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Correlation between classical Fisher information and quantum squeezing properties of Gaussian pure states

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Abstract. The Fisher information of Gaussian pure states is studied in this work. Based on the definition of joint non-classical properties, we calculate the non-classical properties of Gaussian pure states. The results show that the Fisher information and Fisher length are efficacious tools to study the non-classical properties of quantum states. And the non-classical properties of states can be used to calculate the quantum properties quantificationally. Making use of Fisher information, one can obtain the correlation between the Fisher information and quantum squeezing properties of Gaussian pure states. Especially, it is significative that one can quantificationally describes the fluctuation of quantum states by an alternative new method.

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Key words: Gaussian pure states, Fisher information, Fisher length, joint non-classical properties

1 Introduction

Gaussian pure states is an important quantum states in quantum optics domains, it can describe the coherent output of laser and the squeezing optics field of parametric process. In mathematical field Gaussian pure states represent a category distribution namely Gaussian distribution [1]. The Hamiltonian operator of Gaussian pure states is a simple quantum function. So some pursuer have been interested in Gaussian pure states over the past few years and obtained some significative results. For example, Walls and Milburn gave some results of standard forms and entanglement engineering of multimode Gaussian states [2] and Xia *et al.* studied the higher-order squeezing and information entropy for Gaussian pure states [3].

Fisher information was originally introduced by Fisher, as a measure of "intrinsic accuracy" in statistical estimation theory. It provides in particular a bound on the degree to

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which members of a family of probability distributions can be distinguished [4]. Quantum generalizations of Fisher information may be also provide corresponding bounds on the degree to which members of a family of quantum states can be distinguisher by measurement [5–9]. These results show that Fisher information closely correlates to the quantum properties of states. It is well-known that further studies on the quantum states and properties of quantum states are very important to the creation of quantum optics fields and entangled states [10, 11], which are the basic of quantum communication and quantum computation [12].

In this work, based on the definitions of Fisher information and Fisher length, we study the Fisher information and joint non-classical properties of Gaussian pure states. We calculate the Fisher information and Fisher length of Gaussian pure states. The results show that there has been a close relationship between classical Fisher information and quantum squeezing properties of Gaussian pure states. Thus we can investigate the quantum properties of Gaussian pure states in a fire-new way.

2 Function of Gaussian pure states

In coordinate representation single mode Gaussian pure states is given by [1,3]

$$\psi_g(x)=\langle x|\mu_g\rangle=N_g\exp\Big(\frac{1}{2}i\sigma_x+\frac{1}{2}ix_0p_0+ip_0x\Big)\exp\Big[-\frac{r}{2}(x-x_0)^2\Big], \tag{1}$$

where x_0 and p_0 are given by the function

$$p_0=\bar{p}=\langle p\rangle=\int_{-\infty}^{+\infty}dx\langle\mu_g|x\rangle\Big(-i\frac{\partial}{\partial x}\Big)\langle x|\mu_g\rangle, \tag{2}$$

$$x_0=\bar{x}=\langle x\rangle=\int_{-\infty}^{+\infty}dx|\langle x|\mu_g\rangle|^2x, \tag{3}$$

where we assume $\hbar=1$ and $\omega=1$ for the convenience of the study. σ_x denotes the immeasurable phase angle, $r=r_1+ir_2$ is a complex number, in which r exerts a determining influence on a specific Gaussian pure states and $\mathrm{Re}(r)=r_1>0$. The normalized constant N_g is defined as

$$N_g=(\pi/r_1)^{-1/4}. \tag{4}$$

In momentum representation Gaussian pure states is given by

$$\psi_g(p)=\langle p|\mu_g\rangle=n_g\exp\Big(-\frac{1}{2}i\sigma_p+\frac{1}{2}ix_0p_0-ipx_0\Big)\exp\Big[-\frac{r}{2}(p-p_0)^2\Big], \tag{5}$$

where the normalized constant n_g is given by

$$n_g=(\pi|r|^2/r_1)^{-1/4}. \tag{6}$$

The phase angle σ_x and σ_p satisfy the flowing function

$$\exp(i\sigma_p)=\frac{r}{|r|}\exp(-i\sigma_g). \tag{7}$$

3 Classical Fisher information of Gaussian pure states

3.1 Fisher information and Fisher length

. The classical Fisher information associated with translations of a one-dimensional x with corresponding probability density $p(x)$ is given by [5,6]

$$F_x=\int_{-\infty}^{+\infty} dxp(x)[d\ln p(x)/dx]^2. \tag{8}$$

The primary application of this quantity in classical estimation theory is the lower bound.

One may also define a corresponding Fisher length for x

$$\delta_x=\left\{\int_{-\infty}^{+\infty} dxp(x)[d\ln p(x)/dx]^2\right\}^{-1/2}=F_x^{-1/2}, \tag{9}$$

where δ_x is the length scale over which $p(x)$ varies appreciably, known as the Cramer-Rao inequality.

For the quantum states function $\psi(x)$, we can calculate the Fisher information with Eq.(8) simply

$$F_x=\int_{-\infty}^{+\infty} dx|\psi(x)|^2[d\ln|\psi(x)|^2/dx]^2=16[\langle p^2\rangle_\psi-\langle p_{ci}^2\rangle_\psi]^2, \tag{10}$$

where

$$p_{ci}=\frac{1}{2i}\Big(\psi'(x)/\psi(x)-\psi^{*'}(x)/\psi^*(x)\Big), \tag{11}$$

where, $p=p_{ci}+p_{nc}$ denotes the momentum conjugate to x, and p_{ci} is the classical part, namely the system with momentum p has the classical and non-classical properties simultaneously. Using the seam definition we can obtain the Fisher information and Fisher length in momentum space

$$F_p=\int_{-\infty}^{+\infty} dpp(p)[d\ln p(p)/dp]^2, \tag{12}$$

$$\delta_p=\left\{\int_{-\infty}^{+\infty} dpp(p)[d\ln p(p)/dp]^2\right\}^{-1/2}=F_p^{-1/2}. \tag{13}$$

3.2 Fisher information and joint non-classical properties of Gaussian pure states

Making use of the definition Eqs. (1), (5), (8) and (12), we can simply obtain the Fisher information of Gaussian pure states in coordinate representation and momentum representation

$$F_x^g = \int_{-\infty}^{+\infty} dx N_g^2 \exp[-r(x-x_0)^2]\left\{\frac{d}{dx}\ln N_g^2 \exp[-r(x-x_0)^2]\right\}^2 = \frac{2|r|^2}{r_1}, \tag{14}$$

$$F_p^g = \int_{-\infty}^{+\infty} dp n_g^2 \exp[-r(p-p_0)^2]\left\{\frac{d}{dp}\ln n_g^2 \exp[-r(p-p_0)^2]\right\}^2 = \frac{2}{r_1}. \tag{15}$$

The parameter r can determine the properties of a Gaussian pure states uniquely. Consequently, Eqs. (14)-(15) indicate clearly that the Fisher information of a Gaussian pure states only relates to the parameter r. Similarly we can obtain the Fisher length of the Gaussian pure states determined by the parameter r

$$\delta_x^g = \frac{\sqrt{2r_1}}{2|r|}, \quad \delta_p^g = \frac{\sqrt{2r_1}}{2}. \tag{16}$$

According to the definition of Hall [5], the joint non-classical properties of quantum states is given by

$$J_{nc} = \frac{1}{2\delta_x\delta_p} = \frac{1}{2}\sqrt{F_x F_p}, \quad (\hbar = 1). \tag{17}$$

So we can see that joint non-classical properties J_{nc} is related to the Fisher information or Fisher length, namely J_{nc} direct ratio to product of Fisher length of a couple conjugate variables and inverse ratio to its square root of Fisher information. So making use of Eqs. (14), (15) and (17), or Eqs. (16)-(17), we can obtain the joint non-classical properties of a Gaussian pure states

$$J_{nc}^g = \frac{1}{2\delta_x\delta_p} = \frac{1}{2}\sqrt{F_x F_p} = \sqrt{1+\left(\frac{r_2}{r_1}\right)^2}. \tag{18}$$

According to the Eq. (18), we can see that the joint non-classical properties of a Gaussian pure states only relate to the parameter r, if only a Gaussian pure states has been determined by a parameter r, its non-classical properties has a determined value. The result given by Eq. (18) also shows that J_{nc}^g direct ratio to $|r|$ and inverse ratio t r_1 that is the real part of the parameter r. Here we point out emphatically that the joint non-classical properties of a Gaussian pure states is not less than 1, namely, $J_{nc} \geq 1$. When $r_2 = 0$ Gaussian pure states degenerate into coherent states and the $J_{nc} = 1$ accordingly, here it is the minimum uncertain states of quantum fluctuation. The joint states with smaller real part r_1 and bigger imaginary part r_2 have more non-classical properties. So these results indicate evidently that the joint non-classical properties of Gaussian pure states can measure its quantum properties quantificationally.

3.3 Quantum squeezing of Gaussian pure states and its Fisher length

One of the most striking features of quantum mechanics is the property that certain observables cannot simultaneously be assigned arbitrarily precise values. This property does not compromise claims of completeness for the theory, since it may consistently be asserted that such observables cannot simultaneously be measured to an arbitrary accuracy [6]. For example the quantum fluctuation of coordinate x and momentum p flow the Heisenberg inequality

$$\triangle x_{nc}\triangle p_{nc}\geq\frac{1}{2},\quad(\hbar=1),\tag{19}$$

where $\triangle x_{nc}$ and $\triangle p_{nc}$ denote the quantum fluctuation of coordinate x and momentum p accordingly. Eq. (19) describes the intrinsic relation of a couple of conjugated physical quantities. The intrinsic quantum fluctuation stem from the essential principle of quantum mechanics and the characteristic of unmeasured synchronously can not be overcome. Making use of the Fisher information and Fisher length we can rewrite the Heisenberg inequality as quality

$$\delta x\triangle p_{nc}=\frac{1}{2}\ \text{or}\ \delta p\triangle x_{nc}=\frac{1}{2}.\tag{20}$$

For all wave functions, we can regard Eq. (20) as an exact uncertainty relation. Thus, the uncertainty principle of quantum mechanics may be given by a precise form. So for a determined Gaussian pure states, we can describe the quantum fluctuation of conjugated physical quantities just making use of Fisher length and joint non-classical properties of Gaussian pure states obtained above. When $\delta x>\sqrt{0.5}$, conjugated momentum is squeezed, on the other hand, $\delta p>\sqrt{0.5}$, conjugated coordinate is squeezed. Thus the Fisher length and joint non-classical properties of a state provide alternative new method for us to describe the quantum squeezing properties of conjugated physical quantities. While $\delta x=\delta p=\sqrt{0.5}$, accordingly $\triangle x_{nc}=\triangle p_{nc}=\sqrt{0.5}$, it is indicated that the quantum states is a coherent states. For the Gaussian pure states we can obtain obviously

$$\triangle x_{nc}=\frac{1}{2}\frac{1}{\delta p}=\frac{1}{2}\sqrt{F_p}=\sqrt{\frac{1}{2r_1}},\tag{21}$$

$$\triangle p_{nc}=\frac{1}{2}\frac{1}{\delta x}=\frac{1}{2}\sqrt{F_x}=|r|\sqrt{\frac{1}{2r_1}}.\tag{22}$$

So from the result above, it is shown that we can calculate the quantum fluctuation exactly by the use of the Fisher length we have obtained, and we obtain an unanimous result with Refs. [3,6] in a new way. For other characters and use of Fisher information and Fisher length of some quantum states we will give further research.

4 Conclusions

We have calculated the Fisher information and Fisher length of Gaussian pure states. From general Gaussian pure states wave function joint non-classical properties have been studied. The results show that the Fisher information and Fisher length are determined by the parameter r of a determined Gaussian pure state. Joint non-classical property of states is an advantageous tool to study the non-classical properties of quantum states quantitatively. Especially, joint non-classical properties can be used for the mixed states that have non-classical properties and classical properties simultaneously. Fisher information defined can be used to obtain the correlation between Fisher length and quantum squeezing properties of Gaussian pure states. Moreover, we can quantitatively describe the fluctuation of quantum states in an alternative new approach.

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