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The Coulomb attraction in hydrogen may not be of long range

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Abstract. A quantum stationary wave has been examined in an exchange field, which induces the force of the form $F(r)=f_2(1/r^2-f_1/r)$. For the Coulomb attraction in hydrogen atom, the inexplicable discrepancy (0.0023 MHz) between the theoretical and experimental frequencies for its ${}^1S_{1/2}$ has been verified. It is found that the tiny f_1 is $7.45\times10^{-12}{a_1}^{-1}$($a_1$ is the 1st Bohr radius). Meanwhile, when such an f_1 is considered in the $n=2$ Lamb shift, it causes -0.034 MHz difference, which is in good agreement with the deviation of -0.039 MHz between the experimental and one of the theoretical predictions. Similar of searchings are made for the Lande g factor for the H_β spectrum. This f_1 contributes a ratio $\Delta g/g=5.58\times10^{-11}$ and makes both the experiment and theory well agreed within the experimental relative uncertainty $\pm4\times10^{-12}$. In other words, these phenomena can be treated as the reliable physical evidences for the existence of the same repulsion. More importantly, they consistently and strongly imply that the maximum radius for the Coulomb attraction in hydrogen atom can not exceed 7.11 m (if extrapolated). In addition, this analysis prompts us similar cases probably occur in the gravitation because it is also an exchange field, and the repulsion at remote distance may be one kind of dark energy that may have been ignored.

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Key words: hydrogen atom, hyperfine structure sectra, Lamb shift, Lande g factor, Coulomb force, gravitation, dark energy

1 Introduction

Hydrogen atom has been well studied as the simplest quantum system. However, there still exist deviations between the carefully corrected quantum electrodynamics (QED)

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predictions and the most precise experiments. The remarkable issues might be the inexplicable frequency discrepancy on the HFS spectrum for the state $1S_{1/2}$ (at 21cm), and the $n=2$ Lamb shift ($2S_{1/2}-2P_{1/2}$) as well as the Lande g factor for electron. This paper will discuss the electromagnetic interaction and try to find out some new results.

2 The prompt from a quantum stationary wave

The Coulomb interaction belongs to an exchange field. Typically, two electrons interact via exchanging the virtual particle (γ) and scattered, as illustrated in Fig. 1.

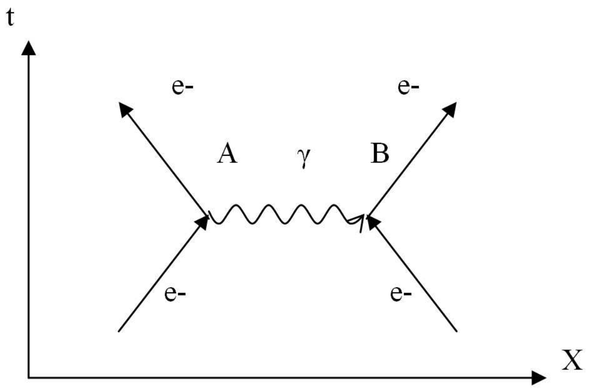

Figure 1: The Feynman diagram for the scattering of electrons in time-space (t-X) plot. The waved line represents the virtual particle γ, which is radiated at A and absorbed at B.

Consider the mode and the number distribution of the exchanger (the virtual particle γ). First, the exchanger is a wave, probably, the simplest mode may correlate to the quantum stationary wave, and each component has an energy $h\nu_n$ (h is the Planck constant). Second, for a stable stationary wave, as we have known, their wavelengths λ_n for the possible components keep the integer relationship: $\lambda_n=\lambda_{max}/n$, in which $\lambda_{max}=\lambda_1=2L$ and $n=1,2,3,\cdots$. Therefore, the frequencies $\nu_n=c/\lambda_n$ must be ν_1, $2\nu_1$, $3\nu_1$, $\cdots$, $n\nu_1$, $\cdots$

Consider the ratio of the n^{th} component ρ_n, with $\sum_n \rho_n=1$. Note that a transverse wave has two degrees of freedom. So the interactive energy may be written as

$$\begin{aligned} E(L) &= 2\sum_n \rho_n h\nu_n = 2\sum_n \rho_n h(n\nu_1) = 2h\nu_1\sum_n n\rho_n \\ &= 2h\nu_1\langle n\rangle \end{aligned} \tag{1}$$

where $\langle n\rangle=\sum_n n\rho_n$ is the average number. Noticing the minimum frequency $\nu_{min}=\nu_1=c/2L$, we may have

$$E(L)=\langle n\rangle hc/L. \tag{2}$$

By considering a particular distribution (e.g., the classical, or Fermi - Dirac, or Bose - Einstein distribution), we can still get a similar relation. As it is valid for arbitrary length

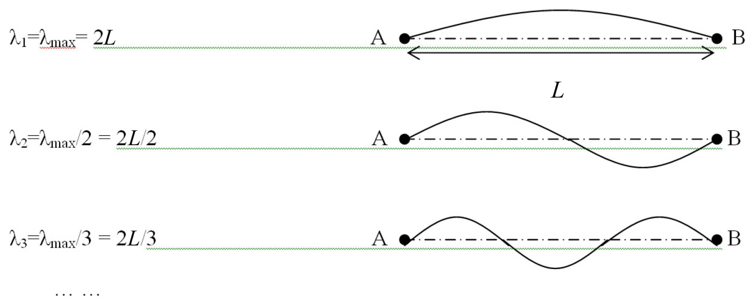

Figure 2: Schematic of the stable stationary wave modes (L, the distance)

and direction, it is better to label L as the spatial distance r. In addition, the internal energy $V(r)$ will be taking as minus in the physical convention

$$V(r) = -\langle n \rangle hc/r. \tag{3}$$

In a conservative system, the interactive force will be $\mathbf{F} = -\nabla V(r)$, which gives

$$\begin{aligned} \mathbf{F} &= -\nabla[-\langle n \rangle hc/r] \\ &= -\langle n \rangle hc/r^2 \mathbf{r}_0 + hc \cdot (\delta \langle n \rangle / \delta r) \cdot 1/r \mathbf{r}_0, \end{aligned} \tag{4}$$

where $\mathbf{r}_0$ is the unit vector. We point out that preferring to take the variation $\delta\langle n \rangle/\delta r$ may refer to the property of the system, other than the kinetic behaviour of $d\langle n \rangle/dr$ (noticing the possible term $\delta c/\delta r$ being omitted).

Obviously, if $\delta \langle n \rangle / \delta r = 0$, then we have the square-inverse force

$$\mathbf{F} = -\nabla V(r) = -\langle n \rangle hc/r^2 \mathbf{r}_0. \tag{5}$$

However, we can get more information from Eq. (4). When the system is in motion, i.e., the distance r increases, it is naturally expected that the average number $\langle n \rangle$ would increase, which it means the space may contain more wave components. This will cause $\delta \langle n \rangle / \delta r \neq 0$. Thus, an additional force $\mathbf{F}_a$ will occur

$$\mathbf{F}_a = hc \cdot (\delta \langle n \rangle / \delta r) \cdot 1/r \mathbf{r}_0. \tag{6}$$

Generally speaking, as the distance r is increasing, the average number $\langle n \rangle$ will increase, i.e., $\delta \langle n \rangle / \delta r > 0$. Therefore, the direction of $\mathbf{F}_a$ will be opposite to the former term. It is very useful to realize this point.

3 The correlations

As lacking the knowledge of the quantum stationary wave, it needs to build up the correlation. In the case of Coulomb attraction in atoms, $F = -kZe^2/r^2$ ($K = 1/4\pi\varepsilon_0$, the Coulomb constant, e, the electric charge), so it will lead to

$$-kZe^2 = -\langle n \rangle hc. \tag{7}$$

It is easy to see that the electric charge e, has now the new physical meaning $-$ it characterizes the wave properties ($\langle n \rangle$, h, and c).

In addition, when correlated to the Newtonian gravitation $F=-GmM/r^2$, we may have

$$-GmM = -\langle n \rangle hc. \tag{8}$$

Also, the gravitational mass becomes a new measurement of the wave properties.

4 The form of the force and the critical radius r_c

For easy of discussions, we let

$$\begin{aligned}\mathbf{F} &= -\langle n \rangle hc/r^2 + hc(\delta \langle n \rangle / \delta r)\cdot 1/r \\ &\equiv f_2 \cdot (1/r^2 - f_1/r),\end{aligned} \tag{9}$$

where, $f_2 \equiv -\langle n \rangle hc$, and $f_1 \equiv 1/f_2 \cdot \delta f_2/\delta r \equiv 1/\langle n \rangle \cdot (\delta \langle n \rangle / \delta r)$. This leads to the following forms for atoms and gravitation, respectively,

$$F_{atom} \equiv -Ze^2/4\pi\varepsilon_0 \cdot (1/r^2 - f_1/r), \tag{10a}$$

$$F_{grav.} \equiv -GmM \cdot (1/r^2 - f_1/r). \tag{10b}$$

We consider the special case of $1/r^2 - f_1/r = 0$. It occurs at the critical radius $r_c = 1/f_1$. i.e., in the range $0 < r < r_c$, the square-inverse force f_2/r^2 will dominate the behaviour, but out of the range, the additional force $F_a = f_2 f_1/r$ will dominate it.

Up to now, such an analysis is just a correlation, no more than a prompt, because many phenomena can be well described by the square-inverse force. It is expected that f_1 may be very small, even not zero. However, it may provide a new clue to some interesting issues.

5 The hyperfine structure (HFS) spectrum of hydrogen $1S_{1/2}$

The hydrogen atom is a pure two-body system, which is an extremely relevant issue to check the main idea. Moreover, we have accumulated much of the most intensive spectrum data both in experiments and theory.

More precisely, the issue is the inexplicable discrepancy between the most precise QED theoretical and experimental frequencies. This spectrum comes from the transition of the HFS $1S_{1/2}$ substates $F=1$ to $F=0$ (Fig. 3). The experimental frequency $\nu_{exp.} = 1.4204057517667(10)$ GHz, and its relative uncertainty $\sigma_{exp} = \pm 0.7 \times 10^{-12}$. In fact, it is the frequency standard in the hydrogen atom clock, also one of the most precise data in atomic physics, and a very important spectrum in radio astronomy (21cm).

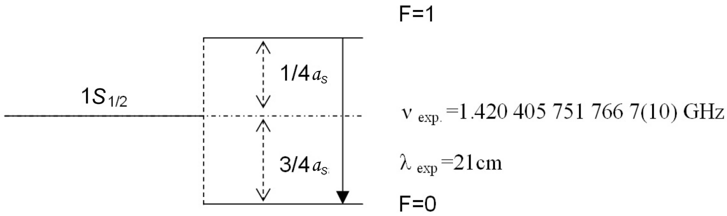

Figure 3: The energy levels of HFS $1S_{1/2}$ spectrum for hydrogen, in which, a_s, the splitting interval, and $\nu_{theory}=a_s/h$, the theoretical frequency.

The problem is so happened to the most precise theory. Though all efforts have been made (including the corrections up to the motion and the charge distribution of the proton), however, the best QED modification is $\nu_{theory}=1.4204034(13)$ GHz [1–3]. The frequency $\nu_{exp.}$ is still higher than ν_{theory}; their discrepancy is given by $\nu_{exp.}-\nu_{theory}=0.002$ 3(13) MHz.

By noting the relative deviation $\sigma=(\nu_{exp.}-\nu_{theory})/\nu_{exp.}=1.66\times10^{-6}$, it seems not bad at the first look. However, comparing with the experimental precision 0.7×10^{-12}, this relative error is 2.37×10^{6} times bigger.

It is really an incredible deviation for the carefully corrected QED prediction, but a good chance to check the f_1 term.

6 Determining the repulsion f_1

Now it may become a little clear − it is not an issue of the calculation precision but probably a matter unconscious of the tiny repulsion. Completely describing the details requires a little space, here we will explain it.

The potential has changed as the existence of f_1. Its zero point can be only at a finite critical radius r_c. For simplification, we start the discussion from the force, and take f_2 and f_1 as constants. This gives

$$\begin{aligned}\mathbf{V}(r)&=-\int_{rc}^{r}\mathbf{F}\cdot d\mathbf{r}=-\int_{rc}^{r}\Big(-Ze^2/4\pi\varepsilon_0[1/r^2-f_1/r]\Big)\cdot dr\\&=-Ze^2/4\pi\varepsilon_0\cdot1/r+-Ze^2/4\pi\varepsilon_0\cdot f_1\cdot[\mathrm{Ln}(r/r_c)-1]\equiv\mathbf{V}_0(r)+\Delta\mathbf{V}(r),\end{aligned}\tag{11}$$

where $\mathbf{V}_0=-Ze^2/4\pi\varepsilon_0\cdot1/r$, and $\Delta\mathbf{V}(r)=-Ze^2/4\pi\varepsilon_0\cdot f_1\cdot[\mathrm{Ln}(r/r_c)-1]$.

Secondly, we use the perturbation theory to deal with HFS $1S_{1/2}$ spectrum for determining the energy E

$$[\mathbf{H}_0(r)+\Delta\mathbf{V}(r)]\Psi_{n,l,F,M_f}=E\Psi_{n,l,F,M_f}.\tag{12}$$

Actually, it is equivalent to the Dirac equation. $\mathbf{H}_0(r)$ is the 0th Hamilton operator, which contains the kinetic, Coulomb potential, Darwin recoil, relativistic effect and other necessary corrections of QED (including vacuum polarization, self energy of the electron, and the contributions from the proton). Ψ_{n,l,F,M_f} represents the wave function, and n, l, F and M_f are a group of good quantum numbers in describing the HFS substates of $1S_{1/2}$ ($F=1$, $M_f=-1,0,1$ and $F=0$, $M_f=0$, respectively).

Expecting f_1 very tiny, and approximately expressing the perturbed wave function Φ on the bases of $F=1$ and $F=0$ [4], we may simply link them to the existing relative deviation σ

$$\Phi = c_1\Psi_1 + c_0\Psi_0 \tag{13}$$

By so doing, the perturbed energy levels are obtained

$$E_\pm = \frac{1}{2}\cdot(E_1+V_{11}+E_0+V_{00}) \pm \frac{1}{2}\cdot\sqrt{(E_1-E_0+v_{11}-V_{00})^2+4V_{10}^2}, \tag{14}$$

where E_1 and E_0 are the QED theoretical energy levels ($F=1$ and $F=0$), respectively. Symbols $V_{11}=\langle\Psi_1|\Delta\mathbf{V}|\Psi_1\rangle$, $V_{00}=\langle\Psi_0|\Delta\mathbf{V}|\Psi_0\rangle$, and $V_{10}=V_{01}{}^*=\langle\Psi_1|\Delta\mathbf{V}|\Psi_0\rangle$ represent the integrals to the wave functions.

Note that $\Delta\mathbf{V}(r) = -Ze^2/4\pi\varepsilon_0\cdot f_1\cdot[\mathrm{Ln}(r/r_c)-1]$ is only the function of the spatial distance r, and take the 0^{th} order spatial wave function $R_{1,0}(r)$ in the calculation. Its precision $(R-R_{1,0})/R_{1,0}$ would be of the order of α^2 ($\alpha \approx 1/137$, the fine structure constant) as the main errors coming from the relativistic effects. We may get

$$V_{10} = V_{00} = V_{10} = \langle R_{1,0}(r)| - Ze^2/4\pi\varepsilon_0\cdot f_1\cdot[\mathrm{Ln}(r/r_c)-1]|R_{1,0}(r)\rangle$$

because both basic functions Ψ_1 and Ψ_0 have the same spatial parts $R_{1,0}(r)$. By using the approximation $(1+x^2)^{1/2} \approx 1+1/2\cdot x^2$, the new energy levels will be

$$\begin{aligned} E_\pm &= \frac{1}{2}\cdot(E_1+E_0)+V_{10} \pm \frac{1}{2}\cdot\sqrt{(E_1-E_0)^2+4V_{10}^2} \\ &\approx \frac{1}{2}\cdot(E_1+E_0)+V_{10} \pm \frac{1}{2}\cdot\Big((E_1-E_0)\cdot[1+2V_{10}^2/(E_1-E_0)^2]\Big), \end{aligned} \tag{15}$$

or equivalently,

$$E_+ = E_1 + V_{00} + \Big((E_1-E_0)\cdot V_{10}^2/(E_1-E_0)^2\Big), \tag{16a}$$

$$E_- = E_1 + V_{00} - \Big((E_1-E_0)\cdot V_{10}^2/(E_1-E_0)^2\Big). \tag{16b}$$

Therefore, the new energy difference ΔE for the substates of $1S_{1/2}$ is

$$\Delta E = E_+ - E_- = \Delta E_0\cdot\Big(1+2V_{10}^2/\Delta E_0^2\Big). \tag{17}$$

Here, $\Delta E = E_+ - E_-$ represents the actual splitting $h\nu_{exp.}$, and $\Delta E_0 = E_1 - E_0 = h\nu_{theory}$ is the QED prediction without counting the repulsion. Noticing the ratio

$$\sigma \approx \frac{\nu_{exp.}-\nu_{theory}}{\nu_{theory}} = \frac{\Delta E - \Delta E_0}{\Delta E_0} = \frac{2V_{10}^2}{\Delta E_0^2}. \tag{18a}$$

Thus we may have

$$V_{10} = \langle R_{1,0}(r)| - Ze^2/4\pi\varepsilon_0\cdot f_1\cdot[\mathrm{Ln}(r/r_c)-1]|R_{1,0}(r)\rangle = \pm\sqrt{(\sigma/2)}\Delta E_0. \tag{18b}$$

Table 1: The determination f_1 by using the relative deviation σ between the experimental and the theoretical frequencies (a.u.: atomic unit, 1 a.e.u.= 27.2eV, $1a_1=0.529\times10^{-10}$m).

State	Relative deviation σ	$\langle\|\mathrm{Ln}(r/r_c)-1\|$	$V_{10}=V_{11}=V_{00}$		$f_1=1/r_c$	$\Delta\nu=\sigma\Delta E_{HFS}/h$
$1S_{1/2}$	1.66×10^{-6}	-26.41	1.97×10^{-10}	5.35×10^{-9}	7.45×10^{-12}	0.0023

Note: the 0th order spatial wave function $R_{10}(r)=2(Z/a_1)^{3/2}\exp(-Zr/a_1)$, for hydrogen atom, $Z=1$.

Obviously, $-\sqrt{(\sigma/2)}$ inducing $\mathrm{Ln}(-|f_1|r)$, which is meaningless. This f_1 must be positive as $r<r_c$ and $[\mathrm{Ln}(r/r_c)-1]<0$. The above equation needs a numerical calculation. We have already known $\sigma=1.66\times10^{-6}$ and $\Delta E_0/h=\Delta E_{HFS}/h=1.42$ GHz for $1S_{1/2}$. The calculation indicates when $f_1=1/r_c=7.45\times10^{-12}(a_1)^{-1}$, the integral

$$\langle|\mathrm{Ln}(r/r_c)-1|\rangle\equiv\int[\mathrm{Ln}(r/r_c)-1]R^2_{1,0}(r)\cdot r^2dr=-26.41,$$

and Eq. (18b) gets satisfied (Table 1).

Therefore, based on the HFS $1S_{1/2}$ spectrum of hydrogen, the force between a proton and an electron should be

$$F(r)=-\frac{1}{r^2}+\frac{7.45\times10^{-12}}{r}, \tag{19a}$$

or in the form of the potential

$$V_r=-\Big(\frac{1}{r}+7.45\times10^{-12}\cdot[\mathrm{Ln}(7.45\times10^{-12}\cdot r)-1]\Big). \tag{19b}$$

That is to say, if the repulsion does really exist, its ratio to the Coulomb attraction will be 7.45×10^{-12} at the 1st Bohr radius $(r=a_1=1)$. It is, indeed, too tiny to be detected in common experiments.

The $1S_{1/2}$ frequency difficulty, therefore, is completely solved by confirming such an f_1 term. However, such an analysis provides a new clue - the electromagnetic interaction what we have understood might be an approximation with the high precision (early, Lundeen and Pipkin proposed their doubt [7]), but this approximation is not in the form of $1/r^{(2\pm n)}$ once widely worried about. Also, it implies the Coulomb attraction (at least, between an electron and a proton in hydrogen atom) maybe not of long range, and the extrapolated critical radius is $r_c=1/f_1=1.30\times10^{11}(a_1)=7.11$ M. If really so, there must exist other inexplicable deviations because we have never corrected similar effect in QED predictions before.

7 Another case − the frequency deviation of the Lamb Shift in hydrogen, $n=2$, $2S_{1/2}(F=0)-2P_{1/2}(F=1)$ transition

The $n=2$ Lamb shift (Fig. 4), though divided into the range of fine structure(FS) spectroscopy [5.6], is significant to QED theory - it was this Lamb shift that promoted the development of the concepts such as the vacuum polarization and the self energy of the electron.

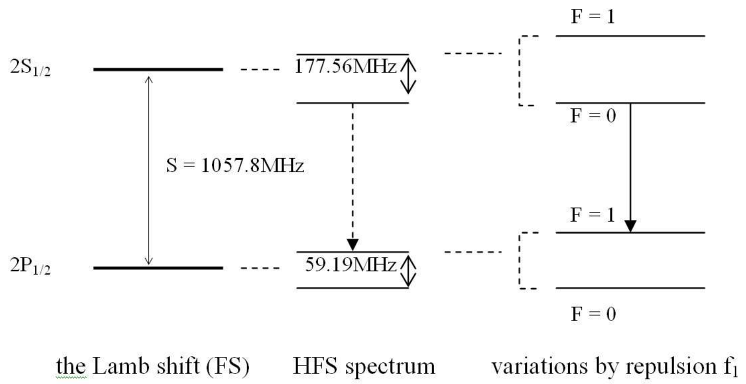

Figure 4: Repulsion induced the frequency difference in the $n=2$ Lamb shift $(2S_{1/2}-2P_{1/2})$.

The deviation still exists. The most precise experimental frequency is 1057.845(9) MHz [7], but the best modified theoretical results were obtained by Erickson [8] and Mohr [9], they are 1057.930(10) MHz and 1057.884(13) MHz, respectively. Their deviations from the experiment are 0.085 MHz and 0.039 MHz, remarkably larger than the experimental uncertainty ± 0.009 MHz. But this time, unlike the $1S_{1/2}$ spectrum, the theoretical frequencies are systematically higher than the experiment. Of course, there is still a later reported QED prediction reached to 1057.855(14) MHz [3], it seemed to be solved (at least, it is so in appearance).

Logically, if the f_1 really exists, it will naturally affect other QED predictions. In other words, the QED predictions before should contain the systematic uncertainties because they omitted the f_1 effect.

The succeeding analysis is something like an inverse process as used in $1S_{1/2}$. First, we calculate the energy differences on $2S_{1/2}$ and $2P_{1/2}$ by using the obtained f_1, then determine the transition frequency from $F=0\to F=1$.

Combining Eq. (16) and the data in Table 2, the variations of ΔE and $\Delta\nu$ will be

$$\Delta E = E - E_{theory} \approx \Delta\Big(V_{00} \pm 1/2\cdot(E_1-E_0)\cdot\Delta[2V_{10}^2/(E_1-E_0)^2]\Big)\Big|_{2P}^{2S}, \tag{20a}$$

$$\begin{aligned}\Delta\nu &= \Delta E/h = \Big(V_{00}(2S_{1/2}) - V_{00}(2P_{1/2})\Big)/h + 1/2\cdot\Big(\pm\Delta\nu(2P_{1/2}) - \pm\Delta\nu(2P_{1/2})\Big)\\ &= (-0.008) + 1/2\cdot(\pm 0.0168 - \pm 0.0511)\text{MHz}.\end{aligned} \tag{20b}$$

Such calculations provide some information on the substates. The HFS frequency differences in the substates $2S_{1/2}$ and $2P_{1/2}$ will increase if the repulsion ignored. They will reach to 0.0168MHz and 0.0511MHz, respectively (Table 2). i.e., their theoretical HFS frequencies will be lower than those in experiments once again. In other words, the repulsive effect will increase as the $\langle r\rangle$ increased.

Table 2: The estimated relative uncertainties σ and frequency deviations $\Delta\nu$ in the substates $2S_{1/2}$ and $2P_{1/2}$.

State	$f_1=\frac{1}{r_c}:10^{-12}(a_1^{-1})$	$\left\langle \left\|\mathrm{Ln}(r/r_c)-1\right\| \right\rangle$	$V_{00}=V_{01}=V_{11}$ (eV)	$\sigma=2\left(\frac{V_{11}}{\Delta E_{HFS}}\right)^2$	$\nu_{HFS}=\frac{\Delta E_{HFS}}{h}$	$\Delta\nu=\frac{\sigma\Delta\nu_{HFS}}{h}$
$2S_{1/2}$	7.45	-24.95	5.053×10^{-9}	9.47×10^{-5}	177.56	0.0168
$2P_{1/2}$	7.45	-25.12	5.086×10^{-9}	8.63×10^{-4}	59.19	0.0511

Note: The spatial wave functions of the 0th order approximation

$$R_{2,0}(r)=2(Z/2a_1)^{3/2}(1-Zr/2a_1)\cdot\exp(-Zr/2a_1),$$
$$R_{2,1}(r)-1/\sqrt{3}(Z/2a_1)^{3/2}(Zr/a_1)\cdot\exp(-Zr/2a_1).$$

The $n=2$ Lamb shift, is just the transition ($2S_{1/2}$, $F=0\rightarrow 2P_{1/2}$, $F=1$). Arranging the HFS energy differences to the $F=1$ and $F=0$ substates by the ratio $1/4\cdot a_s$, $-3/4\cdot a_s$ as in the HFS theory, the estimated deviation $\Delta\nu$ will be

$$\Delta\nu=(-0.008)-\frac{3}{4}\cdot 0.0168-\frac{1}{4}\cdot 0.0511=-0.034\ \text{MHz} \tag{21}$$

Clearly, such a negative deviation is found in good agreement with one of the existing discrepancy -0.039 MHz (Mohr's result [9]) within the experimental uncertainty $\pm$ 0.009MHz [7]. In other words, $f_1=7.45\times10^{-12}(a_1{}^{-1})$ is also valid for both substates $2S_{1/2}$ and $2P_{1/2}$ and the both mutually contribute -0.034 MHz in $n=2$ Lamb shift.

8 The precision of the Lande g factor for the electron

The anomalous magnetic moment (g-2) of the electron is with the same importance as the Lamb shift in QED theory.

Similar deviation happened to the Lande g factor. A modern experimental value [2,3, 10] $g_{exp.}=2\times(1+0.001159652193(4))$, and its relative uncertainty is

$$\sigma=\pm\Delta g_{exp.}/g_{exp.}=\pm4\times10^{-12}. \tag{22a}$$

However, the best calculated result [1, 3] $g_{theory}=2\times(1+001159652140(28))$. Their relative deviation σ_g

$$\sigma_g=(g_{exp.}-g_{theory})/g_{exp.}=5.29\times10^{-11}. \tag{22b}$$

It is one order higher than the experiment. And up to now, no further improved prediction reported.

Unfortunately, considering f_1 and recalculating the g factor will be an extremely hard job. However, a physical viewpoint may simplify the consideration - starting from the correspondence of the force, by comparing the following forms

$$F=e_0^2/4\pi\varepsilon_0\cdot 1/r^2,\quad F=e^2/4\pi\varepsilon_0\cdot 1/r^2\cdot(1-f_1\cdot r), \tag{23}$$

in which e_0 and e are the electronic charges before and after considering the repulsion, respectively. That is to say, the existence of f_1 term will cause a little change of the electric charge when measuring it by the force. The force, of course, is unique in the measurement - it is not depending upon which formula we take, so the correlation will be

$$e^2(1-f_1r)=e_0^2, \tag{24}$$

namely, $e\approx e_0(1+1/2\cdot f_1r)$, as $1/(1-f_1r)^{-\frac{1}{2}}\approx(1+1/2\cdot f_1r)$. Hence,

$$\Delta e/e\approx\frac{1}{2}\cdot f_1\Delta r. \tag{25}$$

This clue is also helpful to improve the precision $\Delta e/e$ in measurements, but back to the farm. The Lande g factor is two times of the average in the unit of the Bohr magnetic moment, i.e., $g=\langle\mu\rangle/[\langle\mathbf{s}\rangle(e\hbar/2m)]$, so the extra deviation induced by the electronic charge e (actually by the existence of f_1) will be

$$\Delta g=-\langle\mu\rangle/[\langle\mathbf{s}\rangle(e\hbar/2m)]\cdot\Delta e/e\approx-g\cdot(1/2\cdot f_1\cdot\Delta r).$$

Taking the average $\langle\rangle$, it becomes

$$\langle\Delta g\rangle/g=-\frac{1}{2}\cdot f_1\cdot\Delta\langle r\rangle \tag{26}$$

This indicates the precision of g factor would be slightly depending upon the initial and final states in transitions. Generally speaking, there will be $\Delta\langle r\rangle>0$ in the photon absorption, as well as $\Delta\langle r\rangle<0$ in the emission.

Hydrogen atom is still a good system. Examining its H_β line (486.1 nm, $n=4\rightarrow n=2$). The theoretical average distance $\langle r\rangle_{n,l}=a_1/2\cdot[3n^2-l(l+1)]$ (where $l=0, 1, \cdots, n-1$) and the weight is $(2l+1)/\sum_{l=0}^{n-1}(2l+1)$. Hence

$$\langle r\rangle_n=\sum_{l=0}^{n-1}\frac{a_1}{2}\cdot[3n^2-l(l+1)]\cdot\frac{(2l+1)}{\sum\limits_{l=0}^{n-1}(2l+1)}. \tag{27}$$

It is easy to get $\langle r\rangle_{n=2}=5.25a_1$ and $\langle r\rangle_{n=4}=20.25a_1$, so we have

$$\Delta\langle r\rangle=\langle r\rangle_{n=2}-\langle r\rangle_{n=4}=-15a_1, \tag{28a}$$

$$\langle\Delta g\rangle/g=15/2\times7.45\times10^{-12}=5.58\times10^{-11} \tag{28b}$$

The present deviation $\sigma_g=(g_{exp.}-g_{theory})/g_{exp.}=5.29\times10^{-11}$, and f_1 will contribute to the theory $\langle\Delta g\rangle/g=5.58\times10^{-11}$. So the improved results will be

$$\sigma=5.29\times10^{-11}-5.58\times10^{-11}=-2.87\times10^{-12}. \tag{28c}$$

Once again, the f_1 effect makes both the experiment and the theory in good agreement within the experimental uncertainty $\pm4\times10^{-12}$.

9 Discussion and conclusion

Three different issues in appearance, now they are completely in good agreement within the experimental precisions, and they consistently point to the same effect − the electromagnetic interaction between a proton and an electron occurs a deviation from the Coulomb force. Its influences to higher HFS states $2S_{1/2}$ and $2P_{1/2}$, are estimated reaching to 0.0168 MHz and 0.0511 MHz, respectively. They are much larger than the 0.0023 MHz at $1S_{1/2}$, hence, easier to be observed. Also, it is expecting that ^{2}D and ^{3}He$^+$ could further check the similar f_1.

Although the repulsion seems tiny at 1st Bohr radius, however, it strongly implies that the Coulomb attraction in hydrogen atom might be not of long range, and the extrapolated critical radius $r_c = 1/f_1 = 1.30\times10^{11}(a_1) = 7.11$ M. As lacking data in the cases of $n>4$, however, there exists the possibility that the f_1 may slightly change with the states, i.e., $f_1 = f_1(r)$. If so, it would cause a smaller r_c (a smaller atom).

Also, it would be very interesting to explore the probability that the f_1 effect would occur for the same kind electric charges (e.g., a proton vs. a proton, or an electron vs. an electron). In these cases, it is expected to exhibit a tiny attraction.

In addition, for the gravitation, it is worthy noting the tiny repulsion would possibly exist, too. The main problem is that it is difficult to find out a pure two-body system in the gravitation, and the precisions for observation could not match up with those in the hydrogen spectra. However, the features are interesting. First, a perturbation analysis indicates it would cause an extra but negative contribution to the precession of the planet's perihelion, and the precession angle per circle $\Delta \approx -\pi a(1-e^2)f_1$ (a and e, the orbital parameters, f_1 the repulsion). That is to say, the outer planet (e.g., Neptune, or Pluto), with larger a, would behave larger anomalous but negative precession, hence, easier to be checked. (on contrast, the general relativity effect is a positive contribution to the precession and becomes smaller for outer planet, as the precession angle per circle $\Delta\approx 6\pi GM/[c^2a(1-e^2)]$ [11]). More important matter maybe the critical radius r_c, because within the scope, those celestial bodies attract each other, but out of such a distance, the Solar system, or galaxies, maybe black holes, would exhibit their net repulsion. Perhaps, it is such a repulsion that drives remote galaxies further apart. In other words, it may be a kind of dark energy once we have ignored.

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