/16$, the two-mode squeezing is said to exist whenever $Z_i = 4\langle(\Delta U_i)^2\rangle - 1 < 0$, $(i=1,2)$ and $Z_i = -1$ shows the biggest squeezing.

After some algebra

$$\begin{aligned} Z_2 &= \langle a^+a + b^+b + ab^+ + a^+b\rangle - \frac{1}{2}\langle a^2 + b^2 + a^{+^2} + b^{+^2} + 2ab + 2a^+b^+\rangle \\ &= \frac{-16\mu\nu R^2 e^{-4R^2}}{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}. \end{aligned} \tag{11}$$

For state (1), system 1 and system 2 are symmetrical, it's antiquating and Sub-Poissonian photon statistics are the same, to obtain the antiquating of state (1), we can compute the Mandel factor Q [17]

$$Q = \frac{\langle a^+a^+aa\rangle}{\langle a^+a\rangle} - \langle a^+a\rangle,$$

$Q<0$ shows the antiquating. After some algebra

$$Q=R^2\left(\frac{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}{\mu^2+\nu^2-2\mu\nu e^{-4R^2}}-\frac{\mu^2+\nu^2-2\mu\nu e^{-4R^2}}{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}\right). \tag{12}$$

From Eqs. (11) and (12) we can get when taking μ, ν as positive or negative, $Z_2<0$, $Q>0$, there exists squeezing, does not show antiquating; when taking μ, ν as opposite mark, $Z_2>0$, $Q<0$, there shows antiquating , does not exist squeezing and from Eq. (9), it can easily be gotten that the state is always entangled with the condition $\mu\neq 0$ or $\nu\neq 0$. It shows that the entanglement always follows one of squeezing and antiquating.

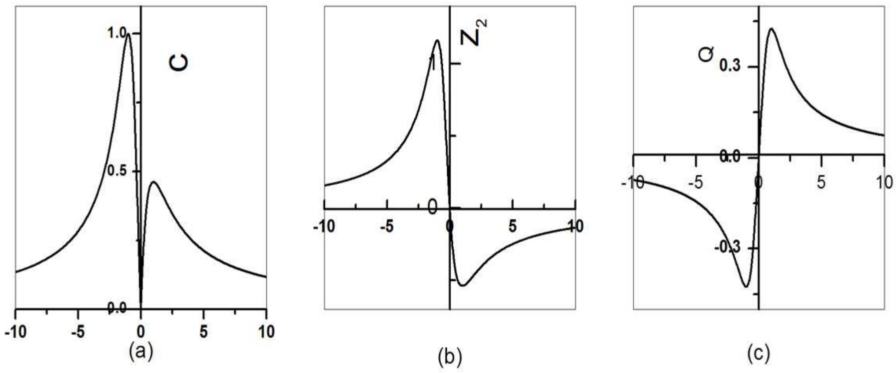

Figure 1: $\mu=1$, $R=0.5$. (a) Concurrence. (b) Z_2. (c) Q as a function of ν.

When $\mu=1$, $R=0.5$, entanglement C, Squeezing parameter Z_2, and Mandel factor Q as a function of ν are in Fig. 1, we can clearly see the relationship as above between the concurrence and Z_2, Q, when $\nu>0$, the change trend of entanglement is the same with that of squeezing, when $\nu<0$, the change trend of entanglement is the same with that of antiquating. The bigger a nonclassical effect is, the stronger entanglement is.

We know that θ in $\alpha=Re^{i\theta}$ is influence to the nonclassical effects, so when $\theta\neq 0$, we research the relationship between entanglement and nonclassical effects of the state (1).

With the same method, we can obtain Z_2

$$\begin{aligned}Z_2=&\langle a^+a+b^+b+ab^++ba^+\rangle-\frac{1}{2}\langle a^2+b^2+a^{+^2}+b^{+^2}+2ab+2a^+b^+\rangle\\&+\frac{1}{2}\Big(\langle a\rangle+\langle b\rangle-\langle a^+\rangle-\langle b^+\rangle\Big)^2\\=&4R^2\left(\frac{\mu^2+\nu^2-2\mu\nu e^{-4R^2}}{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}-\cos2\theta\right)-\frac{8R^2(\mu^2-\nu^2)^2\sin^2\theta}{(\mu^2+\nu^2+2\mu\nu e^{-4R^2})}.\end{aligned} \tag{13}$$

When $\mu=\nu$,

$$Z_2=4R^2\left(\frac{1-e^{-4R^2}}{1+e^{-4R^2}}-\cos2\theta\right), \tag{14}$$

which is negative under some condition, so there exists squeezing.
When $\mu = -\nu$,

$$Z_2 = 4R^2\left(\frac{1+e^{-4R^2}}{1-e^{-4R^2}} - \cos 2\theta\right), \tag{15}$$

which is always positive with any condition, so there do not exist squeezing, Eqs. (14) and (15) equate with the criteria Eqs. (30) and (31) in [10], so we verify the result is right in that paper by another method.

The Mandel factor Q

$$Q = R^2\left(\frac{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}{\mu^2+\nu^2-2\mu\nu e^{-4R^2}} - \frac{\mu^2+\nu^2-2\mu\nu e^{-4R^2}}{\mu^2+\nu^2+2\mu\nu e^{-4R^2}}\right), \tag{16}$$

which is the same with Eq. (12). When $\mu = \nu$, $Q > 0$, the state does not show antiquating, when $\mu = -\nu$, $Q < 0$, the state shows antiquating.

We can see that when taking $\mu = -\nu$, the state (1) has the strongest entanglement, there exists antiquating, not squeezing. And when taking $\mu = \nu$, the state (1) exists entanglement, there exists squeezing under some condition, not antiquating. So for the first kind of entangled coherent states, it can always be gotten while not existing squeezing, there must exist antiquating, while not existing antiquating, there exist squeezing, no matter $\theta \neq 0$ or $\theta = 0$, the entanglement always follows one of squeezing and antiquating.

As we all know, the nonclassical effects of superposition light field have the close relationship with the phase space interference effect and also the relative phase plays an important role on the entanglement. In order to clearly see the influence of the phase difference of two states to light field nonclassical effects and the entanglement, we consider the following entangled coherent states [11]

$$|\psi\rangle = \frac{1}{N}\Big(a|\alpha,-\alpha\rangle + be^{i\phi}|-\alpha,\alpha\rangle\Big), \tag{17}$$

with $N^2 = a^2+b^2+2abe^{-4R^2}\cos\phi$ and $a|\alpha\rangle = Re^{i\theta}|\alpha\rangle$, $a|-\alpha\rangle = -Re^{i\theta}|-\alpha\rangle$ are coherent state, ϕ is the relative phase between a and b.

The entanglement of the state (17) can be computed by the same way as above

$$c = \frac{2|a||b|(1-e^{-4R^2})}{a^2+b^2+2abe^{-4R^2}\cos\phi}. \tag{18}$$

With the same way, the Z_2 can be obtained

$$\begin{aligned} Z_2 = {} & \langle a^+a + b^+b + ab^+ + ba^+\rangle - \frac{1}{2}\langle a^2+b^2+a^{+^2}+b^{+^2}+2ab+2a^+b^+\rangle \\ & + \frac{1}{2}\Big(\langle a\rangle + \langle b\rangle - \langle a^+\rangle - \langle b^+\rangle\Big)^2 = 0. \end{aligned} \tag{19}$$

The state (17) does not exist squeezing. And the Mandel factor Q

$$Q=\frac{4R^2abe^{-4R^2}\cos\phi}{a^2+b^2+2abe^{-4R^2}\cos\phi}. \tag{20}$$

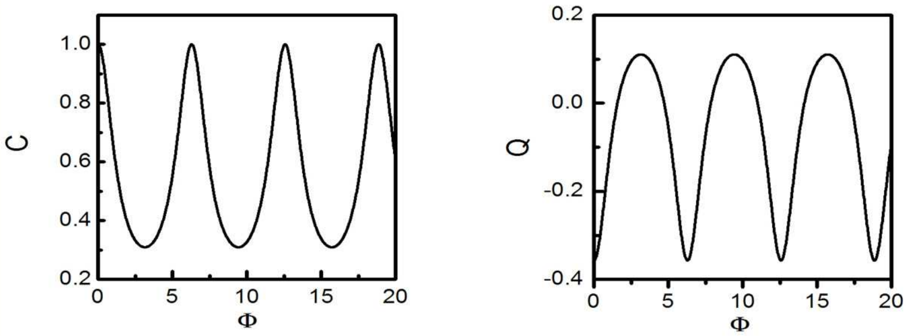

Figure 2: $a=-1$, $b=1$, $R=0.2$. (a) Concurrence. (b) Q as a function of ϕ.

In Fig. 2, we plot the concurrence, Mandel function Q versus ϕ when $a=-1$, $b=1$, $R=0.2$, and can see that the two functions are a periodic function of ϕ, and can find when taking $a=-1$, $b=1$, $R=0.2$, there are photon antiquating but not squeezing, and entanglement always exists. Furthermore, we can see when ϕ are some numbers, there is not antiquating. However, the biggest antiquating corresponds with the strongest entanglement. So for the second kind of entangled coherent states, there does not exist squeezing, but exists antibunching and the antibunching increases along with the increase of entanglement. It attracts our attention that when there are not nonclassical effects, the entanglement exists; it shows that the entanglement is an independent quantum effect and it has lots of unknown property that should be further studied in future.

3 Conclusion

We study the concurrence, squeezing and antiquating of two kinds of entangled coherent states and find an interesting phenomenon. the entangled coherent states in [10] are just as the state (2) with $\mu=1$, $\nu=-1$; $\mu=1$, $\nu=1$ and the authors find the entanglement does not have a relationship with the squeezing as mentioned in that paper and find when having entanglement, the state do not exist squeezing or exist squeezing only under some suitable conditions (we also get this). In this paper, we get there exists photon antiquating when there are not squeezing under some suitable conditions, so in the continuous variable systems, for the entangled coherent states, the entanglement exists some relationship with the photon antiquating of two-mode fields, the entanglement always follows one of squeezing and antiquating and the entanglement increases along with the increase of a nonclassical effect.

In conclusion, for two kinds of entangled coherent states, we get the relationship between the entanglement and photon antiquating of two-mode fields by computing the concurrence and the Mandel factor. Whether we can get this relationship by basic theory just as the relationship between the entanglement and squeezing can be gotten by the variance of some operators, this is the most important remaining problem we will consider.

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