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Symmetric remote two-qubit preparation via positive operator-valued measure

Zhang-Yin Wang[a,*] and Xing-Qiang Yang[b,*]

[a] *Key Laboratory of Optoelectronic Information Acquisition & Manipulation of Ministry of Education of China, School of Physics & Material Science, Anhui University, Hefei 230039, China*
[b] *College of Physics & Electronic Engineering, Nanyang Normal University, Nanyang 473061, China*

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Abstract. We present a novel tripartite scheme for remotely preparing an arbitrary two-qubit state with two three-qubit entanglements. By using a proper positive operator-valued measure (POVM), it is shown that the remote two-qubit preparation can be realized in either distant ministrant's place in a probabilistic manner via their collaboration. We also explore its applications to six special ensembles of state in detail. The extensive investigations show that the remote preparation can be achieved with higher probability provided that the prepared state belongs to the six special ensembles.

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Key words: remote state preparation, three-qubit entanglement, positive operator-valued measure, unitary operationd

1 Introduction

Applying the theory of quantum mechanics in the field of information, many interesting developments have been produced in last decades [1–15], such as quantum teleportation [1], quantum dense coding [2], quantum secret sharing [3], remote state preparation [4], and so on. Quantum teleportation was first proposed by Bennett *et al.* [1] in 1993. It is a method for interchanging quantum resources between different places. In 2000 Lo [4] formally presented another interesting novel method to transmit pure quantum states.

*Corresponding author. *Email addresses:* `zywang@ahu.edu.cn` (Z. Y. Wang); `yxingqiang@yahoo.com.cn` (X. Q. Yang)

It also utilizes a prior shared entanglement and some classical communication. Conventionally, this new communication protocol is termed as remote state preparation (RSP) and viewed as ”teleportation of a known state”. In RSP the prepared state is assumed to be completely known by the sender. In contrast, the teleported state is not required to be known by the sender in quantum teleportation. Moreover, due to the prior knowledge about the original state, to some extend the classical communication and entanglement cost can be reduced in RSP process. For an example, Pati [12] showed that for a qubit chosen from equatorial or polar great circles on a Bloch sphere, RSP requires only one forward classical bit, exactly half that of quantum teleportation. However, for general states RSP procedure requires as much communication cost as quantum teleportation. The detailed trade-off between the classical communication cost and the required entanglement in RSP procedure can be studied distinctly in the protocol proposed by Bennett *et al.* [13].

In the last decade, after Lo's pioneering work [4], RSP has attracted much attention [16–33]. Also some RSP schemes are investigated by using different entangled states as quantum channel [34–42]. In terms of entanglements in quantum channel, these RSP schemes can be classified into two types. One takes pure entangled states as the quantum channel [34–38] while the other utilizes partly pure entangled states [39–42]. In the latter case, usually people need to introduce one or more auxiliary qubits and then entangle them with his/her original qubits. By performing proper measurements on his/her qubits including the ancillas the prepared state can be collapsed to one of the eligible states. Then conditioned on the measurement results on the auxiliary qubits, the receiver performs an appropriate unitary operations on the eligible state to properly retrieve the prepared state. Note that, the so-called proper measurements are projective measurements in the latter type of existing RSP schemes [39–42]. As a matter of fact, there lies another type of measurement named positive operator-valued measure (POVM) [43], which was also called generalized measurement [44]. Since in the RSP schemes, the post-measurement state of the auxiliary system is of little interest. In contrast, the main item of interest is the probability of respective measurement results. Therefore, one may conjecture that it is quite possible to use positive operator-valued measure (POVM) [43] instead of usual projective measurement to realize RSP protocols. As a matter of fact, POVM has already attracted much attention and been employed in various quantum information processing [45–49].

However, to our best knowledge, so far there has been no proposal for remote preparation of an arbitrary two-qubit entangled state via positive operator-valued measure and three-qubit entanglements. In this contribution we show that it is indeed possible to construct such RSP protocol. That is, by using positive operator-valued measure, we propose a tripartite scheme for symmetrically preparing an arbitrary two-qubit state via two non-maximally entangled three-qubit states.

This paper is organized as follows: in Section 2, a symmetric tripartite RSP scheme is amply presented. Then its applications to six special ensembles of states are investigated in Section 3. At last, concise discussions and brief summaries are given in Section 4.

2 Symmetric tripartite RSP scheme

Now let us present the symmetric RSP scheme. Suppose Alice is the state preparer, Bob and Charlie are the two remote ministrants. Alice, Bob and Charlie share in priori two non-maximally entangled three-qubit states

$$\begin{aligned}&|\Psi\rangle_{123}=a|000\rangle_{123}+b|111\rangle_{123}(|a|^2+|b|^2=1),\\&|\Phi\rangle_{456}=c|000\rangle_{456}+d|111\rangle_{456}(|c|^2+|d|^2=1),\end{aligned}\tag{1}$$

where a, b, c and d are nonzero real numbers, and satisfy $|a|\geq|b|$ and $|c|\geq|d|$. Qubits (1, 4) belong to Alice while qubits (2, 5) and (3, 6) to Bob and Charlie, respectively. Alice wants to prepare remotely a state with the two ministrants' help. The prepared state is

$$|V\rangle=\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle,$$

where α,β,γ,δ are arbitrary complex numbers and satisfy

$$|\alpha|^2+|\beta|^2+|\gamma|^2+|\delta|^2=1.$$

Alice knows it exactly while Bob and Charlie do not. Owing to the channel symmetry for Bob and Charlie, each of them has the chance to construct the state $|V\rangle$ with another's assistance. Specifically, Charlie can retrieve it with Bob's help and vice versa. Without loss of generality, suppose Charlie is assigned to construct the prepared state. To fulfill the state preparation, Alice carries out a two-qubit projective measurement on her qubits 1, 2 in a set of mutually orthogonal basis vectors $\{|\lambda_1\rangle,|\lambda_2\rangle,|\lambda_3\rangle,|\lambda_4\rangle\}$, which are given as

$$\begin{aligned}&|\lambda_1\rangle=\alpha|00\rangle+\beta|01\rangle+\gamma|10\rangle+\delta|11\rangle,\\&|\lambda_2\rangle=\eta\alpha|00\rangle+\eta\beta|01\rangle-\eta^{-1}\gamma|10\rangle-\eta^{-1}\delta|11\rangle,\\&|\lambda_3\rangle=\beta^*|00\rangle-\alpha^*|01\rangle+\delta^*|10\rangle-\gamma^*|11\rangle,\\&|\lambda_4\rangle=\eta\beta^*|00\rangle-\eta\alpha^*|01\rangle-\eta^{-1}\delta^*|10\rangle+\eta^{-1}\gamma^*|11\rangle,\end{aligned}\tag{2}$$

where

$$\eta=\sqrt{\frac{1-p}{1+p}},\quad p=|\alpha|^2+|\beta|^2-|\gamma|^2-|\delta|^2.$$

This four non-maximally entangled basis states are related to the computation basis vector $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ and form a complete orthogonal basis set in a four-dimensional Hilbert space, i.e., $\langle\lambda_i|\lambda_j\rangle=\delta_{ij}$.

Note that the entangled state $|\Psi\rangle_{123}|\Phi\rangle_{456}$ in the basis $\{|\lambda_1\rangle,|\lambda_2\rangle,|\lambda_3\rangle,|\lambda_4\rangle\}$ can be written as

$$\begin{aligned}|\Psi\rangle_{123}|\Phi\rangle_{456}=&(ac|000000\rangle_{123456}+ad|000111\rangle_{123456}+bc|111000\rangle_{123456}+bd|111111\rangle_{123456})\\=&|\lambda_1\rangle_{14}\otimes|\Gamma_1\rangle_{2356}+|\lambda_2\rangle_{14}\otimes|\Gamma_2\rangle_{2356}+|\lambda_3\rangle_{14}\otimes|\Gamma_3\rangle_{2356}+|\lambda_4\rangle_{14}\otimes|\Gamma_4\rangle_{2356},\end{aligned}\tag{3}$$

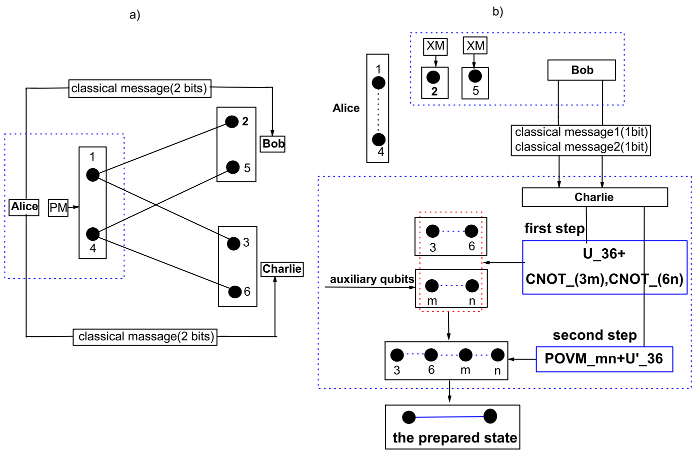

Figure 1: a) Alice, Bob and Charlie each keeps two qubits from the two non-maximally entangled three-qubit states. Alice makes a two-qubit projective measurements (PM) and then broadcasts her measurement results (2 bits). b) Bob is assigned to measure his two qubits in the X bases (XM), respectively, and then tells Charlie his measurement results (2 bits). Conditioned on Alice and Bob's classical message, Charlie constructs the prepared state by incorporating two auxiliary qubits and executing appropriate operations (U, CNOT, POVM).

where

$$\begin{aligned}
|\Gamma_1\rangle_{2356} &= \alpha^* ac|0000\rangle_{2356} + \beta^* ad|0011\rangle_{2356} + \gamma^* bc|1100\rangle_{2356} + \delta^* bd|1111\rangle_{2356},\\
|\Gamma_2\rangle_{2356} &= \eta\alpha^* ac|0000\rangle_{2356} + \eta\beta^* ad|0011\rangle_{2356} - \eta^{-1}\gamma^* bc|1100\rangle_{2356} - \eta^{-1}\delta^* bd|1111\rangle_{2356},\\
|\Gamma_3\rangle_{2356} &= \beta ac|0000\rangle_{2356} - \alpha ad|0011\rangle_{2356} + \delta bc|1100\rangle_{2356} - \gamma bd|1111\rangle_{2356},\\
|\Gamma_4\rangle_{2356} &= \eta\beta ac|0000\rangle_{2356} - \eta\alpha ad|0011\rangle_{2356} - \eta^{-1}\delta bc|1100\rangle_{2356} + \eta^{-1}\gamma bd|1111\rangle_{2356}. \qquad (4)
\end{aligned}$$

After this measurement, Alice broadcasts her measurement result in terms of a prior agreements (shown in Fig. 1(a)), i.e., ″00″ correspond to $|\lambda_1\rangle$, ″01″ to $|\lambda_2\rangle$, ″10″ to $|\lambda_3\rangle$ and ″11″ to $|\lambda_4\rangle$, respectively. According to the above equation, one can see that Alice's measurement result should be one of the four states defined in Eq. (2). Without loss of generality, suppose Alice measures $|\lambda_3\rangle_{14}$. So she sends two classical bits ″10″ to publish her measurement result. Then according to Eq. (3), it is known the joint state of qubit pairs 2, 3 and 5, 6 will collapse to $|\Gamma_3\rangle_{2356}$. As mentioned just, Charlie is assigned to construct the prepared state with Bob's help. Hence, after Alice's publication, Bob is asked to measure his two qubits 2 and 5 in the X bases $\{|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}, |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}\}$, respectively. After Bob's measurements, in order to construct the original state, Charlie cooperates with Bob to get his help. Provided Bob agrees to cooperate with Charlie, he will communicate his measurement results to Charlie over a public channel (shown in Fig. 1(b)). To be specifical, Bob sends two one-bit classical message to Charlie corresponding to his measurement results. Incidentally, Bob and Charlie agree in advance that, the one classical bit ″0″ corresponds to the state $|+\rangle$, ″1″ to $|-\rangle$, respectively.

In the X bases, the collapsed state $|\Gamma_3\rangle_{2356}$ can be reexpressed as

$$\begin{aligned}|\Gamma_3\rangle_{2356}=&\frac{1}{2}|+\rangle_2|+\rangle_5(\beta ac|00\rangle_{36}-\alpha ad|01\rangle_{36}+\delta bc|10\rangle_{36}-\gamma bd|11\rangle_{36})\\&+\frac{1}{2}|+\rangle_2|-\rangle_5(\beta ac|00\rangle_{36}+\alpha ad|01\rangle_{36}+\delta bc|10\rangle_{36}+\gamma bd|11\rangle_{36})\\&+\frac{1}{2}|-\rangle_2|+\rangle_5(\beta ac|00\rangle_{36}-\alpha ad|01\rangle_{36}-\delta bc|10\rangle_{36}+\gamma bd|11\rangle_{36})\\&+\frac{1}{2}|-\rangle_2|-\rangle_5(\beta ac|00\rangle_{36}+\alpha ad|01\rangle_{36}-\delta bc|10\rangle_{36}-\gamma bd|11\rangle_{36}).\end{aligned}\tag{5}$$

Naturally, Bob's measurement result should be one of the two states $|+\rangle$ and $|-\rangle$. Without loss of generality, suppose further Bob's measurement results are $|+\rangle_2|-\rangle_5$. So Bob sends '0' and '1' to Charlie. After receiving the classical bits, according to Eq. (5), Charlie knows the state of his qubits (3, 6) is

$$|\Delta_1\rangle_{36}=\frac{1}{2}\Big(\beta ac|00\rangle_{36}+\alpha ad|01\rangle_{36}+\delta bc|10\rangle_{36}+\gamma bd|11\rangle_{36}\Big).\tag{6}$$

At this stage, Charlie performs $U_1=I_3\otimes\sigma_6^x$ on her qubits 3 and 6, which transforms $|\Delta_1\rangle_{36}$ into

$$|\Theta_1\rangle_{36}=I_3\otimes\sigma_6^x|\Delta_1\rangle_{36}=\frac{1}{2}\Big(ad\alpha|00\rangle_{36}+ac\beta|01\rangle_{36}+bd\gamma|10\rangle_{36}+bc\delta|11\rangle_{36}\Big).\tag{7}$$

After this, Charlie introduces two auxiliary qubits m and n in the initial state $|00\rangle_{mn}$, and then performs two controlled-not (CNOT) operations with qubits 3 and 6 as the controlled qubits while the auxiliary qubits m and n as the target ones, respectively. These two CNOT operations transform the state $|\Theta_1\rangle_{36}|00\rangle_{mn}$ into the following form

$$\begin{aligned}|T\rangle_{36mn}=&\frac{1}{2}(\alpha ad|0000\rangle_{36mn}+\beta ac|0101\rangle_{36mn}+\gamma bd|1010\rangle_{36mn}+\delta bc|1111\rangle_{36mn})\\=&\frac{1}{8}(|K_1\rangle_{36}|Q_1\rangle_{mn}+|K_2\rangle_{36}|Q_2\rangle_{mn}+|K_3\rangle_{36}|Q_3\rangle_{mn}+|K_4\rangle_{36}|Q_4\rangle_{mn}),\end{aligned}\tag{8}$$

where

$$\begin{aligned}&|K_1\rangle_{36}=\alpha|00\rangle_{36}+\beta|01\rangle_{36}+\gamma|10\rangle_{36}+\delta|11\rangle_{36}\equiv|V\rangle,\\&|Q_1\rangle_{mn}=ad|00\rangle_{mn}+ac|01\rangle_{mn}+bd|10\rangle_{mn}+bc|11\rangle_{mn},\\&|K_2\rangle_{36}=(\alpha|00\rangle_{36}+\beta|01\rangle_{36}-\gamma|10\rangle_{36}-\delta|11\rangle_{36},\\&|Q_2\rangle_{mn}=ad|00\rangle_{mn}+ac|01\rangle_{mn}-bd|10\rangle_{mn}-bc|11\rangle_{mn},\\&|K_3\rangle_{36}=\alpha|00\rangle_{36}-\beta|01\rangle_{36}+\gamma|10\rangle_{36}-\delta|11\rangle_{36},\\&|Q_3\rangle_{mn}=ad|00\rangle_{mn}-ac|01\rangle_{mn}+bd|10\rangle_{mn}-bc|11\rangle_{mn},\\&|K_4\rangle_{36}=\alpha|00\rangle_{36}-\beta|01\rangle_{36}-\gamma|10\rangle_{36}+\delta|11\rangle_{36},\\&|Q_4\rangle_{mn}=ad|00\rangle_{mn}-ac|01\rangle_{mn}-bd|10\rangle_{mn}+bc|11\rangle_{mn}.\end{aligned}$$

From Eq. (8), one can see that, Charlie can get the state $|K_i\rangle_{36}$ $(i=1,2,3,4)$ provided that the states $|Q_i\rangle_{mn}$ $(i=1,2,3,4)$ are distinguished. Note that $|K_1\rangle$ is exactly the prepared state. Readily, the prepared state can be further retrieved from $|K_2\rangle$, $|K_3\rangle$ and $|K_4\rangle$. Unfortunately, the four states $|Q_i\rangle_{mn}$ $(i=1,2,3,4)$ are not orthogonal in general. As a consequence, they can not be differentiated deterministically by using a usual projective measurement. Nevertheless, the discrimination can be achieved in a probabilistic manner by making an optimal POVM measurement [43, 44] on the ancillary qubits m and n as follows

$$\begin{aligned}
&P_1=\frac{1}{x}|M_1\rangle\langle M_1|, \quad P_2=\frac{1}{x}|M_2\rangle\langle M_2|, \quad P_3=\frac{1}{x}|M_3\rangle\langle M_3|,\\
&P_4=\frac{1}{x}|M_4\rangle\langle M_4|, \quad P_5=I-\frac{1}{x}\sum_{i=1}^{4}|M_i\rangle\langle M_i|.
\end{aligned} \tag{9}$$

Here

$$\begin{aligned}
|M_1\rangle&=\frac{1}{\sqrt{\xi}}\left(\frac{1}{ad}|00\rangle+\frac{1}{ac}|01\rangle+\frac{1}{bd}|10\rangle+\frac{1}{bc}|11\rangle\right)_{mn},\\
|M_2\rangle&=\frac{1}{\sqrt{\xi}}\left(\frac{1}{ad}|00\rangle+\frac{1}{ac}|01\rangle-\frac{1}{bd}|10\rangle-\frac{1}{bc}|11\rangle\right)_{mn},\\
|M_3\rangle&=\frac{1}{\sqrt{\xi}}\left(\frac{1}{ad}|00\rangle-\frac{1}{ac}|01\rangle+\frac{1}{bd}|10\rangle-\frac{1}{bc}|11\rangle\right)_{mn},\\
|M_4\rangle&=\frac{1}{\sqrt{\xi}}\left(\frac{1}{ad}|00\rangle-\frac{1}{ac}|01\rangle-\frac{1}{bd}|10\rangle+\frac{1}{bc}|11\rangle\right)_{mn},\\
\xi&=\frac{1}{(ad)^2}+\frac{1}{(ac)^2}+\frac{1}{(bd)^2}+\frac{1}{(bc)^2}=\frac{1}{(1-b^2)b^2(1-d^2)d^2},
\end{aligned}$$

I is an identity operator, x is a coefficient relating to a, b, c and d, which satisfies $1\le x\le 4$ and should be able to assure P_5 to be a positive operator. To exactly determine x, we can rewrite the five elements P_1, P_2, P_3 and P_4 in the matrix form

$$\begin{aligned}
P_1&=\frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ad)^2} & \frac{1}{acad} & \frac{1}{adbd} & \frac{1}{acbd}\\ \frac{1}{acad} & \frac{1}{(ac)^2} & \frac{1}{adbc} & \frac{1}{acbc}\\ \frac{1}{adbd} & \frac{1}{adbc} & \frac{1}{(bd)^2} & \frac{1}{bcbd}\\ \frac{1}{acbd} & \frac{1}{acbc} & \frac{1}{bdbc} & \frac{1}{(bc)^2}\end{pmatrix}, \quad
P_2=\frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ad)^2} & \frac{1}{acad} & -\frac{1}{adbd} & -\frac{1}{acbd}\\ \frac{1}{acad} & \frac{1}{(ac)^2} & -\frac{1}{adbc} & -\frac{1}{acbc}\\ -\frac{1}{adbd} & -\frac{1}{adbc} & \frac{1}{(bd)^2} & \frac{1}{bcbd}\\ -\frac{1}{acbd} & -\frac{1}{acbc} & \frac{1}{bdbc} & \frac{1}{(bc)^2}\end{pmatrix},\\
P_3&=\frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ad)^2} & -\frac{1}{acad} & \frac{1}{adbd} & -\frac{1}{acbd}\\ -\frac{1}{acad} & \frac{1}{(ac)^2} & -\frac{1}{adbc} & \frac{1}{acbc}\\ \frac{1}{adbd} & -\frac{1}{adbc} & \frac{1}{(bd)^2} & -\frac{1}{bcbd}\\ -\frac{1}{acbd} & \frac{1}{acbc} & -\frac{1}{bdbc} & \frac{1}{(bc)^2}\end{pmatrix}, \quad
P_4=\frac{1}{x\xi}\begin{pmatrix} \frac{1}{(ad)^2} & -\frac{1}{acad} & -\frac{1}{adbd} & \frac{1}{acbd}\\ -\frac{1}{acad} & \frac{1}{(ac)^2} & \frac{1}{adbc} & -\frac{1}{acbc}\\ \frac{1}{adbd} & -\frac{1}{adbc} & -\frac{1}{(bd)^2} & \frac{1}{bcbd}\\ -\frac{1}{acbd} & \frac{1}{acbc} & \frac{1}{bdbc} & -\frac{1}{(bc)^2}\end{pmatrix},
\end{aligned} \tag{10}$$

Moreover, $P_5 = \mathrm{diag}(A,B,C,D)$, where

$$A = 1 - \frac{4}{x\xi(ad)^2}, \quad B = 1 - \frac{4}{x\xi(ac)^2},$$
$$C = 1 - \frac{4}{x\xi(bd)^2}, \quad D = 1 - \frac{4}{x\xi(bc)^2}.$$

Obviously, the coefficient x should be chosen such that all the diagonal elements A,B,C,D are nonnegative. Charlie performs this POVM operation on his auxiliary qubits m and n. Then in terms of the POVM's value Charlie can positively conclude the state of qubits m and n. To be specifical, the element P_1's value corresponds to the state $|Q_1\rangle_{mn}$, P_2's value to $|Q_2\rangle_{mn}$, P_3's value to $|Q_3\rangle_{mn}$ and P_4's value to $|Q_4\rangle_{mn}$, respectively. After the measurement, Charlie can obtain the value of P_i $(i=1,2,3,4)$ with probability p, where

$$p = {}_{36mn}\langle T|P_i|T\rangle_{36mn} = {}_{mn}\langle Q_i|P_i|Q_i\rangle_{mn}/64 = \frac{1}{4x\xi}, \quad (i=1,2,3,4). \tag{11}$$

However, if Charlie gets P_5's value (such probability is $1-\frac{1}{x\xi}$), then he can not infer which state the qubits m and n are in. Once Charlie determines the $|Q_i\rangle_{mn}$ $(i=1,2,3,4)$, this means he also knows the state $|K_i\rangle_{36}$ $(i=1,2,3,4)$ of her qubits 3 and 6. As a consequence, Charlie can construct the prepared state $|V\rangle$ on his qubits 3 and 6 by performing an appropriate unitary operation (see Fig. 1(b)). Explicitly, if Charlie knows the state of his qubits (3, 6) is $|K_1\rangle_{36}(|K_2\rangle_{36},|K_3\rangle_{36},|K_4\rangle_{36})$, he retrieves the prepared state $|V\rangle$ in his place by performing the unitary operation I $(\sigma_z^3 I_6, I_3\sigma_z^6, \sigma_z^3\sigma_z^6)$. So far we have depicted the case that Bob measures $|+\rangle_2|-\rangle_5$. As mentioned before Bob may get $|+\rangle_2|+\rangle_5$, $|-\rangle_2|+\rangle_5$ or $|-\rangle_2|-\rangle_5$. In each latter case, the RSP process is trivially similar to that in the former one. Alternatively, Charlie can also construct the prepared state on his qubits (3, 6) with the same success probability. All possible cases are listed in Table 1 and here we do not depict them anymore. Thus, the total success probability of the tripartite RSP scheme is

$$4\times 4\times p = \frac{4}{x\xi} = \frac{4}{x}\times(1-b^2)(1-d^2)b^2d^2. \tag{12}$$

3 The application of the symmetric RSP scheme to six special ensembles of states

It is already shown that the tripartite symmetric RSP scheme of an arbitrary two-qubit state (demonstrated in Fig. 1). In this section, its applications to some special ensembles of states will be explored. As depicted previously, it is possible that Alice measures $\lambda_1\rangle_{14}, \lambda_2\rangle_{14}$ or $\lambda_4\rangle_{14}$. According to Eq. (3), the collapsed state of the qubit pairs 2, 3 and 5, 6 will be $|\Gamma_1\rangle_{2356}$, $|\Gamma_2\rangle_{2356}$ and $|\Gamma_4\rangle_{2356}$, respectively. Since Bob and Charlie have no knowledge of the four coefficients α,β, γ and δ, they can not convert the above three states into

Table 1: Bob's measurement results (BM) and classical bits (C_B), the unitary operation (U) on qubits (3, 6) before Charlie's CNOT operations. The collapsed states (CS) of qubits (3, 6) after Charlie's POVM operation and the succedent unitary operation (U') performed on qubits (3, 6). See text for more details.

BM	C_B	U	CS	U'
$\|+\rangle_2\|+\rangle_5$	00	$I_3\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|+\rangle_5$	00	$I_3\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|+\rangle_5$	00	$I_3\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|+\rangle_5$	00	$I_3\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|-\rangle_5$	01	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|-\rangle_5$	01	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|+\rangle_5$	10	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|+\rangle_5$	10	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|-\rangle_5$	11	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|-\rangle_5$	11	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$

the prepared state $|V\rangle$ by performing certain unitary operations. Apparently, the symmetric tripartite RSP fails in the latter three cases. Nonetheless, it should be noted that the coefficients $\alpha,\beta,\gamma,\delta$ are assumed to be complex in the beginning. Then it is intriguing to ask whether the conversion can be unitarily realized if $\alpha,\beta,\gamma,\delta$ are some special values in the latter three cases. After our extensive investigations we get the positive answer and find out six special ensembles, which are given as:

Ensemble I: α, β, γ and δ are real

In this case, if Alice's measurement result is $|\lambda_1\rangle_{14}$, then according to the Eq. (3), the joint state of qubits 2, 3, 5, 6 will be

$$|\Gamma_1'\rangle_{2356}=\alpha ac|0000\rangle_{2356}+\beta ad|0011\rangle_{2356}+\gamma bc|1100\rangle_{2356}+\delta bd|1111\rangle_{2356}. \tag{13}$$

Similarly, Alice then assigns Bob to measure his two qubits 2 and 5 in the X bases proposed above, respectively. In the X bases, the collapsed state $|\Gamma_1\rangle_{2356}$ can be reexpressed as

$$\begin{aligned}|\Gamma_1'\rangle_{2356}=&\frac{1}{2}|+\rangle_2|+\rangle_5(\alpha ac|00\rangle_{36}+\beta ad|01\rangle_{36}+\gamma bc|10\rangle_{36}+\delta bd|11\rangle_{36})\\&+\frac{1}{2}|+\rangle_2|-\rangle_5(\alpha ac|00\rangle_{36}-\beta ad|01\rangle_{36}+\gamma bc|10\rangle_{36}-\delta bd|11\rangle_{36})\\&+\frac{1}{2}|-\rangle_2|+\rangle_5(\alpha ac|00\rangle_{36}+\beta ad|01\rangle_{36}-\gamma bc|10\rangle_{36}-\delta bd|11\rangle_{36})\\&+\frac{1}{2}|-\rangle_2|-\rangle_5(\alpha ac|00\rangle_{36}-\beta ad|01\rangle_{36}-\gamma bc|10\rangle_{36}+\delta bd|11\rangle_{36}).\end{aligned}\tag{14}$$

After his measurements, Bob communicates his measurement results to Charlie over a public channel. Without loss of generality, suppose Bob measures $|-\rangle_2|+\rangle_5$. Then two classical bits '1' and '0' are sent to Charlie. After receiving the classical bits, according to Eq. (14), Charlie knows the state of his qubits (3, 6) is

$$|\Delta_2\rangle_{36}=\frac{1}{2}\Big(\alpha ac|00\rangle_{36}+\beta ad|01\rangle_{36}-\gamma bc|10\rangle_{36}-\delta bd|11\rangle_{36}\Big).\tag{15}$$

In order to retrieve the prepared state, Charlie first performs $U_2=\sigma_3^z\otimes I_6$ on her qubits 3 and 6, which transforms $|\Delta_2\rangle_{36}$ into

$$|\Theta_2\rangle_{36}=\sigma_3^z\otimes I_6|\Delta_2\rangle_{36}=\frac{1}{2}\Big(ac\alpha|00\rangle_{36}+ad\beta|01\rangle_{36}+bc\gamma|10\rangle_{36}+bd\delta|11\rangle_{36}\Big).\tag{16}$$

Charlie then introduces two auxiliary qubits m and n which are in the state $|00\rangle_{mn}$ and performs two controlled-not (CNOT) operations with qubits 3 and 6 as the controlled qubits while the auxiliary qubits m and n as the target ones, respectively. The two CNOT operations transform the state of qubits (3, 6, m, n) into the following form

$$\begin{aligned}|R\rangle_{36mn}=&\frac{1}{2}\Big(\alpha ac|0000\rangle_{36mn}+\beta ad|0101\rangle_{36mn}+\gamma bc|1010\rangle_{36mn}+\delta bd|1111\rangle_{36mn}\Big)\\=&\frac{1}{8}\Big(|K_1\rangle_{36}|H_1\rangle_{mn}+|K_2\rangle_{36}|H_2\rangle_{mn}+|K_3\rangle_{36}|H_3\rangle_{mn}+|K_4\rangle_{36}|H_4\rangle_{mn}\Big),\end{aligned}\tag{17}$$

where

$$\begin{aligned}|H_1\rangle_{mn}&=ac|00\rangle_{mn}+ad|01\rangle_{mn}+bc|10\rangle_{mn}+bd|11\rangle_{mn},\\|H_2\rangle_{mn}&=ac|00\rangle_{mn}+ad|01\rangle_{mn}-bc|10\rangle_{mn}-bd|11\rangle_{mn},\\|H_3\rangle_{mn}&=ac|00\rangle_{mn}-ad|01\rangle_{mn}+bc|10\rangle_{mn}-bd|11\rangle_{mn},\\|H_4\rangle_{mn}&=ac|00\rangle_{mn}-ad|01\rangle_{mn}-bc|10\rangle_{mn}+bd|11\rangle_{mn}.\end{aligned}$$

Obviously, if the non-orthogonal states $|H_i\rangle_{mn}$ $(i=1,2,3,4)$ can be distinguished, Charlie will know exactly the state $|K_i\rangle_{36}$ $(i=1,2,3,4)$. To achieve the goal, Charlie then performs an optimal POVM measurement [43,44] on the ancillary qubits m and n, which takes the following matrix form

$$W_1=\frac{1}{x\xi}\begin{pmatrix}\frac{1}{(ac)^2} & \frac{1}{acad} & \frac{1}{acbc} & \frac{1}{acbd}\\ \frac{1}{acad} & \frac{1}{(ad)^2} & \frac{1}{adbc} & \frac{1}{adbd}\\ \frac{1}{acbc} & \frac{1}{adbc} & \frac{1}{(bc)^2} & \frac{1}{bcbd}\\ \frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{bdbc} & \frac{1}{(bd)^2}\end{pmatrix},\qquad W_2=\frac{1}{x\xi}\begin{pmatrix}\frac{1}{(ac)^2} & \frac{1}{acad} & -\frac{1}{acbc} & -\frac{1}{acbd}\\ \frac{1}{acad} & \frac{1}{(ad)^2} & -\frac{1}{adbc} & -\frac{1}{adbd}\\ -\frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(bc)^2} & \frac{1}{bcbd}\\ -\frac{1}{acbd} & -\frac{1}{adbd} & \frac{1}{bdbc} & \frac{1}{(bd)^2}\end{pmatrix},$$

$$W_3=\frac{1}{x\xi}\begin{pmatrix}\frac{1}{(ac)^2} & -\frac{1}{acad} & \frac{1}{acbc} & -\frac{1}{acbd}\\ -\frac{1}{acad} & \frac{1}{(ad)^2} & -\frac{1}{adbc} & \frac{1}{adbd}\\ \frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(bc)^2} & -\frac{1}{bcbd}\\ -\frac{1}{acbd} & \frac{1}{adbd} & -\frac{1}{bdbc} & \frac{1}{(bd)^2}\end{pmatrix},\quad W_4=\frac{1}{x\xi}\begin{pmatrix}\frac{1}{(ac)^2} & -\frac{1}{acad} & -\frac{1}{acbc} & \frac{1}{acbd}\\ -\frac{1}{acad} & \frac{1}{(ad)^2} & \frac{1}{adbc} & -\frac{1}{adbd}\\ \frac{1}{acbc} & -\frac{1}{adbc} & -\frac{1}{(bc)^2} & \frac{1}{bcbd}\\ -\frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{bdbc} & -\frac{1}{(bd)^2}\end{pmatrix},$$

$$W_5=\mathrm{diag}(B,A,D,C). \tag{18}$$

After this, in terms of the POVM's value Charlie can positively conclude the state of qubits m and n. The probability in either case is also p. Similarly, if Charlie gets W_5's value, then he can not infer which state the qubits m and n are in. Consequently, the remote preparation fails in this case and such probability is also $1-\frac{1}{x\xi}$. Once Charlie determines the $|H_i\rangle_{mn}$ $(i=1,2,3,4)$, according to Eq. (17), he also knows the state of her qubits 3 and 6. In this way as above, Charlie constructs the prepared state $|V\rangle$ by performing an appropriate unitary operation on his qubits (3, 6) (see Table 2). Analogously, Bob's measurement result may be $|+\rangle_2|+\rangle_5$, $|+\rangle_2|-\rangle_5$ or $|-\rangle_2|-\rangle_5$. In each latter case, the RSP process is trivially similar to that in the former one. In other words, Charlie can also construct the prepared state on his qubits (3, 6) with the same success probability. We summarize all possible cases in Table 2 and do not depict them hereafter.

If Alice's measurement result is $|\lambda_2\rangle_{14}$ or $|\lambda_4\rangle_{14}$, then according to Eq. (3), the joint state of qubits 2, 3, 5, 6 will be $\eta\alpha ac|0000\rangle_{2356}+\eta\beta ad|0011\rangle_{2356}-\eta^{-1}\gamma bc|1100\rangle_{2356}-\eta^{-1}\delta bd|1111\rangle_{2356}$ and $|\Gamma_4\rangle_{2356}$, respectively. Apparently, the two states can not be unitarily converted into the prepared state $|V\rangle$ via Bob and Charlie's collaboration. Thus, the total success probability of the tripartite RSP scheme in **Ensemble I** is

$$2\times\frac{4}{x}\times(1-b^2)(1-d^2)b^2d^2=\frac{8}{x}\times(1-b^2)(1-d^2)b^2d^2. \tag{19}$$

Ensemble II: α, β, γ and δ satisfy $\eta=1$

In this case, if Alice's measurement result is $|\lambda_4\rangle_{14}$, then according to the Eq. (3), the joint state of qubits 2, 3, 5, 6 will be

$$\begin{aligned}|\Gamma_4'\rangle_{2356}=&\beta ac|0000\rangle_{2356}-\alpha ad|0011\rangle_{2356}-\delta bc|1100\rangle_{2356}+\gamma bd|1111\rangle_{2356}\\ =&\sigma_3^z|\Gamma_3\rangle_{2356}.\end{aligned} \tag{20}$$

Table 2: Same as Table 1.

BM	C_B	U	CS	U'
$\vert+\rangle_2\vert+\rangle_5$	00	I_3I_6	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	I_3I_6
$\vert+\rangle_2\vert+\rangle_5$	00	I_3I_6	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$\sigma_3^zI_6$
$\vert+\rangle_2\vert+\rangle_5$	00	I_3I_6	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$I_3\sigma_6^z$
$\vert+\rangle_2\vert+\rangle_5$	00	I_3I_6	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert+\rangle_2\vert-\rangle_5$	01	$I_3\sigma_6^z$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	I_3I_6
$\vert+\rangle_2\vert-\rangle_5$	01	$I_3\sigma_6^z$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$\sigma_3^zI_6$
$\vert+\rangle_2\vert-\rangle_5$	01	$I_3\sigma_6^z$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$I_3\sigma_6^z$
$\vert+\rangle_2\vert-\rangle_5$	01	$I_3\sigma_6^z$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^zI_6$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	I_3I_6
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^zI_6$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$\sigma_3^zI_6$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^zI_6$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$I_3\sigma_6^z$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^zI_6$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^z\sigma_6^z$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	I_3I_6
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^z\sigma_6^z$	$\alpha\vert 00\rangle_{36}+\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$\sigma_3^zI_6$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^z\sigma_6^z$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}+\gamma\vert 10\rangle_{36}-\delta\vert 11\rangle_{36}$	$I_3\sigma_6^z$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^z\sigma_6^z$	$\alpha\vert 00\rangle_{36}-\beta\vert 01\rangle_{36}-\gamma\vert 10\rangle_{36}+\delta\vert 11\rangle_{36}$	$\sigma_3^z\sigma_6^z$

According to the above equation, it can be concluded directly that while Alice gets $|\lambda_4\rangle_{14}$, the whole process of realizing the tripartite RSP is nearly the same as the case that Alice's measurement result is $|\lambda_3\rangle_{14}$. In other words, Charlie can also retrieve the prepared state with the same probability in **Ensemble II** while Alice's measurement result is $|\lambda_4\rangle_{14}$. The explicit correspondence relations among the measurement results, the unitary operations and the collapsed state are listed in the Table 3.

If Alice's measurement result is $|\lambda_1\rangle_{14}$ or $|\lambda_2\rangle_{14}$, then according to the Eq. (3), the joint state of qubits 2, 3, 5, 6 will be $|\Gamma_1\rangle_{2356}$ and $|\Gamma_2'\rangle_{2356}=\alpha^*ac|0000\rangle_{2356}+\beta^*ad|0011\rangle_{2356}-\gamma^*bc|1100\rangle_{2356}-\delta^*bd|1111\rangle_{2356}$, respectively. Apparently, the two states can not be unitarily converted into the prepared state $|V\rangle$ via Bob and Charlie's collaboration. Conclusively, the total success probability of the tripartite RSP scheme in **Ensemble II** is also

$$2\times\frac{4}{x}\times(1-b^2)(1-d^2)b^2d^2=\frac{8}{x}\times(1-b^2)(1-d^2)b^2d^2. \tag{21}$$

Ensemble III: $\alpha,\beta,\gamma,\delta$ are real and satisfy $\eta=1$

In this case, if Alice measures $|\lambda_1\rangle_{14}$ or $|\lambda_4\rangle_{14}$, then according to the Eq. (3), the joint state of qubits 2, 3, 5, 6 will be$|\Gamma_1'\rangle_{2356}$ and $|\Gamma_4'\rangle_{2356}$, respectively. In these two cases the whole process of realizing the tripartite RSP is the same as that proposed in **Ensemble I** and **Ensemble II**, respectively. While Alice measures $|\lambda_2\rangle_{14}$, then according to the Eq. (3), the joint state of qubits 2, 3, 5, 6 will be $|\Gamma_2'\rangle_{2356}=\sigma_3^z|\Gamma_1'\rangle_{2356}$. In like manner, the whole process of realizing the tripartite RSP is nearly the same as that proposed in **Ensemble I**. We also give out the the explicit correspondence relations among the measurement results, the unitary operations and the collapsed state in Table 4 when Alice's measurement

Table 3: Same as Table 1.

BM	C_B	U	CS	U'
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^z\sigma_6^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	$I_3\sigma_6^z\sigma_6^x\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|+\rangle_5$	10	$I_3\sigma_6^z\sigma_6^x\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|+\rangle_5$	10	$I_3\sigma_6^z\sigma_6^x\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	$I_3\sigma_6^z\sigma_6^x\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^x$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$

Table 4: Same as Table 1.

BM	C_B	U	CS	U'
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^zI_6$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^zI_6$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^zI_6$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|+\rangle_5$	00	$\sigma_3^zI_6$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^z$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^z$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^z$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|+\rangle_2\|-\rangle_5$	01	$\sigma_3^z\sigma_6^z$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	I_3I_6	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|+\rangle_5$	10	I_3I_6	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|+\rangle_5$	10	I_3I_6	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|+\rangle_5$	10	I_3I_6	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^z$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	I_3I_6
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^z$	$\alpha\|00\rangle_{36}+\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$\sigma_3^zI_6$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^z$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}+\gamma\|10\rangle_{36}-\delta\|11\rangle_{36}$	$I_3\sigma_6^z$
$\|-\rangle_2\|-\rangle_5$	11	$I_3\sigma_6^z$	$\alpha\|00\rangle_{36}-\beta\|01\rangle_{36}-\gamma\|10\rangle_{36}+\delta\|11\rangle_{36}$	$\sigma_3^z\sigma_6^z$

result is $|\lambda_2\rangle_{14}$ in this ensemble. As a result, Charlie can construct the prepared state $|V\rangle$ on his qubits (3, 6) in **Ensemble IV**, and the total success probability of the tripartite RSP scheme is

$$4\times\frac{4}{x}\times(1-b^2)(1-d^2)b^2d^2=\frac{16}{x}\times(1-b^2)(1-d^2)b^2d^2. \tag{22}$$

Ensemble IV: $|\alpha|=|\beta|=|\gamma|=|\delta|=\frac{1}{2}$ and $\alpha\gamma=\beta\delta$

In terms of $|\alpha|=|\beta|=|\gamma|=|\delta|=\frac{1}{2}$ and $\alpha\gamma=\beta\delta$, it can be easily obtained $\eta=1$, $(\alpha^*)^{-1}=4\alpha$, $(\beta^*)^{-1}=4\beta$, $(\gamma^*)^{-1}=4\gamma$, $(\delta^*)^{-1}=4\delta$ and $\alpha^*\gamma^*=\beta^*\delta^*$. In this case, if Alice measures $|\lambda_4\rangle_{14}$, then the four qubits 2, 3, 5 and 6 are left in $|\Gamma_4'\rangle_{2356}$. Applying the same analysis method proposed in **Ensemble II**, it can be known Charlie can construct the prepared state $|V\rangle$ with the probability $16p$ via Bob's help.

If Alice gets $|\lambda_1\rangle_{14}$ or $|\lambda_2\rangle_{14}$, then the four qubits 2, 3, 5 and 6 are left in $|\Gamma_1\rangle_{2356}$ and $|\Gamma_2'\rangle_{2356}=\sigma_3^z|\Gamma_1\rangle_{2356}$. Evidently, the treatment of preparation in each case is very similar. As enumerations, the case that Alice measures $|\lambda_2\rangle_{14}$ is taken to show the whole process of preparation hereafter. In this case, the joint state of the four qubits 2, 3, 5 and 6 is

$$\begin{aligned}|\Gamma_2'\rangle_{2356}=&\alpha^*ac|0000\rangle_{2356}+\beta^*ad|0011\rangle_{2356}-\gamma^*bc|1100\rangle_{2356}-\delta^*bd|1111\rangle_{2356}\\=&\beta^*\delta^*\Big(\frac{\alpha^*}{\beta^*\delta^*}ac|0000\rangle_{2356}+\frac{1}{\delta^*}ad|0011\rangle_{2356}-\frac{\gamma^*}{\beta^*\delta^*}bc|1100\rangle_{2356}-\frac{1}{\beta^*}bd|1111\rangle_{2356}\Big)\\=&\beta^*\delta^*\Big(\frac{1}{\gamma^*}ac|0000\rangle_{2356}+\frac{1}{\delta^*}ad|0011\rangle_{2356}-\frac{1}{\alpha^*}bc|1100\rangle_{2356}-\frac{1}{\beta^*}bd|1111\rangle_{2356}\Big)\\=&4\beta^*\delta^*\Big(\gamma ac|0000\rangle_{2356}+\delta ad|0011\rangle_{2356}-\alpha bc|1100\rangle_{2356}-\beta bd|1111\rangle_{2356}\Big).\end{aligned}\tag{23}$$

To construct the prepared state in Charlie's place, Bob is asked to measure the qubits (2, 5) in the X bases and then tell Charlie his measurement results via a classical channel. In the X bases, $|\Gamma_2'\rangle_{2356}$ can be reexpressed as

$$\begin{aligned}|\Gamma_2'\rangle_{2356}=&2\beta^*\delta^*|+\rangle_2|+\rangle_5(\gamma ac|00\rangle_{36}+\delta ad|01\rangle_{36}-\alpha bc|10\rangle_{36}-\beta bd|11\rangle_{36})\\&+2\beta^*\delta^*|+\rangle_2|-\rangle_5(\gamma ac|00\rangle_{36}-\delta ad|01\rangle_{36}-\alpha bc|10\rangle_{36}+\beta bd|11\rangle_{36})\\&+2\beta^*\delta^*|-\rangle_2|+\rangle_5(\gamma ac|00\rangle_{36}+\delta ad|01\rangle_{36}+\alpha bc|10\rangle_{36}+\beta bd|11\rangle_{36})\\&+2\beta^*\delta^*|-\rangle_2|-\rangle_5(\gamma ac|00\rangle_{36}-\delta ad|01\rangle_{36}+\alpha bc|10\rangle_{36}-\beta bd|11\rangle_{36}).\end{aligned}\tag{24}$$

Without loss of generality, suppose Bob measures $|+\rangle_2|+\rangle_5$. Then two classical bits '0' and '0' are broadcasted. According to the above equation, with Bob's classical bits Charlie knows his two qubits (3, 6) are left in the following state

$$|\Delta_3\rangle_{36}=2\beta^*\delta^*\Big(\gamma ac|00\rangle_{36}+\delta ad|01\rangle_{36}-\alpha bc|10\rangle_{36}-\beta bd|11\rangle_{36}\Big).\tag{25}$$

Very similarly, to retrieve the prepared state, Charlie first performs the unitary operation $U_3=\sigma_3^x\sigma_3^z I_6$ on his qubits (3, 6), which transforms the state

$$|\Theta_3\rangle_{36}=\sigma_3^x\sigma_3^z I_6|\Delta_3\rangle_{36}=2\beta^*\delta^*\Big(\alpha bc|00\rangle_{36}+\beta bd|01\rangle_{36}+\gamma ac|10\rangle_{36}+\delta ad|11\rangle_{36}\Big).\tag{26}$$

Charlie then introduces two auxiliary qubits m and n in the state $|00\rangle_{mn}$, and then performs two controlled-not (CNOT) gate operations with the qubits 3 and 6 as the controlled qubits while the auxiliary qubits m and n as the target ones, respectively. The

CNOT operations transform the state of the qubits (3, 6, m, n) into

$$\begin{aligned}|G\rangle_{36mn} =& 2\beta^*\delta^*(\alpha bc|0000\rangle_{36mn}+\beta bd|0101\rangle_{36mn}+\gamma ac|1010\rangle_{36mn}+\delta ad|1111\rangle_{36mn})\\ =& \frac{1}{2}\beta^*\delta^*(|K_1\rangle_{36}|S_1\rangle_{mn}+|K_2\rangle_{36}|S_2\rangle_{mn}+|K_3\rangle_{36}|S_3\rangle_{mn}+|K_4\rangle_{36}|S_4\rangle_{mn}),\end{aligned} \tag{27}$$

where

$$\begin{aligned}|S_1\rangle_{mn} &= bc|00\rangle_{mn}+bd|01\rangle_{mn}+ac|10\rangle_{mn}+ad|11\rangle_{mn},\\ |S_2\rangle_{mn} &= bc|00\rangle_{mn}+bd|01\rangle_{mn}-ac|10\rangle_{mn}-ad|11\rangle_{mn},\\ |S_3\rangle_{mn} &= bc|00\rangle_{mn}-bd|01\rangle_{mn}+ac|10\rangle_{mn}-ad|11\rangle_{mn},\\ |S_4\rangle_{mn} &= bc|00\rangle_{mn}-bd|01\rangle_{mn}-ac|10\rangle_{mn}+ad|11\rangle_{mn}.\end{aligned}$$

From Eq. (27), one can see if $|S_i\rangle_{mn}$, $(i=1,2,3,4)$ are distinguished, the state $|V\rangle$ can be constructed via an appropriate unitary operation on qubits (3, 6). Likewise, the discrimination of the states $|S_i\rangle_{mn}$, $(i=1,2,3,4)$ can be achieved in a probabilistic manner by making an optimal POVM measurement [43, 44]. Forasmuch, Charlie then performs an optimal POVM measurement on the auxiliary qubits m and n, which takes the following matrix form

$$\begin{aligned}Q_1 &= \frac{1}{x\xi}\begin{pmatrix}\frac{1}{(bc)^2} & \frac{1}{bcbd} & \frac{1}{acbc} & \frac{1}{acbd}\\ \frac{1}{bcbd} & \frac{1}{(bd)^2} & \frac{1}{adbc} & \frac{1}{adbd}\\ \frac{1}{acbc} & \frac{1}{adbc} & \frac{1}{(ac)^2} & \frac{1}{acad}\\ \frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{adac} & \frac{1}{(ad)^2}\end{pmatrix}, \quad Q_2 = \frac{1}{x\xi}\begin{pmatrix}\frac{1}{(bc)^2} & \frac{1}{bcbd} & -\frac{1}{acbc} & -\frac{1}{acbd}\\ \frac{1}{bcbd} & \frac{1}{(bd)^2} & -\frac{1}{adbc} & -\frac{1}{adbd}\\ -\frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(ac)^2} & \frac{1}{acad}\\ -\frac{1}{acbd} & -\frac{1}{adbd} & \frac{1}{adac} & \frac{1}{(ad)^2}\end{pmatrix},\\ Q_3 &= \frac{1}{x\xi}\begin{pmatrix}\frac{1}{(bc)^2} & -\frac{1}{bcbd} & \frac{1}{acbc} & -\frac{1}{acbd}\\ -\frac{1}{bcbd} & \frac{1}{(bd)^2} & -\frac{1}{adbc} & \frac{1}{adbd}\\ \frac{1}{acbc} & -\frac{1}{adbc} & \frac{1}{(ac)^2} & -\frac{1}{acad}\\ -\frac{1}{acbd} & \frac{1}{adbd} & -\frac{1}{adac} & \frac{1}{(ad)^2}\end{pmatrix}, \quad Q_4 = \frac{1}{x\xi}\begin{pmatrix}\frac{1}{(bc)^2} & -\frac{1}{bcbd} & -\frac{1}{acbc} & \frac{1}{acbd}\\ -\frac{1}{bcbd} & \frac{1}{(bd)^2} & \frac{1}{adbc} & -\frac{1}{adbd}\\ \frac{1}{acbc} & -\frac{1}{adbc} & -\frac{1}{(ac)^2} & \frac{1}{acad}\\ -\frac{1}{acbd} & \frac{1}{adbd} & \frac{1}{adac} & -\frac{1}{(ad)^2}\end{pmatrix},\\ Q_5 &= \mathrm{diag}(D,C,B,A).\end{aligned} \tag{28}$$

After the manipulation, in terms of each POVM's value Charlie concludes the corresponding state of qubits m and n with the probability p. Further, he knows the state $|K_i\rangle_{36}$ $(i=1,2,3,4)$ of her qubits 3 and 6. Then Charlie constructs the prepared state by performing an appropriate unitary operation on his qubits (3, 6) (see Table 5). Similarly, Charlie gets Q_5's value with the probability $1-\frac{1}{x\xi}$. It means he can not infer which state the qubits m and n are in. Then RSP protocol fails in this case. As same as that proposed before Bob's measurement results may be $|+\rangle_2|-\rangle_5$, $|-\rangle_2|+\rangle_5$ or $|-\rangle_2|-\rangle_5$. In each latter case, the RSP process is trivially similar to that in the former one. Alternatively, Charlie can also construct the prepared state on his qubits (3, 6) with the same success probability. We summarize all possible cases in Table 5 and do not depict them anymore hereafter. Hence, the success probability of the RSP protocol in **Ensemble IV** is also $\frac{16}{x}\times(1-b^2)(1-d^2)b^2d^2$.

Table 5: Same as Table 1.

BM	C_B	U	CS	U'
$\vert+\rangle_2\vert+\rangle_5$	00	$\sigma_3^x\sigma_3^z I_6$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$I_3 I_6$
$\vert+\rangle_2\vert+\rangle_5$	00	$\sigma_3^x\sigma_3^z I_6$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$\sigma_3^z I_6$
$\vert+\rangle_2\vert+\rangle_5$	00	$\sigma_3^x\sigma_3^z I_6$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$I_3\sigma_6^z$
$\vert+\rangle_2\vert+\rangle_5$	00	$\sigma_3^x\sigma_3^z I_6$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert+\rangle_2\vert-\rangle_5$	01	$\sigma_3^x\sigma_3^z\sigma_6^z$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$I_3 I_6$
$\vert+\rangle_2\vert-\rangle_5$	01	$\sigma_3^x\sigma_3^z\sigma_6^z$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$\sigma_3^z I_6$
$\vert+\rangle_2\vert-\rangle_5$	01	$\sigma_3^x\sigma_3^z\sigma_6^z$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$I_3\sigma_6^z$
$\vert+\rangle_2\vert-\rangle_5$	01	$\sigma_3^x\sigma_3^z\sigma_6^z$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^x I_6$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$I_3 I_6$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^x I_6$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$\sigma_3^z I_6$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^x I_6$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$I_3\sigma_6^z$
$\vert-\rangle_2\vert+\rangle_5$	10	$\sigma_3^x I_6$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$\sigma_3^z\sigma_6^z$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^x\sigma_6^z$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$I_3 I_6$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^x\sigma_6^z$	$\alpha\vert00\rangle_{36}+\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$\sigma_3^z I_6$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^x\sigma_6^z$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}+\gamma\vert10\rangle_{36}-\delta\vert11\rangle_{36}$	$I_3\sigma_6^z$
$\vert-\rangle_2\vert-\rangle_5$	11	$\sigma_3^x\sigma_6^z$	$\alpha\vert00\rangle_{36}-\beta\vert01\rangle_{36}-\gamma\vert10\rangle_{36}+\delta\vert11\rangle_{36}$	$\sigma_3^z\sigma_6^z$

Ensemble V: $|\alpha|=|\beta|=|\gamma|=|\delta|=\frac{1}{2}$ and $\alpha\beta=\delta\gamma$; **Ensemble VI**: $|\alpha|=|\beta|=|\gamma|=|\delta|=\frac{1}{2}$ and $\alpha\delta=\beta\gamma$

In both cases, it can be noted $\eta=1$. So applying the same analysis method proposed in **Ensemble II**, the two-qubit RSP protocol can be realized when Alice measures $|\lambda_4\rangle_{14}$. While Alice's measurement result is $|\lambda_1\rangle_{14}$ or $|\lambda_2\rangle_{14}$, applying the very similar method proposed in **Ensemble IV**, the prepared state $|V\rangle$ can also be constructed in Charlie's place with the same success probability provided that the prepared state belongs to **Ensemble V** or **Ensemble VI**. We will not recount the whole process of preparation in each case because of the very similar treatment. Accordingly, both the success probabilities in these two cases are $\frac{16}{x}\times(1-b^2)(1-d^2)b^2d^2$.

4 Discussion and conclusion

In summary, one can see, in general the tripartite RSP can be fulfilled in one ministrant's place with the other's help. The total success probability is $\frac{4}{x}\times(1-b^2)(1-d^2)b^2d^2$, which is only determined by the smaller coefficients of the two non-maximally entangled states taken as the quantum channel. Nonetheless, if the state to be prepared is chosen from six special ensembles proposed above, then the success probability can be enhanced to $\frac{8}{x}\times(1-b^2)(1-d^2)b^2d^2$ (**Ensemble I-II**) or even to $\frac{16}{x}\times(1-b^2)(1-d^2)b^2d^2$ (**Ensemble III-VI**). The additional cost is that some more classical bits should be consume by Alice to show which ensemble the prepared state belongs to.

To summarize, we have explicitly presented a symmetric tripartite scheme for remotely preparing an arbitrary two-qubit state. In the scheme the quantum channel employed by the involved parties is two non-maximally entangled three-qubit states. To achieve the state preparation, the preparer Alice performs a two-qubit projective measurement and then publishes her measurement result via a classical channel. Due to the symmetry, either Bob or Charlie can construct the prepared state. Once one person is assigned to retrieve the prepared state, another one acts as an assistant. Then by collaboration, the tripartite RSP protocol can be realized with a certain probability by incorporating two auxiliary qubits and executing appropriate operations including a proper POVM. Furthermore, we have also explored its applications to six special ensembles of states in detail. Our extensive investigations show that the two-qubit RSP can be achieved with higher probability provided that the prepared state belongs to the six special ensembles. Incidentally, it is easy and forthright to generalize our present tripartite RSP scheme to a multiparty case.

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