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Influence of homogeneous weak electric field on the chemical potential and heat capacity of charged fermion system

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Abstract. In light of the semiclassical (Thomas-Fermi) approximation method, the thermodynamic properties of the charged fermion system in homogeneous weak electric field are studied. After deriving the relationship of chemical potential and heat capacity of charged fermion system changed with the external electric field, then the analytic formula of the chemical potential and heat capacity of charged fermion system in the high and low temperature approximation condition is desired. Moreover, the influence of the homogeneous weak electric field on the chemical potential and heat capacity are analyzed.

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Key words: charged fermion, chemical potential,heat capacity, homogeneous weak electric field

1 Introduction

The Bose-Einstein condensation (BEC) was originally conceived in 1925 by Albert Einstein, who calculated that if a gas of atoms could be cooled below a transition temperature, it should suddenly condense into a remarkable state in which all the atoms have exactly the same location and energy. In other words, the wave-function of each atom in a Bose-Einstein condensate should extend across the entire sample of gas. In 1995, Eric A. Cornell, Carl E. Wieman and Wolfgang Ketterle directly proved BEC by using the optic cooling and magnetic trap techniques in experiment [1–7]. The abnormal properties of BEC possess potential appliance value in the fields of chip technology, precision measurement, nano-technology, etc. With the development and improvement of experimental techniques, extensive research on the thermodynamic properties of Fermi gas in an external electric field there has been conducted and a large number of meaningful results have been obtained [8–21].

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Uniform electric field is easy to realize and control experimentally. In uniform electric field, the charged system contains particles has different properties compared with the free-particle system [22]. Therefore, it is very meaningful to study the thermodynamic properties of charged fermion system in uniform electric field and compare with the one of free-particle system. The study on the effect of weak electric field with the simplest space distribution on the thermodynamic properties of matter system will lay a foundation for further study on the external electric field with the complex time-space distribution.

In this paper, the thermodynamic properties of charged fermion system in homogeneous weak electric field are studied by using the semiclassical approximation of statistical physics. After deriving the relationship of the chemical potential and heat capacity of charged fermion system changed with external electric field, then the analytic formula of the chemical potential and heat capacity of charged fermion system at high and low temperature, and the influence of the homogeneous weak electric field on the chemical potential and heat capacity are analyzed. In addition, the thermodynamic properties of charged and free fermion system in homogeneous weak electric field are also compared.

2 Total population and total energy of fermion system

It is assumed that N fermion system are restricted in a cylindrical chamber with a basal area of S and height of L, the charge and mass of each fermion system are q and m respectively. When a uniform electric field is external along the axial direction of the cylindrical chamber regarded as x axis, the energy of a single fermion in the case of ignoring the influence of gravity can be expressed as

$$\varepsilon=\frac{p_x^2+p_y^2+p_z^2}{2m}-qEx, \tag{1}$$

where E is the strength of homogeneous weak electric field, and then the electric potential energy $u(x)$ in this electric field is $-qEx$.

The N rarefied charged fermion with the energy of ε occupy the space volume of V in a weak uniform electric field, where the population N tends to infinity. If neglecting the interparticle weak interaction and the kinetic energy of fermion being much larger than its potential energy, the semiclassical (Thomas-Fermi) approximate method is applicable. Let g be the possible spin degeneracy of the charged fermion (as the electronic spin degeneracy $g=2$), the possible quantum state density with the consideration of all above can be represented as

$$\begin{aligned} D(\varepsilon)=&\frac{2\pi g(2m)^{3/2}V}{h^3L}\int_0^L\sqrt{\varepsilon+qEx}\,dx\\ \approx&\frac{2\pi g(2m)^{3/2}V\varepsilon^{1/2}}{h^3}\left(1+\frac{qEL}{4\varepsilon}\right). \end{aligned} \tag{2}$$

Defining the system fugacity $z=e^{\mu/kT}$, the particle occupation number of the energy state ε

can be written as

$$f(\varepsilon)=\frac{1}{1+e^{(\varepsilon-\mu)/(kT)}}=\frac{1}{1+z^{-1}e^{\varepsilon/(kT)}}, \tag{3}$$

where μ is the chemical potential, T is the Kelvin degree, and $k=1.38065\times10^{-23}$ J/K, is the the Boltzmann constant. Let T be 0.01 K, 100 K, 1000 K and 2000 K, the dependences of $f(\varepsilon)$ on the variable of $(\varepsilon-\mu)$ are drawn, as shown in the figure 1. It reveales that the value of $f(\varepsilon)$ varies from 1 to 0 when the variable of $(\varepsilon-\mu)$ varies from -2 eV to 2 eV. Moreover, the lower the temperature T, the bigger the gradients in the transition region. Especially for $T=0.01$ K, the corresponding curve is in rectangular distribution. Namely,

$$f(\varepsilon)=1 \text{ for } (\varepsilon-\mu)<0 \text{ and } f(\varepsilon)=0 \text{ for } (\varepsilon-\mu)>0.$$

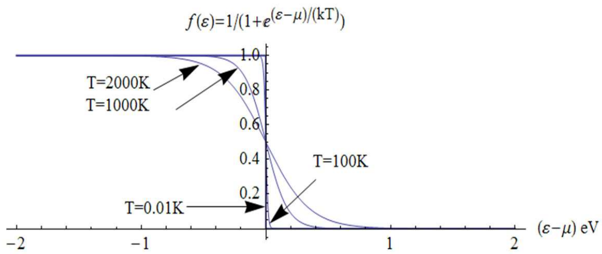

Figure 1: The distribution function curve of charged fermion system

For convenience, it is supposed that

$$G=\frac{2\pi g(2m)^{3/2}V}{h^3}. \tag{4}$$

The combination of Eqs. (2)−(4) leads to

$$N\approx G\int_0^{\infty}\frac{\varepsilon^{1/2}+qEL\varepsilon^{-1/2}/4}{e^{(\varepsilon-\mu)/kT}+1}d\varepsilon, \tag{5a}$$

$$U\approx G\int_0^{\infty}\frac{\varepsilon^{3/2}+qEL\varepsilon^{1/2}/4}{e^{(\varepsilon-\mu)/kT}+1}d\varepsilon, \tag{5b}$$

where N and U is respectively the total number and total energy of particles in the system. Let $y=\mu/(kT)$ and defining the fermion integral function by using the Gamma function, namely

$$fm_n(y)=\frac{1}{\Gamma(n)}\int_0^{\infty}\frac{x^{n-1}dx}{e^{x-y}+1}=-PolyLog(n,-e^y). \tag{6}$$

Thus Eq. (5) can be rewritten as

$$N=\left(\frac{\sqrt{\pi}}{2}\right)(kT)^{3/2}G\left(fm_{3/2}(y)+\frac{qEL}{2kT}fm_{1/2}(y)\right), \tag{7a}$$

$$U=\left(\frac{3\sqrt{\pi}}{4}\right)G(kT)^{5/2}\left(fm_{5/2}(y)+\frac{qEL}{6kT}fm_{3/2}(y)\right). \tag{7b}$$

3 Low temperature properties of charged fermion system

3.1 Fermion energy and ground state energy

Let $\mu(0)$ be the chemical potential of charged fermion system, the fermion distribution function shows a rectangle distribution for $T\to 0$ K (seen in Fig. 1), namely

$$f(\varepsilon)=\begin{cases}1, & \varepsilon<\mu(0),\\ 0, & \varepsilon>\mu(0).\end{cases} \tag{8}$$

In the energy lowest theory, particles firstly take up the level with lowest energy and are restricted by the Pauli principle. Then each particle fills a quantum state whose energy is from 0 to $\mu(0)$. Thus

$$N=G\int_0^{\mu(0)}\left(\varepsilon^{1/2}+\frac{qEL}{4}\varepsilon^{-1/2}\right)d\varepsilon=\frac{2G}{3}\mu^{3/2}(0)\left(1+\frac{3qEL}{4\mu(0)}\right), \tag{9}$$

where $\mu(0)=\varepsilon_F$, is defined as fermion energy, ε_{F0} is the fermion energy of free-system without the external electric field. Due the very small external electric field leads to $qEL\ll\mu(0)$, solving Eq. (9) gives by

$$\mu(0)=\varepsilon_F=\varepsilon_{F0}-qEL/2,$$
$$\varepsilon_{F0}=(3N/2G)^{2/3}=\frac{h^2}{2m}\left(\frac{3}{4\pi g}\frac{N}{V}\right)^{2/3}=\frac{\hbar}{2m}\left(\frac{6\pi^2}{g}\frac{N}{V}\right)^{2/3}. \tag{10}$$

Set $L=1$ m, $q=1.6\times10^{-19}$ C, $E=1\times10^{-3}$ V/m, $N/V=(6.02214/2.2414)10^{25}$ and $g=1$ in Eq. (10). It is obtained that the value of ε_{F0} is an 8.31933×10^{-21} J $=0.051925$ eV. This value is much larger than the adding energy caused by the weak electric field ($qEL/2=8.011\times10^{-23}$ J $=5\times10^{-4}$ eV and $\mu(0)=\varepsilon_F=8.23922\times10^{-21}$ J $=0.051425$ eV). The fermion energy slightly decreases because of the influence of external electric field. Especially the chemical potential is greater than zero.

According to Eqs. (6)−(8), the ground state energy of charged fermion system for $T\to 0$ K can be expressed as

$$U(0)=G\int_0^{\mu(0)}\left(\varepsilon^{3/2}+\frac{qEL}{4}\varepsilon^{1/2}\right)d\varepsilon$$
$$=(2G/5)\mu^{5/2}(0)\left(1+\frac{5qEL}{12\mu(0)}\right)=\left(\frac{3}{5}\varepsilon_{F0}-\frac{1}{2}qEL\right)N. \tag{11}$$

It presents that the influence of homogeneous weak electric field makes the ground state energy decrease. In particular for $T \to 0$ K, the average energy of charged fermion system $U(0)/N = 0.030655$ eV is smaller than the chemical potential $\mu(0)$ and then $U = 131961$ J, which is the inrenal energy of 44.615 mol material.

3.2 Heat capacity and chemical potential

Define fermion temperature $T_F = \mu(0)/k \approx 600$ K. If the temperature is very low, namely $T \ll T_F$, it is obtained that $kT/\mu \ll 1$ in Eq. (5). Thus the fermion distribution function is well approximate to be in rectangle distribution. The truncation and approximate solution of integral equation can be given by

$$\begin{aligned}&\int_0^\infty \frac{\varepsilon^n}{e^{(\varepsilon-\mu)/kT}+1}d\varepsilon \\ =&\mu^{n+1}\left(\frac{1}{n+1}+\frac{\pi^2 n}{6}\left(\frac{kT}{\mu}\right)^2+\frac{7\pi^2 n(n-1)(n-2)}{360}\left(\frac{kT}{\mu}\right)^4+\cdots\right).\end{aligned} \tag{12}$$

It is noted that the electric potential energy caused by the weak external electric field is a first-order small quantity. Neglecting the third- and higher-order small quantities in Eq. (12) and taking the first two terms into Eq. (5a), the population equation of fermion system at low temperature can be expressed as

$$N \approx \frac{2}{3}G\mu^{3/2}\left(1+\frac{3qEL}{4\mu}+\frac{\pi^2}{8}\left(\frac{kT}{\mu}\right)^2\right), \tag{13}$$

By using Eq. (10) and the method of progressive approximation, the heat capacity of fermion system at low temperature can be solved and given by

$$\mu_0 \approx \left(\frac{3N}{2G}\right)^{2/3} = \varepsilon_{F0}, \tag{14a}$$

$$\mu_1 \approx \varepsilon_{F0}\left(1-\frac{qEL}{2\varepsilon_{F0}}\right) = \varepsilon_{F0}-\frac{qEL}{2} = \varepsilon_F, \tag{14b}$$

$$\begin{aligned}\mu(T)=\mu_2 &\approx \varepsilon_{F0}\left(1+\frac{3qEL}{4\varepsilon_F}+\frac{\pi^2}{8}\left(1-\frac{qEL}{4\varepsilon_F}\right)\left(\frac{kT}{\varepsilon_F}\right)^2\right)^{-2/3} \\ &\approx \varepsilon_{F0}\left(1-\frac{qEL}{2\varepsilon_{F0}}+\frac{1}{16}\left(\frac{qEL}{\varepsilon_{F0}}\right)^2-\frac{\pi^2}{12}\left(\frac{kT}{\varepsilon_{F0}}\right)^2\right).\end{aligned} \tag{14c}$$

Let $L = 1$ m, $q = 1.6\times10^{-19}$ C, $N/V = (6.02214/2.2414)10^{25}$, $g = 1$, $k = 1.38\times10^{-23}$ J/K, $T =$ 50 K and $E = 1\times10^{-3}$ V/m. It is obtained that $\varepsilon_F = 8.23922\times10^{-21}$ J=0.0514251 eV, $\mu(T) =$ 8.1923×10^{-21} J $= 0.0511322$ eV. Through comparison, it is found that the heat capacity of charged fermion system at low temperature decrease slightly with T.

Combining Eqs. (5b), (10) and (12) and neglecting the higher-order small quantities, the internal energy of charged fermion system at low temperature can be obtained

$$U=\frac{2G\mu^{5/2}}{5}\left(1+\frac{5\pi^2}{8}\left(\frac{kT}{\mu}\right)^2\right)+\frac{qELG\mu^{3/2}}{6}$$
$$\approx\frac{3\varepsilon_{F0}}{5}\left(1-\frac{5qEL}{6\varepsilon_{F0}}+\frac{5}{16}\left(\frac{qEL}{\varepsilon_{F0}}\right)^2+\frac{5\pi^2}{8}\left(\frac{kT}{\varepsilon_{F0}}\right)^2\right)N, \tag{15}$$

By keeping the external electric field constant and calculating the first derivative with respect to T, the heat capacity of N charged fermion system can be obtained and expressed as

$$C=\frac{\partial U}{\partial T}\approx\frac{3}{4}\frac{(k\pi)^2}{\varepsilon_{F0}}NT. \tag{16}$$

Eq. (16) shows that the heat capacity of charged fermion system with weak external field at low temperature increases linearly with T. Let $N=2.6867\times10^{25}$, $k=1.38\times10^{-23}$ J/K, $g=1$ and $T=50$ K. It is obtained that $U=137672$ J, $C=1293.6$ J/K mol.

4 Heat capacity and chemical potential at high temperature

For the case of high temperature, the series development of fermion integral function with respect to $y=\mu/(kT)\ll1$ can be expressed as

$$fm_{1/2}(y)=-\mathrm{PolyLog}(1/2,-e^y)\approx0.604899+0.380105y,$$
$$fm_{3/2}(y)=-\mathrm{PolyLog}(3/2,-e^y)\approx0.765147+0.604899y,$$
$$fm_{5/2}(y)=-\mathrm{PolyLog}(5/2,-e^y)\approx0.8672+0.765147y. \tag{17}$$

Then substituting Eq. (17) into Eq. (7a) and neglecting the second-order and above driblets, one obtains

$$N=(kT)^{3/2}G\left(0.678094+0.536078\frac{\mu+qEL/2}{kT}\right). \tag{18}$$

By solving Eq. (18), the chemical potential is given by

$$\mu=1.2436\left(\frac{\varepsilon_{F0}}{kT}\right)^{1/2}-1.26492kT-qEL/2. \tag{19}$$

Let $N=2.6867\times10^{25}$, $k=1.38\times10^{-23}$ J/K, $L=1$ m, $q=1.6\times10^{-19}$ C, $E=1\times10^{-3}$ V/m, $T=3000$ K and $g=1$, it is obtained that $\varepsilon_{F0}=8.31933\times10^{-21}$ J, $\varepsilon_F=8.23922\times10^{-21}$ J, $\mu=-4.78357\times10^{-20}$ J $=-0.298567$ eV. This result shows that the chemical potential of charged fermion system is less than zero at high temperature and its absolute value is 6 times of the one at low temperature. Through comparison, it is found that the chemical potential changes from positive to negative and becomes lower and lower when the variation of temperature is from low to high.

Considering Eqs. (7b) and (17) and neglecting the second and higher-order small quantities, the heat capacity and chemical potential of system at high temperature can be respectively written as

$$U=\left(\frac{kT}{\varepsilon_{F0}}\right)^{3/2}\left(1.72927+1.52571\left(\frac{\mu}{kT}+\frac{qEl}{6kT}\right)\right)NkT, \tag{20}$$

$$C=\frac{\partial U}{\partial T}=\left(\frac{kT}{\varepsilon_{F0}}\right)^{3/2}\left(4.32318+2.28856\left(\frac{\mu}{kT}+\frac{qEL}{6kT}\right)\right)Nk. \tag{21}$$

Let $N=2.6867\times10^{25}$, $k=1.38\times10^{-23}$ J/K, $L=1$ m, $q=1.6\times10^{-19}$ C, $E=1\times10^{-3}$ V/m, $T=3000$ K. It is obtained that $U=-393158$ J, $C=155.319$ J/(K mol).

5 Conclusions

The chemical potential, average internal energy, heat capacity and relative variation of charged fermion system under different temperature are listed in Table 1. Compared to the situation without external electric field, the chemical potential, average internal energy and heat capacity of charged fermion system with external electric field are all decrease. However, the chemical potential and average internal energy show considerable change, while the weak external electric field has no effect on the heat capacity of charged fermion system at low temperature.

Table 1: The chemical potential, average internal energy, heat capacity and relative variation of charged fermion system under different temperature.

	$\mu(T)$ (eV)			$U(T)/N$ (eV)			C (J/K mol)	
Conditions	$T\to0$ K	$T=50$ K	$T=3000$ K	$T\to0$ K	$T=50$ K	$T=3000$ K	$T=50$ K	$T=3000$ K
Without external field	0.051925	0.051631	−0.29067	0.031155	0.0324783	−0.0856827	1293.6	155.592
With external field	0.051425	0.051132	−0.298567	0.030655	0.0319819	−0.0913324	1293.6	155.319
Relative variation	−0.96%	−0.97%	−0.17%	−1.6%	−1.5%	−6.6%	0	−0.18%

The chemical potential, internal energy and heat capacity of a given charged fermion system without external electric field are only considered as function of temperature T. However, the chemical potential, internal energy and heat capacity are relative to T and the external electric field when the given charged fermion system is in an external electric field. Eqs. (10) and (11) show that the chemical potential $\mu(0)$ and ground state energy $U(0)$ of charged fermion system decrease with electric intensity for $T\to0K$. At low temperature, the chemical potential of fermion system with weak external electric field is lower than the one of free-system. With temperature increasing, the chemical potential continues to decrease slightly. When the weak external electric field satisfies the condition of $KT\gg qEL$ for $T\gg T_F$, it has little effect on the heat capacity.

In summary, we investigate the effect of weak external electric field with the simplest spatial distribution on the thermodynamic properties of matter system, which will lay a foundation of further study on the effect of complex external electric field on the thermodynamic properties of the matter system.

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References

[1] M. H. Anderson, J. R. Ensher, M. R. Matthews, *et al*., Science 269 (1995) 198.
[2] J. R. Ensher, D. S. Jin, M. R. Matthews, *et al.*, Phys. Rev. Lett.77 (1996) 4984.
[3] M. Bayindir and B.Tanatar, Phys. Rev. A 58 (1998) 3134.
[4] K. B. Davis, M. O. Mewes, M. R. Andrews, *et al.*, Phys. Rev. Lett. 75 (1995) 3969.
[5] W. Petrich, M. H. Anderson, J. R. Ensher, *et al.*, Phys. Rev. Lett. 74 (1995) 3352.
[6] Y. Takasu, K. Maki, K. Komori, *et al.*, Phys. Rev. Lett. 91 (2003) 040404.
[7] Z. Y. Zhang, H. X. Fu, J. C. Chen, *et al.*, Phys. Rev. A79 (2009) 055603.
[8] H. Liu, J. At. Mol. Phys. 25 (2008) 646 (in Chinese).
[9] G. M. Bruum and K. Burnrtt, Phys. Rev. A 58 (1998) 2427.
[10] C. A. Regal, M. Greiner, and D. S. Jin, Phys. Rev. Lett. 92 (2004) 040403.
[11] G. Z. Su, J. C. Chen, and L. X. Chen, Phys. Lett. A 315 (2003) 109.
[12] F. D. Men, H. Liu, and H. Y. Zhu, Chinese Phys. B 17 (2008) 3236.
[13] F. D. Men and H. Liu, Chinese Phys. 15 (2006) 2856.
[14] Y. Shin, C. H. Schunck, A. Schirotzek, *et al.*, Nature 45 (2008) 689.
[15] M. Bradley, C. A. Sackett, and R. G. Hulet, Phys. Rev. Lett. 78 (1997) 985.
[16] M. Tezuka and M. Ueda, Phys. Rev. Lett. 100 (2008) 110403.
[17] M. V. Ivanov, M. B. Barbaro, J. A. Caballero, *et al.*, Phys. Rev. C 77 (2008) 034612.
[18] R. Roldan, M. P. Lopez-Sancho, and F. Guinea, Phys. Rev. B 77 (2008) 115410.
[19] A. V. Chubukov, D. L. Maslov, and R Saha, Phys. Rev. B 77 (2008) 085109.
[20] S. K. Cai, G. Z. Su, and J. C. Chen, J. Phys. A: Math. Theor. 40 (2007) 11245.
[21] S. R. Granade, M. E. Gehm, K. M. O'Hara, *et al.*, Phys. Rev. Lett. 88 (2002) 120405.
[22] Y. Q. Sun, C. J. Huang, S. M. Long, *et al.*, Acta Phys. Sin. 58 (2009) 7502 (in Chinese).