J. At. Mol. Sci.
doi: 10.4208/jams.030410.032810a

Vol. **1**, No. 3, pp. 268-274
August 2010

Entanglement dynamics of two trapped ions in the intermediate excitation regime

Zhong-Jie Wang* and Chao-Yang Fan

Department of Physics, Anhui Normal University, Wuhu 241000, China

Received 4 March 2010; Accepted (in revised version) 28 March 2010
Published online 28 June 2010

Abstract. We have studied the model for interaction of two trapped ions in a trap with a laser beam in the intermediate excitation regime. By applying unitary transformations, the system can be transformed into the Tavis-Cummings model. The entanglement dynamics of two trapped ions in this system has been investigated. With computation of the concurrence, unlike that in Tavis-Cummings model, we find that the entanglement of two ions in our model undergoes periodic death and revivals.

PACS: 03.65Yz, 03.67Mn

Key words: trapped ion, entanglement sudden death, concurrence

1 Introduction

The entanglement in quantum systems has received great attentation not only because it is of fundamental interest in quantum mechanics but also because it plays very important role in processing of quantum information such as quantum cryptography [1], quantum key distribution [2], quantum superdense coding [3], quantum teleportation [4] and quantum computation [5]. The entanglement of two particles has been demonstrated experimentally using ultra cold trapped ions [6] and cavity quantum electrodynamics schemes [7].However, one has found that the entanglement of the bipartite system being open to environment will be decay exponentially as similar as quantum decoherence. This kind of decay is called as the loss of entanglement. Quite recently, one found that the entanglement between the bipartite coupled with independent reservoirs terminates abruptly in finite time, which is called the entanglement sudden death (ESD). It has been shown that ESD effect is not only sensitive to the initial states of systems, but also is dependent on property of noise [8–11]. For a special initial state, the entanglement of two particles will disappear and then revive.

*Corresponding author. *Email address:* `wuliwzj@mail.ahnu.edu.cn` (Z. J. Wang)

There is growing interest in the entanglement of trapped ions. On the one hand, it has been realized that the trapped ions can be located at a known and controllable distance from one another. On the other hand, it was discovered that the trapped ions can be prepared in maximally entangled states that are isolated from its environment [12]. In Ref. [13], the authors show that ESD can be influenced by the Stark shift and long-lived entanglement can be produced between the two trapped ions that are driven by a classical light field in the weak excitation region.

In this paper, we investigate the dynamics of the entanglement between two trapped ions in the intermediate excitation regime. We find that the entanglement of two ions undergoes periodic death and revives. An explicit connection between the entanglement dynamics of two trapped ions and their initial states is presented.

2 Theoretical model

We consider two two-level ions trapped in a harmonic potential traps, which interacts with a laser field with frequency ω.We restrict our consideration to the quantum mechanical motion of the ion in the x direction and omit the breathe mode of the ions. The Hamiltonian of system is given by [14]

$$H=H_t+\hbar\frac{\Delta}{2}J_z+\frac{\hbar\Omega}{2}(e^{i\eta(a+a^+)}J_+ +e^{i\eta(a+a^+)}J_-), \quad (1)$$

where $H_t=\hbar\nu a^+a$,

$$J_z=\sum_{i=1}^{2}\sigma_z^{(i)},\ J_+=\sum_{i=1}^{2}\sigma_+^{(i)},\ J_-=\sum_{i=1}^{2}\sigma_-^{(i)}$$

and $a(a^+)$ is the annihilation (creation) operator of the center-of-mass vibrational mode of the ion, $\Delta=\omega_a-\omega$ is the detuning of the ionic transition from the laser frequency, which is set $\Delta=0$, the Lamb-Dick parameter $\eta=\pi a_0/\lambda$, with a_0 the amplitude of the ground state of the trap and the optical wavelength, $\sigma_z^{(i)}$, $\sigma_+^{(i)}$ and $\sigma_-^{(i)}$ are pseudospin inversion, raising, lowering operators of the ith ion and set $\eta \ll 1$. Applying the Lamb-Dicke approximation, neglecting the higher order terms on η, Eq.(1) can be represented in the form of the matrix as ($\hbar=1$)

$$H=H_t+\hbar\frac{\Delta}{2}J_z+\frac{\Omega}{2}J_x-\frac{\eta^2\Omega}{4}(a+a^+)^2J_x+\frac{\eta\Omega}{2}(a+a^+)J_y, \quad (2)$$

where $J_x=J^++J^-$, $J_y=-i(J^++J^-)$. In order to investigate entanglement dynamics of two trapped ions in the intermediate excitation regime ($\Omega\Box\nu$), we cannot make directly rotating wave approximation. We firstly make a unitary transformation $H'=THT^+=e^{i\pi J_y/4}He^{-i\pi J_y/4}$. The transformed Hamiltonian reads

$$H'=H_t+\frac{\Omega}{2}J_z-\frac{\eta^2\Omega}{4}(a+a^+)^2J_z-i\frac{\eta\Omega}{2}(a+a^+)(J^+-J^-). \quad (3)$$

Furthermore we perform a rotating wave approximation, Eq.(3) is then turned into

$$H' = H_t + \frac{\Omega}{2} J_z - \frac{\eta^2\Omega}{4}(ae^{-i\nu t} + a^+ e^{i\nu t})^2 J_z - i\frac{\eta\Omega}{2}(ae^{-i\nu t} + a^+ e^{i\nu t})(J^+ e^{i\Omega t} - J^- e^{-i\Omega t}). \quad (4)$$

We set the parameter $\Omega = \nu$ (so called intermediate excitation regime) and neglect the rapidly oscillating terms, we then obtain the transformed interaction Hamiltonian as

$$H' = H_t + \frac{\Omega}{2} J_z - \frac{\eta^2\Omega}{4}(2a^+a + 1) J_z - i\frac{\eta\Omega}{2}(aJ^+ - a^+ J^-). \quad (5)$$

It is seen that The Hamiltonian (5) is similar to the two atoms Jaynes-Cummings model. In the Hamiltonian (5), the third term denotes Stark effect. In order to solve the evolution operator, we define the dressed states as basic vectors

$$|1>^{(n)} = |1> \otimes |n>, \quad (6a)$$

$$|2>^{(n)} = |2> \otimes |n+1>, \quad (6b)$$

$$|3>^{(n)} = |3> \otimes |n+2>, \quad (6c)$$

$$|4>^{(n)} = |4> \otimes |n+1>, \quad (6d)$$

where

$$|1>|e_1e_2>, \; |2>= \frac{1}{\sqrt{2}}(|e_1g_2> + |g_1e_2>), \; |3>= g_1g_2>, \; |4>= \frac{1}{\sqrt{2}}(|e_1g_2> - |g_1e_2>)$$

are the ionic state vectors. In the subspace spanned by the basic vectors $|l>^{(n)}$ $(l=1,2,3,4)$, the Hamiltonian (5) is represented as

$$H' = \nu(n+1)l + \lambda_0 \begin{bmatrix} -(2n+1) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 2n+5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} - ig \begin{bmatrix} 0 & \sqrt{n+1} & 0 & 0 \\ -\sqrt{n+1} & 0 & \sqrt{n+1} & 0 \\ 0 & -\sqrt{n+2} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \quad (7)$$

where $\lambda_0 = \eta^2\Omega/4$, $g = \eta\Omega/2$.

3 The entanglement between ions

Supposing above ions system only has the phase decoherence induced by laser intensity and phase fluctuation, the master equation of the system can be written in the form [13]

$$\frac{d\rho(t)}{dt} = -i[H,\rho] - \frac{\gamma}{2}[H,[H,\rho]], \quad (8)$$

where γ is the phase decoherence rate. The first term on the righ-hand of Eq. (8) generates a coherent unitary evolution of density operator, while the second term denotes the decoherence effect of the environment on the system. Owing to $H' = THT^{+}$, the master Eq. (8) is rewritten as

$$\frac{d\rho'(t)}{dt} = -i[H',\rho'] - \frac{\gamma}{2}[H',[H',\rho']], \tag{9}$$

where $\rho' = T\rho T^{+}$. The solution to this master equation can be expressed as follows [13]

$$\rho'(t) = \sum_{m=0}^{\infty} \frac{(\gamma t)^m}{m!} M^m(t)\rho'(0)M^{+m}(t), \tag{10a}$$

$$M^m(t) = H^m \exp(-iHt)\exp(-\gamma t H^2/2), \tag{10b}$$

$$\rho'(0) = T\rho(0)T^{+}. \tag{10c}$$

The density operator $\rho'(t)$ can also be represented as follows

$$\rho'(t) = \sum_r \sum_{i,j} \rho'^{(r)}_{i,j}(t)|i>^{(r)(r)}<j|, \tag{11}$$

where $\rho'^{(r)}_{i,j}(t)$ are matrix elements of $\rho'(t)$ in the dressed state subspace. The reduced density operator of ionic system can be represented as

$$\rho^a(t) = Tr_f\{T^{+}\rho'(t)T\}, \tag{12}$$

where $Tr_f\{\}$ indicates the trace operation on the vibrational motion of the trapped ions. We now present a study of the entanglement properties of our model. We choose the initial state

$$\rho(0) = (|1> - |3)(<1| - <3|)/2 \otimes |\alpha><\alpha|,$$

in which $|\alpha>$ is the coherence state for the quantized center-of mass mode of the ions and the ion is maximum entanglement state. The reduced density matrix $\rho^a(t)$ for the ionic system can be computed numerically using Eq. (10a) and Eq. (11). If the matrix elements $\rho'^{(r)}_{i,j}(t)$ are computed, we can obtain the matrix elements $\rho^a_{i,j}(t)$ of the reduced density $\rho^a(t)$ as follows

$$\rho^a_{1,1}(t) = \frac{1}{4}\sum_r \left(\rho'^{(r)}_{1,1}(t) + 2\rho'^{(r+1)}_{2,2}(t) + \rho'^{(r+2)}_{3,3}(t)\right), \tag{13a}$$

$$\rho^a_{1,2}(t) = \frac{1}{4}\sum_r \left(\sqrt{2}\rho'^{(r)}_{1,1}(t) - \sqrt{2}\rho'^{r+2}_{3,3}(t)\right), \tag{13b}$$

$$\rho^a_{1,3}(t) = \frac{1}{4}\sum_r \left(\sqrt{2}\rho'^{(r)}1,1(t) - \sqrt{2}\rho'^{(r+2)}3,3(t)\right), \tag{13c}$$

$$\rho^a_{1,4}(t) = \frac{\sqrt{2}}{2}\sum_r \rho'^{(r+1)}_{2,4}(t), \tag{13d}$$

$$\rho^a_{2,1}(t) = \frac{1}{4}\sum_r \left(\sqrt{2}\rho'^{(r)}_{1,1}(t) - \sqrt{2}\rho'^{(r+2)}_{3,3}(t)\right), \tag{13e}$$

$$\rho^a_{2,2}(t)=\frac{1}{2}\sum_r\left(\rho'^{(r)}_{1,1}(t)+\rho'^{(r+2)}_{3,3}(t)\right), \tag{13f}$$

$$\rho^a_{2,3}(t)=\frac{\sqrt{2}}{4}\sum_r\left(\rho'^{(r)}_{1,1}(t)+\rho'^{(r+2)}_{3,3}(t)\right), \quad \rho^a_{2,4}(t)=0, \tag{13g}$$

$$\rho^a_{3,1}(t)=\frac{1}{4}\sum_r\left(\rho'^{(r)}1,1(t)-2\rho'^{(r+1)}_{2,2}(t)+\rho'^{(r+2)}_{3,3}(t)\right), \tag{13h}$$

$$\rho^a_{3,2}(t)=\frac{\sqrt{2}}{4}\sum_r\left(\rho'^{(r)}_{1,1}(t)-\rho'^{(r+2)}_{3,3}(t)\right), \tag{13i}$$

$$\rho^a_{3,3}(t)=\frac{1}{4}\sum_r\left(\rho'^{(4)}_{1,1}(t)+2\rho'^{(r+1)}_{2,2}(t)-\rho'^{(r+2)}_{3,3}(t)\right), \tag{13j}$$

$$\rho^a_{3,4}(t)=\frac{\sqrt{2}}{2}\sum_r\rho'^{(r+1)}_{2,4}(t), \tag{13k}$$

$$\rho^a_{4,1}(t)=-\frac{\sqrt{2}}{2}\sum_r\rho'^{(r+1)}_{4,2}(t), \quad \rho^a_{4,2}(t)=0, \tag{13l}$$

$$\rho^a_{4,3}(t)=\frac{\sqrt{2}}{2}\sum_r\rho'^{(r+1)}_{4,2}(t), \quad \rho^a_{4,4}(t)=0. \tag{13m}$$

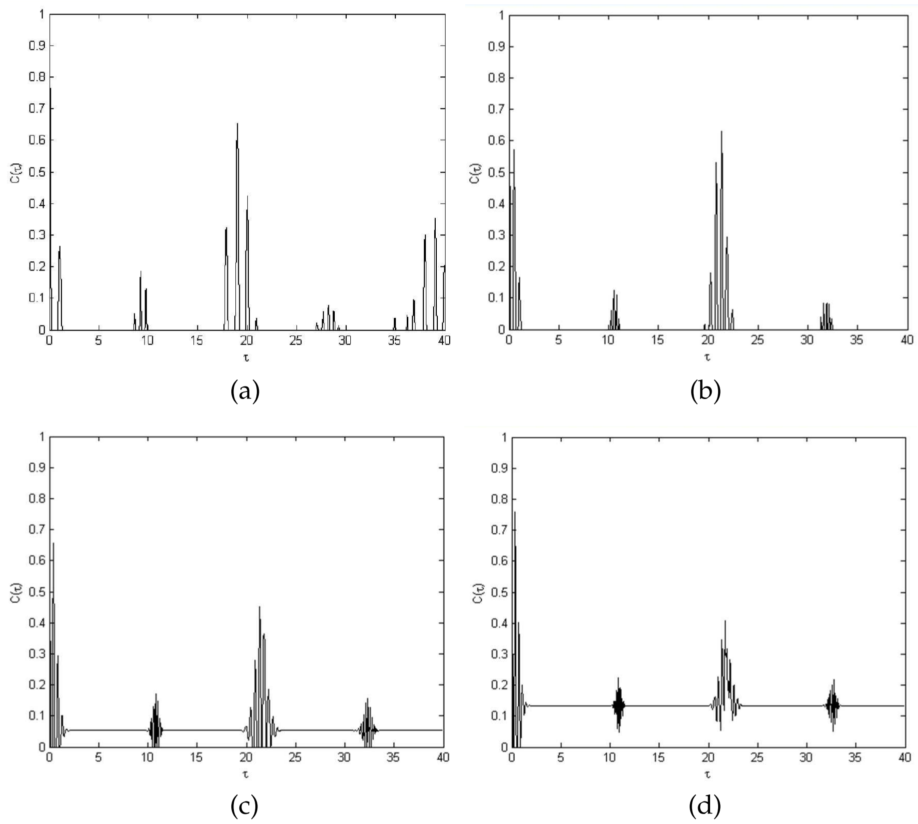

Figure 1: The evolution of the concurrence with the scaled time for the different parameter $\bar{n}$ with $\gamma=0$, $\eta=0.2$, (a)$\bar{n}=10$, (b)$\bar{n}=30$, (c)$\bar{n}=40$, (d)$\bar{n}=50$.

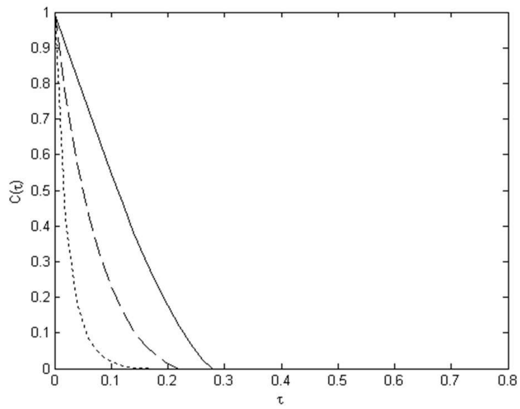

Figure 2: The evolution of the concurrence with the scaled time for the different parameter γ, $\bar{n}$ with $\eta=0.2$, $\gamma=0.001$, solid line: $\bar{n}=5$, dashed line: $\bar{n}=10$, dotted line: $\bar{n}=20$.

In order to study the dynamics of entanglement in our model, we employ the concurrence as a entanglement measure. The concurrence is defined as [15]

$$C(\rho)=\max(0,\lambda_1-\lambda_2-\lambda_3-\lambda_4), \tag{14}$$

where λ_i $(\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$ are the eigenvalues of the time-dependent operator $\rho(\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$. The concurrence ensures the scale between 0 and 1. In particular, $C(\rho)=1$ indicates maximum entanglement between the two qubits, whereas $C(\rho)=0$ represents disentanglement. It should be pointed out that before computing the concurrence by Eq.(11), we must represent the matrix elements $\rho^a_{i,j4}(t)$ in the basic ionic state vectors $|\bar{i}>$ ($|\bar{1}>=|e_1e_2>$, $|\bar{2}>=|e_1g_2>$, $|\bar{3}>=|g_1e_2>$ and $|\bar{4}>=|g_1g_2>$). This can made by a simple unitary transformation.

We firstly investigate entanglement dynamics of two trapped ions without the phase damping (i.e., $\gamma=0$). The time evolution of the concurrence for different average photon number (average vibrational quanta number) $\bar{n}=<a^+a>$ has been calculated, as shown in Fig. 1. The feature of the periodic collapses and revivals of the concurrence is observed. With the increase of the average photon number, the revival periodic of the concurrence does not change basically, but the amplitude of the concurrence becomes smaller. In other hand, When $\bar{n}$is small, the entanglement of two ions can be collapsed to zero. When $\bar{n}$ is large, the entanglement of two ions cannot be collapsed to zero. This makes clear that the entanglement of two ions is much influenced by the Stark effect (which is dependence on the average photon number).

In the condition of the phase damping (i.e., $\gamma \neq 0$), the entanglement of two trapped ions will decay rapidly (see Fig. 2). When the average photon number $\bar{n}=5$, the entanglement will decay into zero suddenly, but for the large average photon number, the entanglement will decay into zero exponentially. The more average photon number is, the faster the entanglement will decay.

4 Conclusions

We have investigated that the entanglement of two trapped ions with a radiation field in the intermediate regime. The entanglement of two trapped ions undergoes periodic fluctuation. When the phase damping is absent, the larger the average photon number is, the smaller the maximal entanglement we can obtain is. When the phase damping is present, the larger the average photon number is, the more rapid the entanglement decays. These denote that the entanglement of two ions much influenced by the Stark effect.

Acknowledgments. This work was supported by the Natural Science Foundation of Anhui Province of China under Grant No. 090412060 and the Natural Science Foundation of the Education Committee of Anhui Province of China under Grant No. KJ2008A029.

References

[1] N. Gisin, G. Riberdy, W. Tittel, and H. Zbinden, Phys. Mod. Rev. 74 (2002) 145.
[2] J. Rigas, O. Ghne, and N. Ltkenhaus, Phys. Rev. Lett. 67 (1991) 661.
[3] A. K. Pati, P. Parashar, and P. Agrawal, Phys. Rev. A 72 (2005) 012329.
[4] G. Rigolin, Phys. Rev. A 71 (2005) 032303.
[5] A. Srensen and K. Mlmer, Phys. Rev. A 62 (2000) 022311.
[6] Q. A. Turchette, Q. A. Turchette, C. S. Wood, B. E. King, C. J. Myatt, D. Leibfried, W. M. Itano, C. Monroe, and D. J. Wineland, Phys. Rev. Lett. 81 (1998) 3631.
[7] G. P. He, S. L. Zhu, Z. D. Wang, and H. Z. Li, Phys. Rev. A 68 (2003) 012315.
[8] T. Yu and J. H. Eberly, Phys. Rev. Lett. 93 (2004) 140404.
[9] T. Yu and J. H. Eberly, Phys. Rev. B 66 (2002) 193306.
[10] C. Pineda and T. H. Seligman, Phys. Rev. A 73 (2006) 012305.
[11] H. T. Cui, K. Li, and X. X. Yi, Phys. Lett. A 365 (2007) 44.
[12] K. Mlmer and A. Srensen, Phys. Rev. Lett. 83 (1999) 022311.
[13] Mahmoud Abdel-Aty and H. Moya-Cessa, Phys. Lett. A 369 (2007) 372.
[14] Z. J. Wang, J. Phys. A: Math. Theor. 40 (2007) 6211.
[15] W. K. Wootters, Phys. Rev. Lett. 80 (1998) 2245.