# Remote preparation of three-particle GHZ-class states 

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#### Abstract

A scheme is presented for remote state preparation of three-particle GHZclass states by using a Bell state and an asymmetric W state as the quantum channel. In the scheme, the success probability of preparation and classical communication cost are calculated. In general, Bob can successfully prepare the initial state with the probability $1 / 4$ and consume $1 / 4$ classical bits. However, in special situations the success probability of preparation can reach $1 / 2$ or even 1 after consuming a little additional classical bits.


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Key words: remote state preparation, asymmetric W state, two-qubit projective measurement, classical communication, unitary operation

## 1 Introduction

In quantum information field, quantum entanglement and classical communication are two elementary resources. Bennett et al. [1] first proposed quantum teleportation protocol in 1993. In the scheme, an arbitrary unknown quantum state can be teleported by utilizing a prior shared entanglement and some classical communication. Very similar to quantum teleportation [2-7], a distinct application of quantum entanglement, i.e., remote state preparation (RSP) is first presented by Lo [8] in 2000. The main common point between teleportation and RSP is that entanglement should inhabit the quantum channel linking two parties. In contrast, the key difference between usual teleportation and RSP is that, in RSP the preparer Alice is assumed to completely know the state to be prepared, while in teleportation schemes the sender Alice needs not to know the state to be transmited.

The principal concern of RSP is to study whether the required classical communication and entanglement cost can be reduced in the case that the sender Alice knows

[^0]the prepared state. So far one has already known that, for general states RSP does not overwhelm quantum teleportation due to its probabilistic success in preparation. However, for some special ensembles of states, Pati [9] has found RSP protocol is more economical than teleportation. Since then, RSP has already attracted many attentions and various kinds of theoretical RSP protocols have been proposed [10-24], such as lowentanglement RSP [10], higher-dimension RSP [11], optimal RSP [12], oblivious RSP [13], RSP without oblivious conditions [14], generalized RSP [15], faithful RSP [16], RSP for multiparties [17], and continuous variable RSP in phase space [18], etc. Meantime, RSP schemes have been implemented experimentally by using Nuclear magnetic resonance (NMR) [25] and spontaneous parametric down-conversion [26].

## 2 The RSP scheme with a Bell state and asymmetric W state

In this paper, we will present a protocol for remotely preparing a three-particle GHZclass state by using a Bell state and a asymmetric $W$ state as the quantum channel. In the scheme, suppose that Alice wants to help Bob remotely prepare a three-particle GHZclass state. The GHZ class state $|M\rangle$ is written as

$$
\begin{equation*}
|M\rangle=q|000\rangle+r|111\rangle+u|001\rangle+v|110\rangle, \tag{1}
\end{equation*}
$$

where $q, r, u$ and $v$ are complex and satisfy $|q|^{2}+|r|^{2}+|u|^{2}+|v|^{2}=1$. Bob does not know the coefficients but Alice does. The prior established quantum channel consisting of a Bell state and a asymmetric $W$ state is represented as

$$
\begin{align*}
& |\varphi\rangle_{a_{1} b_{1}}=\frac{1}{\sqrt{2}}|01\rangle_{a_{1} b_{1}}+\frac{1}{\sqrt{2}}|10\rangle_{a_{1} b_{1}},  \tag{2}\\
& |\varphi\rangle_{a_{2} b_{2} b_{3}}=\frac{1}{2}|001\rangle_{a_{2} b_{2} b_{3}}+\frac{1}{2}|010\rangle_{a_{2} b_{2} b_{3}}+\frac{1}{\sqrt{2}}|100\rangle_{a_{2} b_{2} b_{3}} . \tag{3}
\end{align*}
$$

Obviously, the total state of the five-qubit system can be expressed as

$$
\begin{equation*}
|\psi\rangle_{a_{1} b_{1} a_{2} b_{2} b_{3}}=|\varphi\rangle_{a_{1} b_{1}} \otimes|\varphi\rangle_{a_{2} b_{2} b_{3}} . \tag{4}
\end{equation*}
$$

It is assumed that Alice owns qubits $a_{1}$ and $a_{2}$, and $b_{1}, b_{2}$ and $b_{3}$ belong to Bob. To prepare the state $|M\rangle$ in Bob's site, Alice first carries out a two-qubit projective measurement on her qubit pair ( $a_{1}, a_{2}$ ) in the mutually orthogonal basis vectors $\left\{\left|P_{1}\right\rangle,\left|P_{2}\right\rangle,\left|P_{3}\right\rangle,\left|P_{4}\right\rangle\right\}$. These vectors are defined as

$$
\begin{align*}
& \left|P_{1}\right\rangle=q|00\rangle+r|11\rangle+u|01\rangle+v|10\rangle  \tag{5}\\
& \left|P_{2}\right\rangle=\omega q|00\rangle+\omega r|11\rangle-\omega^{-1} u|01\rangle-\omega^{-1} v|10\rangle  \tag{6}\\
& \left|P_{3}\right\rangle=r^{*}|00\rangle-q^{*}|11\rangle+v^{*}|01\rangle-u^{*}|10\rangle  \tag{7}\\
& \left|P_{4}\right\rangle=\omega r^{*}|00\rangle-\omega q^{*}|11\rangle-\omega^{-1} v^{*}|01\rangle+\omega^{-1} u^{*}|10\rangle \tag{8}
\end{align*}
$$

where

$$
\omega=\frac{\sqrt{|u|^{2}+|v|^{2}}}{\sqrt{|q|^{2}+|r|^{2}}} .
$$

By the way, the above four states are related to the computation basis vectors $\{|00\rangle,|01\rangle,|10\rangle,|11\rangle\}$ and form a complete orthogonal basis set in an four-dimensional Hilbert space, i.e., $\left\langle P_{i} \mid P_{j}\right\rangle=\delta_{i j}$. Then we have

$$
\begin{align*}
&|\psi\rangle_{a_{1} a_{2} b_{1} b_{2} b_{3}} \\
&=\frac{1}{2}\left|P_{1}\right\rangle_{a_{1} a_{2}}\left[\frac{1}{\sqrt{2}} q^{*}(|101\rangle+|110\rangle)+r^{*}|000\rangle+u^{*}|100\rangle+\frac{1}{\sqrt{2}} v^{*}(|001\rangle+|010\rangle)\right]_{b_{1} b_{2} b_{3}} \\
&+\frac{1}{2}\left|P_{2}\right\rangle_{a_{1} a_{2}}\left[\frac{1}{\sqrt{2}} \omega q^{*}(|101\rangle+|110\rangle)+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|100\rangle\right. \\
&\left.-\frac{1}{\sqrt{2}} \omega^{-1} v^{*}(|001\rangle+|010\rangle)\right]_{b_{1} b_{2} b_{3}}-\frac{1}{2}\left|P_{3}\right\rangle_{a_{1} a_{2}}\left[q|000\rangle-\frac{1}{\sqrt{2}} r(|101\rangle+|110\rangle)\right. \\
&\left.+\frac{1}{\sqrt{2}} u(|001\rangle+|010\rangle)-v|100\rangle\right]_{b_{1} b_{2} b_{3}}-\frac{1}{2}\left|P_{4}\right\rangle_{a_{1} a_{2}}[\omega q|000\rangle \\
&\left.-\frac{1}{\sqrt{2}} \omega r(|101\rangle+|110\rangle)-\frac{1}{\sqrt{2}} \omega^{-1} u(|001\rangle+|010\rangle)+\omega^{-1} v|100\rangle\right]_{b_{1} b_{2} b_{3}} . \tag{9}
\end{align*}
$$

One can easily see that Alice's measurement results should be one of the four states defined in Eqs. (5)-(8) and each will occur with equal probability (i.e., $\frac{1}{4}$ ). To recover the quantum information, Bob first performs the following collective unitary operation $D$ on his three qubits $b_{1} b_{2} b_{3}$, and then Bob implements a controlled-not (CNOT) gate operation $C_{b_{1}, b_{2}}$ by taking the qubit $b_{1}$ as a control qubit and the qubit $b_{2}$ as a target one

$$
\begin{align*}
D= & |000\rangle\langle 000|+|100\rangle\langle 100| \\
& +\frac{1}{\sqrt{2}}(|001\rangle\langle 001|+|010\rangle\langle 001|+|001\rangle\langle 010|-|010\rangle\langle 010|) \\
& +\frac{1}{\sqrt{2}}(|101\rangle\langle 101|+|110\rangle\langle 101|+|101\rangle\langle 110|-|110\rangle\langle 110|) \\
& +\frac{1}{\sqrt{2}}(|011\rangle\langle 011|+|111\rangle\langle 011|+|011\rangle\langle 111|-|111\rangle\langle 111|) . \tag{10}
\end{align*}
$$

Thus

$$
\begin{align*}
|\psi\rangle_{a_{1} a_{2} b_{1} b_{2} b_{3}}= & \frac{1}{2}\left|P_{1}\right\rangle_{a_{1} a_{2}}\left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}} \\
& +\frac{1}{2}\left|P_{2}\right\rangle_{a_{1} a_{2}}\left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}} \\
& -\frac{1}{2}\left|P_{3}\right\rangle_{a_{1} a_{2}}(q|000\rangle-r|111\rangle+u|001\rangle-v|110\rangle)_{b_{1} b_{2} b_{3}} \\
& -\frac{1}{2}\left|P_{4}\right\rangle_{a_{1} a_{2}}\left(\omega q|000\rangle-\omega r|111\rangle-\omega^{-1} u|001\rangle+\omega^{-1} v|110\rangle\right)_{b_{1} b_{2} b_{3}} \tag{11}
\end{align*}
$$

Obviously, after Bob performs the unitary transformation $D$ and CNOT gate operation $C_{b_{1}, b_{2}}$, if Alice's measurement outcome is $\left|P_{1}\right\rangle_{a_{1} a_{2}},\left|P_{2}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{4}\right\rangle_{a_{1} a_{2}}$, the state of Bob's qubits $b_{1}, b_{2}$ and $b_{3}$ collapses to

$$
\begin{aligned}
& \left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}^{\prime}} \\
& \left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}
\end{aligned}
$$

or

$$
\left(\omega q|000\rangle-\omega r|111\rangle-\omega^{-1} u|001\rangle+\omega^{-1} v|110\rangle\right)_{b_{1} b_{2} b_{3}} .
$$

In these conditions, Bob can not convert the collapsed states into the original state $|M\rangle$. However, there exist a case which will occur with probability $1 / 4$, that is, Alice's measurement result is $\left|P_{3}\right\rangle_{a_{1} a_{2}}$, and Bob has the state $(q|000\rangle-r|111\rangle+u|001\rangle-v|110\rangle)_{b_{1} b_{2} b_{3}}$. This collapsed state can be recovered the initial state $|M\rangle$ by performing $\sigma_{b_{1}}^{z} I_{b_{2}} I_{b_{3}}$ on Bob's qubits $b_{1}, b_{2}$ and $b_{3}$, where $\sigma^{z}=|0\rangle\langle 0|-|1\rangle\langle 1|, I=|0\rangle\langle 0|+|1\rangle\langle 1|$. That is,

$$
(q|000\rangle-r|111\rangle+u|001\rangle-v|110\rangle)_{b_{1} b_{2} b_{3}}=\left(\sigma_{b_{1}}^{z} I_{b_{2}} I_{b_{3}}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
$$

Hence, if Alice notices Bob this collapse via their classical channel, Bob can reconstruct the original state by performing appropriate unitary operation. In terms of their prior agreements, if Alice's measurement outcome is $\left|P_{3}\right\rangle_{a_{1} a_{2}}$, she sends the classical bit (cbit) ' 0 ' to Bob. Otherwise, Alice publishes nothing. Hence, One can figure out that the total success probability (SP) of RSP is $1 / 4$ and the average classical communication cost (CCC) is $\frac{1}{4} \times(1+0+0+0)=\frac{1}{4}$ bits on average.

As mentioned before, the collapsed states can not be converted into the initial state $|M\rangle$ in three cases that Alice's measurement outcome is $\left|P_{1}\right\rangle_{a_{1} a_{2}},\left|P_{2}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{4}\right\rangle_{a_{1} a_{2}}$. However, it should be noted that the coefficients $q, r, u$ and $v$ are assumed to be complex at the beginning. Actually, this is a very general condition. If these coefficients are some special values, we find even if Alice obtains $\left|P_{1}\right\rangle_{a_{1} a_{2}},\left|P_{2}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{4}\right\rangle_{a_{1} a_{2}}$, the corresponding collapsed state can also be successfully converted into the original state $|M\rangle$ via appropriate local unitary operations. After our extensive investigations, we have found out
the special coefficients and classified them into six types. Since Alice exactly knows the prepared state $|M\rangle$, she can distinguish whether the coefficients belong to the six types. Now let us discuss them concretely as follows.

1. $\omega=1$

In this situation, if Alice gets the measurement result $\left|P_{4}\right\rangle_{a_{1} a_{2}}$, Alice publishes the cbit ' 1 ' in terms of their prior definitions. Once receiving the message ' 1 ', Bob knows his qubits has collapsed to

$$
(q|000\rangle-r|111\rangle-u|001\rangle+v|110\rangle)_{b_{1} b_{2} b_{3}}=\left(I_{b_{1}} I_{b_{2}} \sigma_{b_{3}}^{z}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
$$

As mentioned before, the general SP of RSP is 1/4 (same hereafter, and it is not repeated anymore). Hence, in this case one can work out that the total SP is $1 / 2$ and the average CCC is $\frac{1}{4} \times(1+1+0+0)=\frac{1}{2}$ bits.
2. $q, r, u$ and $v$ are real and $\omega \neq 1$.

In this case, if Alice's measurement result is $\left|P_{1}\right\rangle_{a_{1} a_{2}}$, she sends the cbits ' 01 ' in accord with their prior agreements, Bob is able to obtain the collapsed state

$$
\left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}=\left(\sigma_{b_{1}}^{x} \sigma_{b_{2}}^{x} \sigma_{b_{3}}^{x}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
$$

One can easily figure out that the total SP is $1 / 2$ and the CCC is $\frac{1}{4} \times(1+2+0+0)=\frac{3}{4}$ bits on average.
3. $q, r, u$ and $v$ are real and $\omega=1$.

In this condition, if Alice obtains the measurement outcome $\left|P_{2}\right\rangle_{a_{1} a_{2}}$, she publishes the cbits ' 10 ' to Bob in terms of their prior definitions. In this case, Bob knows that the collapsed state is

$$
\left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}=\left(\sigma_{b_{1}}^{y} \sigma_{b_{2}}^{x} \sigma_{b_{3}}^{y}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
$$

Easily, one can work out that the total SP is 1 and the CCC is $\frac{1}{4} \times(1+1+2+2)=\frac{3}{2}$ bits on average.
4. $|q|=|r|=|u|=|v|=\frac{1}{2}$ and $q v=r u$.

In terms of these relations, one can easily obtain

$$
q^{*} v^{*} r^{*} u^{*}, 4 q q^{*}=1,4 r r^{*}=1,4 u u^{*}=1,4 v v^{*}=1
$$

and

$$
\omega=\frac{\sqrt{|u|^{2}+|v|^{2}}}{\sqrt{|q|^{2}+|r|^{2}}}=1 .
$$

In this situation, Alice can remotely prepare the original state $|M\rangle$ in Bob's site provided that Alice's measurement outcome is $\left|P_{4}\right\rangle_{a_{1} a_{2}}$. Besides, if Alice gets $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{2}\right\rangle_{a_{1} a_{2}}$, the state of the qubits $b_{1}, b_{2}$ and $b_{3}$ collapses to

$$
\left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}=4 q^{*} v^{*}\left(I_{b_{1}} I_{b_{2}} \sigma_{b_{3}}^{x}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}}
$$

or

$$
\begin{aligned}
& \left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}} \\
= & 4 q^{*} v^{*}\left(\sigma_{b_{1}}^{z} I_{b_{2}} \sigma_{b_{3}}^{y}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
\end{aligned}
$$

Obviously, Bob can reconstruct the state $|M\rangle$ except for an overall trivial factor $4 q^{*} v^{*}$. In accord with their prior agreements, Alice sends the cbits ' 010 ' to $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ while the cbits ' 101 ' to $\left|P_{2}\right\rangle_{a_{1} a_{2}}$. Hence, for this type of quantum states the total SP is 1 and the average CCC is $\frac{1}{4} \times(1+1+3+3)=2$ bits.
5. $|q|=|r|=|u|=|v|=\frac{1}{2}$ and $q u=r v$.

In this case, one can easily get

$$
q^{*} u^{*}=r^{*} v^{*}, 4 q q^{*}=1,4 r r^{*}=1,4 u u^{*}=1,4 v v^{*}=1
$$

and

$$
\omega=\frac{\sqrt{|u|^{2}+|v|^{2}}}{\sqrt{|q|^{2}+|r|^{2}}}=1 .
$$

Because of this, if Alice gets the measurement result $\left|P_{4}\right\rangle_{a_{1} a_{2}}$, Bob can recover the initial state $|M\rangle$. Moreover, if Alice obtains $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{2}\right\rangle_{a_{1} a_{2}}$, the collapsed state of Bob's qubits $b_{1}, b_{2}$ and $b_{3}$ is

$$
\left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}=4 q^{*} u^{*}\left(\sigma_{b_{1}}^{x} \sigma_{b_{2}}^{x} I_{b_{3}}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}}
$$

or

$$
\begin{aligned}
& \left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}} \\
= & 4 q^{*} u^{*}\left(\sigma_{b_{1}}^{x} \sigma_{b_{2}}^{y} \sigma_{b_{3}}^{z}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
\end{aligned}
$$

In this situation, in terms of their prior definitions, Alice publishes the cbits ' 001 ' to $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ while the cbits ' $100^{\prime}$ to $\left|P_{2}\right\rangle_{a_{1} a_{2}}$. The same as type 4 , for this types of coefficients the total SP is 1 and the average CCC is $\frac{1}{4} \times(1+1+3+3)=2$ bits.
6. $|q|=|r|=|u|=|v|=\frac{1}{2}$ and $q r=u v$.

In this situation, one can easily obtain

$$
q^{*} r^{*}=u^{*} v^{*}, 4 q q^{*}=1,4 r r^{*}=1,4 u u^{*}=1,4 v v^{*}=1
$$

Table 1: Coefficient's type (CT), the success probability (SP), the classical communication cost (CCC), Alice's measurement results (AR), the classical bits from Alice to Bob according to her measurement results (CB), the collapsed states (CS) and Bob's appropriate unitary operation (BUO). The letter "A" denotes the case of arbitrary coefficients.

| CT | SP | CCC(bits) | AR | CB | CS | BUO |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | $1 / 4$ | $1 / 4$ | $\left\|P_{3}\right\rangle_{a_{1} a_{2}}$ | 0 | $(q\|000\rangle-r\|111\rangle+u\|001\rangle-v\|110\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{1}}^{z} I_{b_{2}} I_{b_{3}}$ |
| 1 | $1 / 2$ | $1 / 2$ | $\left\|P_{4}\right\rangle_{a_{1} a_{2}}$ | 1 | $(q\|000\rangle-r\|111\rangle-u\|001\rangle+v\|110\rangle)_{b_{1} b_{2} b_{3}}$ | $I_{b_{1}} I_{b_{2}} \sigma_{b_{3}}^{z}$ |
| 2 | $1 / 2$ | $3 / 4$ | $\left\|P_{1}\right\rangle_{a_{1} a_{2}}$ | 01 | $(q\|111\rangle+r\|000\rangle+u\|110\rangle+v\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{1}}^{x} \sigma_{b_{2}}^{x} \sigma_{b_{3}}^{x}$ |
| 3 | 1 | $3 / 2$ | $\left\|P_{2}\right\rangle_{a_{1} a_{2}}$ | 10 | $(q\|111\rangle+r\|000\rangle-u\|110\rangle-v\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{1}}^{y} \sigma_{b_{2}}^{x} \sigma_{b_{3}}^{y}$ |
| 4 | 1 | 2 | $\left\|P_{1}\right\rangle_{a_{1} a_{2}}$ | 010 | $4 q^{*} v^{*}(v\|111\rangle+u\|000\rangle+r\|110\rangle+q\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $I_{b_{1}} I_{b_{2}} \sigma_{b_{3}}^{x}$ |
|  |  |  | $\left\|P_{2}\right\rangle_{a_{1} a_{2}}$ | 101 | $4 q^{*} v^{*}(v\|111\rangle+u\|000\rangle-r\|110\rangle-q\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{1}}^{z} I_{b_{2}} \sigma_{b_{3}}^{y}$ |
| 5 | 1 | 2 | $\left\|P_{1}\right\rangle_{a_{1} a_{2}}$ | 001 | $4 q^{*} u^{*}(u\|111\rangle+v\|000\rangle+q\|110\rangle+r\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{b_{2}}} \sigma_{b_{2}}^{x} I_{b_{3}}$ |
|  |  |  | $\left\|P_{2}\right\rangle_{a_{1} a_{2}}$ | 100 | $4 q^{*} u^{*}(u\|111\rangle+v\|000\rangle-q\|110\rangle-r\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $\sigma_{b_{1}}^{x} \sigma_{b_{2}}^{y} \sigma_{b_{3}}^{z}$ |
| 6 | 1 | $3 / 2$ | $\left\|P_{1}\right\rangle_{a_{1} a_{2}}$ | 00 | $4 q^{*} r^{*}(r\|111\rangle+q\|000\rangle+v\|110\rangle+u\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $I_{b_{1}} I_{b_{2}} I_{b_{3}}$ |
|  |  |  | $\left\|P_{2}\right\rangle_{a_{1} a_{2}}$ | 11 | $4 q^{*} r^{*}(r\|111\rangle+q\|000\rangle-v\|110\rangle-u\|001\rangle)_{b_{1} b_{2} b_{3}}$ | $I_{b_{1}} \sigma_{b_{2}}^{z} \sigma_{b_{3}}^{z}$ |

and

$$
\omega=\frac{\sqrt{|u|^{2}+|v|^{2}}}{\sqrt{|q|^{2}+|r|^{2}}}=1
$$

Surely, besides the state $\left|P_{4}\right\rangle_{a_{1} a_{2}}$ if Alice's measurement outcome is $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ or $\left|p_{2}\right\rangle_{a_{1} a_{2}}$, she can remotely prepare the state $|M\rangle$ in Bob's site. In terms of their prior agreements, Alice sends the cbits ' $00^{\prime}$ or ' $11^{\prime}$ ' to Bob corresponding to her measurement result $\left|P_{1}\right\rangle_{a_{1} a_{2}}$ or $\left|P_{2}\right\rangle_{a_{1} a_{2}}$. Hence, Bob knows that the collapsed state is

$$
\left(q^{*}|111\rangle+r^{*}|000\rangle+u^{*}|110\rangle+v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}}=4 q^{*} r^{*}\left(I_{b_{1}} I_{b_{2}} I_{b_{3}}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}}
$$

or

$$
\begin{aligned}
& \left(\omega q^{*}|111\rangle+\omega r^{*}|000\rangle-\omega^{-1} u^{*}|110\rangle-\omega^{-1} v^{*}|001\rangle\right)_{b_{1} b_{2} b_{3}} \\
= & 4 q^{*} r^{*}\left(I_{b_{1}} \sigma_{b_{2}}^{z} \sigma_{b_{3}}^{z}\right)^{+}|M\rangle_{b_{1} b_{2} b_{3}} .
\end{aligned}
$$

One can easily work out that, for this type of coefficients, the total SP is 1 and the CCC is $\frac{1}{4} \times(1+1+2+2)=\frac{3}{2}$ bits on average. From the above analyses, we have a brief summary in Table 1.

## 3 Conclusions

To summarize, in this paper we have proposed a scheme for remotely preparing a threeparticle GHZ-class state by using a Bell state and a asymmetric $W$ states as the quantum channel. In this scheme, Alice is only required to implement a two-qubit projective measurement and Bob is required to perform collective unitary operation on his three qubits
$b_{1} b_{2} b_{3}$ and CNOT gate operation $C_{b_{1}, b_{2}}$. In general situation, applying our protocol, the target state can be prepared with SP at least $1 / 4$ and the necessary CCC is $1 / 4$ bits. However, if the states to be prepared are some special cases as the mentioned types $1,2,3$, 4,5 and 6 , the SP of RSP can be enhanced to $1 / 2$ or even 1 after consuming some extra classical bits. We have extensively discussed the success probability and figured out the classical communication cost.

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