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Modified atomic orbital calculations for $(1s,nl)^3L^\pi$ and ${}_2(1,0)_n{}^{\pm 1,3}S^e$ excited states of He isoelectronic sequence

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Abstract. The singly $(1s,ns)^3S^e$, $(1s,np)^3P^o$, $(1s,nd)^3D^e$ excited states and the doubly ${}_2(1,0)_n^{+1}S^e$ and ${}_2(1,0)_n^{-3}S^e$ $(n \leq 10)$ autoionizing states of the helium isoelectronic sequence are investigated using Modified Atomic Orbital Theory (MAOT). Total energies up to $Z=10$ and excitation energies with $Z=2\sim 5$ are presented and comparison with experimental and theoretical available results indicates a good agreement. In addition, the method is applied in the calculation of accurate results in very high Z - He isoelectronic sequence with $11 \leq Z \leq 58$ and with $Z=60$, 70, 80, 90, 91 and 92 for the $(1s,ns)^{1,3}S^e$, $(1s,np)^{1,3}P^o$, $(1s,nd)^{1,3}D^e$, ${}_2(1,0)_n^{+1}S^e$ and ${}_2(1,0)_n^{-3}S^e$ $(n \leq 7)$ excited states. The results obtained for these high Z - He isoelectronic sequence are in good agreement with double sums over the complete hydrogen spectrum calculations of Ivanov and Safronova.

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Key words: atomic orbital theory, screening constant, semi-empirical calculations, autoionizing, excited states, helium isoelectronic sequence

1 Introduction

Studies of excited states in two electron systems remain an active field of research both experimentally and theoretically since the earlier synchrotron radiation source experiment of Madden and Codling on He [1]. At present moment, the description of the properties of the excited states in the helium isoelectronic series is done in the framework of the new classification scheme with the label ${}_n(K,T)_N^{A\ 2S+1}L^\pi$. In this notation, N and n denote respectively the principal quantum numbers of the inner and of the outer electron, S the total spin, L the total angular momentum, π the parity of the system, K and T are angular correlation quantum numbers and A represents the radial correlation quantum

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number of the two electrons. Besides, this designation incorporates singly excited states and both intrashell and intershell excited states in the helium-like ions. A given channel μ is labelled by: $\mu = (K,T)^{A}_{N}\ ^{2S+1}L^{\pi}$, as originally used by Herrick and Sinanoglu [2,3] with the following assignment of K and T for a given N, L and π

$$T = 0,1,2,\cdots,\min(L,N-1),$$
$$K = N-1-T, N-3-T, \cdots, -(N-1-T).$$

These assigned values of K and T depend not on the total angular spin S. Besides, T is roughly speaking the projection of L onto the interelectronic axis and describes then the orientations between the orbitals of the two electrons and K is related to the cosinus of the interelectronic angle as: $K \approx -\langle r_{<} \cos\theta_{12} \rangle$, where $r_{<}$ denotes the radius of the inner electron. Physically, the larger the positive value of K, the value of $-\langle \cos\theta_{12} \rangle$ is closer to unity. In addition, A can take the values $+1$ or -1 and 0 only [4]. For the $A=+1$ states, the two electrons tend to approach or to move away from the nucleus in phase and for $A=-1$ states, the two electrons have out- of-phase such that, when one electron approaches the nucleus, the other tends to move away from it [5]. For $A=0$ states, they are little radial correlation between the two electrons and they are similar to singly excited states [4].

For the lowest-energy envelope $(N=1)$ containing the $(1s,nl)^{2S+1}L^{\pi}$ singly-excited states and for the ${}_2(1,0)^{+1}_{n}S^{e}$ and ${}_2(1,0)^{-3}_{n}S^{e}$ doubly excited states, various methods of calculation are used, such as the hyperspherical close-coupling method [6], the double sums over the total hydrogen spectrum of product of radials integrals technique [7], the analytical method [8], the spin-dependent localized Hartree-Fock density-functional theory [9], the truncated diagonalization technique [10], the saddle-point complex-rotation method [11], the close-coupling method [12], the discretization technique [13], the complex-coordinate rotation [14–16], the density functional theory [17], and so on.

Recently, we have presented a modification of the atomic orbital theory of Slater (MAOT) and applied it successfully to the calculation of energies for $(1sns)^1S^e$, $(1snp)^1P^o$, $(1snd)^1D^e$ and $(ns^2)^1S^e$, $(np^2)^1D^e$, $(nf^2)^1I^e$, $(nh^2)^1K^e$ excited states of He-like ions up to $Z=12$ [18]. In the present paper, we extend these calculations in the case of the singly $(1s,ns)^3S^e$, $(1s,np)^3P^o$, $(1s,nd)^3D^e$ and of the doubly ${}_2(1,0)^{+1}_{n}S^{e}$ and ${}_2(1,0)^{-3}_{n}S^{e}$ $(n \leq 10)$ excited states of the helium isoelectronic sequence. Calculations are performed for total energies up to $Z=10$ and excitation energies with $Z=2 \sim 5$ and compared to experimental and theoretical available results. In addition, the method referred as MAOT is applied in the calculation of accurate results in very high Z - He isoelectronic sequence with $11 \leq Z \leq 58$ and with $Z=60$, 70, 80, 90 and 92 for the $(1s,ns)^{1,3}S^e$, $(1s,np)^{1,3}P^o$, $(1s,nd)^{1,3}D^e$, ${}_2(1,0)^{+1}_{n}S^{e}$ $(n \leq 7)$ excited states. For these high Z - He -like ions, only the theoretical results from double sums over the complete hydrogen spectrum method of Ivanov and Safronova [7] with $n \leq 4$ are available. Comparison with respect to the forecast of these authors is then made.

2 Theory

2.1 Formalism of MAOT method for excited states of two electron systems

In the framework of the modified atomic orbital theory (MAOT), total energy of a (νl)-given orbital is expressed in the form [18]

$$E(\nu l) = -\frac{1}{\nu_i^2}[Z-\sigma_i(l)]^2. \tag{1}$$

For an atomic system of several electrons M, the total energy is given by (in rydbergs):

$$E = -\sum_{i=1}^{M}\frac{1}{\nu_i^2}[Z-\sigma_i(^{2S+1}L^{\pi})]^2. \tag{2}$$

With regard to the new classification scheme, Eq.(2) takes the form

$$E_N(KT)_n^A = -\sum_{i=1}^{M}\frac{1}{\nu_i^2}[Z-\sigma_i(KTNn,A)]^2.$$

For two electron systems which are the intention of the present work, we obtain

$$E_N(KT)_n^A = -\frac{1}{N^2}[Z-\sigma(KTNn,A)]^2 - \frac{1}{n^2}[Z-\sigma'(KTNn,A)]^2. \tag{3}$$

2.2 Energy for $(1s,nl)^3L^{\pi}$ singly excited states

For $(1s,nl)^3L^{\pi}$ singly excited states, total energy is expressed as follows (in rydbergs)

$$E(1s,nl^3L^{\pi}) = -[Z-\sigma_1(^3L^{\pi})]^2 - \frac{[Z-\sigma_2(^3L^{\pi})]^2}{n^2}. \tag{4}$$

We find for $l=0,1$ and 2, respectively

$$E(1s,ns^3S^e) = -[Z-\sigma_1(^3S^e)]^2 - \frac{[Z-\sigma_2(^3S^e)]^2}{n^2}. \tag{5a}$$

$$E(1s,np^3P^o) = -[Z-\sigma_1(^3P^o)]^2 - \frac{[Z-\sigma_2(^3P^o)]^2}{n^2}. \tag{5b}$$

$$E(1s,nd^3D^e) = -Z^2 - \frac{[Z-\sigma(^3D^e)]^2}{n^2}. \tag{5c}$$

For $(1s,nd)^3D^e$, the empirical value of $\sigma_1(^3D^e)$ is negative. This justifies the form of the total energy for these levels as shown by the last equation above.

Besides, for evaluating empirically the σ_i-screening constants in Eq. (5), we consider the following experimental total energies of the helium atom [19] (in atomic units)

$$E(1s2s)^3S^e = -2.17524,\ \ E(1s3s)^3S^e = -2.06869,$$
$$E(1s2p)^3P^o = -2.13317,\ E(1s3p)^3P^o = -2.05808,\ E(1s3d)^3D^e = -2.05566.$$

Using these experimental data, the σ_i-screening constants in Eq. (5) are determined. We obtain then

$$E(1s,ns^3S^e) = -[Z-0.0082921901]^2 - \frac{1}{n^2}[Z-0.7613232867]^2. \tag{6a}$$

$$E(1s,np^3P^o) = -[Z-0.000996248127]^2 - \frac{1}{n^2}[Z-0.9601461641]^2. \tag{6b}$$

$$E(1s,nd^3D^e) = -Z^2 - \frac{1}{n^2}[Z-0.9990604414]^2. \tag{6c}$$

2.3 Energy for $_2(1,0)_n{}^{\pm1,3}S^e$ doubly excited states

Total energy for $_2(1,0)_n{}^{\pm1,3}S^e$ autoionizing states of two-electron systems is in the shape.

• For $_2(1,0)_n{}^{+1}S^e$:

$$E[_2(1,0,+)_n^{+1}S^e] = -\frac{1}{4}\left[Z-\sigma_1(1,0,+)-\sigma_1(1,0,+)^2\frac{(n-3)(n-4)}{2}\right]^2 - \frac{1}{n^2}\left[Z-\sigma_2(1,0,+)+\sigma_2(1,0,+)^2\frac{(n-3)(n-4)}{2n+1}\right]^2. \tag{7a}$$

• For $_2(1,0)_n{}^{-3}S^e$:

$$E[_2(1,0)_n^{-3}D^e] = -\frac{1}{4}\left[Z-\sigma_3(1,0,-)+\sigma_3(1,0,-)\frac{(n-3)(n-4)}{2n+1}\right]^2 - \frac{1}{n^2}\left[Z-\sigma_4(1,0,-)+\sigma_4(1,0,-)^2\frac{(n-3)(n-4)}{n^2}\right]^2. \tag{7b}$$

The σ_i-screening constants in Eq. (7) are evaluated using the experimental energy results from electron ejected of Hicks and Comer [20] on He and the photoionisation experiment values of Diehl *et al.* [21] on Li^+. From Ref. [20], we find the excitation energies

$$E[_2(1,0)_3{}^{+1}S^e] = 62.96\ \text{eV},\ E[_2(1,0)_4{}^{+1}S^e] = 64.20\ \text{eV}.$$

Similarly, from Ref. [21], we get

$$E[_2(1,0)_3{}^{-3}S^e] = 158.907\ \text{eV},\ E[_2(1,0)_4{}^{-3}S^e] = 163.087\ \text{eV}.$$

For energy conversion, the experimental ground state energy of He equals to -79.01 eV, that of Li^+ equals to -198.09 eV and the infinite Rydberg (13.605698 eV) are used. We

obtain then form Eq. (7): $\sigma_1(1,0,+)=0.0288716522$, $\sigma_2(1,0,+)=0.6307502976$, $\sigma_3(1,0,-)=0.0486141874$, $\sigma_4(1,0,-)=0.4860307961$. Consideration of these semi-empirical screening constants yields

$$E[{}_2(1,0)_n^{+1}S^e]=-\frac{1}{4}\Big[Z-0.0288716522-0.0008335723\frac{(n-3)(n-4)}{2}\Big]^2 -\frac{1}{n^2}\Big[Z-0.6307502976+0.3978459379\frac{(n-3)(n-4)}{2n+1}\Big]^2, \quad (8a)$$

$$E[{}_2(1,0)_n^{-3}S^e]=-\frac{1}{4}\Big[Z-0.486141874+0.0486141874\frac{(n-3)(n-4)}{(n-2)^2}\Big]^2 -\frac{1}{n^2}\Big[Z-0.4860307961+0.2362259348\frac{(n-3)(n-4)}{n^2}\Big]^2. \quad (8b)$$

3 Results and discussion

The results obtained in the present study for total energies of the $(1snp)^3S^e$, $(1snp)^3P^o$ and $(1snd)^3D^e$ singly excited states of He isoelectronic series with $n\leq 4$ and $Z\leq 10$ are displayed in Tables 1-3 and compared to experimental data [19] and other theoretical results [6–8]. The agreement between the present results and the experimental data for the $(1s4s)^1S^e$ and $(1s4p)^1P^o$ states of He is seen to be satisfactory. For the $(1s4d)^3D^e$ level, the agreement is very good. As far as the results for the $(1snp)^3S^e$, $(1snp)^3P^o$ and $(1snd)^3D^e$ are concerned, the agreements between the calculations are generally good. It should be mentioned the very good agreement between our results and the double sums over the total hydrogen spectrum of product of radials integrals results of Ivanov and Safronova [7] and the analytical data of Arias de Saavedra *et al.* [8] for the $(1snp)^3S^e$, $(1snp)^3D^e$ $(2<n\leq 4)$ up to $Z=10$.

Tables 4 and 5 show the comparison of the present results for the ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of the helium atom with the data obtained from truncated-diagonalization method [10], the saddle-point complex-rotation method [11], the close-coupling method [12] and with the discretization technique [13]. In general, the agreements between the calculations are good except for the ${}_2(1,0)_4{}^{-3}S^e$ level where our result (0.5476) is lower than the accurate value (0.5488) obtained by the other authors cited in the table. However, the energy difference between these two data is at 0.0012 a.u., which is very satisfactory. The present results for excitation energies of the ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ levels of Li^+ are listed in Table 6 and compared to the complex-coordinates results of Chung and Lin [14] who include in their computational procedure relativistic corrections to the energy (kinetic energy and Darwin term) and the mass polarization (the spin-orbit, spin-spin and spin-other orbit interaction being neglected as their contribution are very weak for Li^+). Although the present MAOT is for non relativistic theory, the agreements between the calculations are seen to be good up to $n=10$.

We quote in Table 7 the present results for excitation energies of ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of He -like ions up to $Z=10$. Comparison is made with the complex-coordinate rotation values of Ho [15], the computing double sum over the complete hydrogen spectrum results of Ivanov and Safronova [7], the density functional calculations of Roy *et al.* [17] and with the discretization technique data of Macias and Riera [13]. In general, our results agree well with the quoted literature values. However, it should be mentioned that the very good agreement between the present results and those obtained from the complex rotation [15] for ${}_2(1,0)_3{}^{+1}S^e$ level of $Z=2\sim 10$ and that calculated using a discretization technique [13] for ${}_2(1,0)_3{}^{+1}S^e$ state of $Z=3\sim 5$.

In Table 8, the present calculations for excitation energies of ${}_2(1,0)_3{}^{+1}S^e$ and ${}_2(1,0)_3{}^{-3}S^e$ autoionizing states of the helium atom are compared to various *ab initio* calculations such as the complex- coordinates method used by Burgers *et al.* [22] and by Ho [15], the hyperspherical coordinate approaches employed by Koyama *et al.* [23], the truncated-diagonalization method utilized by Conneely and Lipsky [10], the time-dependent variation perturbation theory applied by Ray and Mukherjee [24] and by Das *et al.* [27], the discretization technique used by Macias and Riera [13], the multiconfiguration calculation employed by Lipsky *et al.* [26] and with the many-body perturbation theory to all orders of electron-electron interaction applied by Lindroth [25]. The agreement for the ${}_2(1,0)_3{}^{+1}S^e$ energy level of He with the result of Burgers *et al.* [29] is seen to be excellent. In addition, the present calculations for ${}_2(1,0)_3{}^{+1}S^e$ state of $Z=3\sim 5$ agree also very well with the results of Ho [15]. For the ${}_2(1,0)_3{}^{+1}S^e$ level of He and for the ${}_2(1,0)_3{}^{-3}S^e$ level of Li^+, it is seen that our results are very close to that of Conneely and Lipsky [10]. The agreements between the present calculations for ${}_2(1,0)_3{}^{-3}S^e$ level of $Z=3\sim 5$ with that of Macias and Riera [13] is also seen to be very good as the energy differences are respectively at 0.0003, 0.0013 and 0.0025 respectively for $Z=3$, 4 and 5.

In the intension to examine the upper limit to which the present MAOT formalism can provide accurate results that may be useful references for future experimental and theoretical investigations, we have listed in Tables 9-16 energy positions in very high Z-He isoelectronic sequence with $11\leq Z\leq 58$ and with $Z=60$, 70, 80, 90, 91 and 92 for the $(1s,ns)^{1,3}S^e$, $(1s,np)^{1,3}P^o$, $(1s,nd)^{1,3}D^e$, ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ $(n\leq 7)$ excited states. For very high Z-helium- like ions $(11\leq Z\leq 92)$, no theoretical results are found in the literature for direct comparison. Nevertheless, Ivanov and Safronova [7] present a Z expansion for the diagonal matrix elements of the energy (in a.u.)

$$E(nl,n'l';LS)=-Z^2[1/2n^2+1/2n'^2]+ZE_1+E_2,$$

and compute the values of the E_1 and E_2 energy coefficients for each state. Then, no numerical energy positions are quoted in the paper of Ivanov and Safronova [7]. But, considering the values of E_1 and E_2 for a given state, one can calculate the corresponding energy positions using the above equation. Comparison of the results quoted in both Tables 9-16 indicate a good agreement up to $Z=90$. Although the present modified atomic orbital theory is a non relativistic technique of calculation, the agreement between the presently results and those of Ivanov and Safronova [7] indicates the possibilities to

use the MAOT formalism to study very high Z-helium - like ions properties without having to invoke fastidious computational techniques.

Table 1: Comparison of total energy of $(1sns)^3S^e$, $n\leq 4$ of He-like systems up to $Z=10$ with some experimental and theoretical values. All energies are reported in a.u.

States		Z								
		2	3	4	5	6	7	8	9	10
$1s2s$	$-E^{pw}$	2.175 24	5.101 62	9.277 99	14.704 37	21.380 75	29.307 12	38.483 50	48.909 88	60.586 26
	$-E^{a}$	2.171 55	5.108 63	9.295 70	14.732 77	21.419 84	29.356 91	38.543 98	48.981 05	60.668 13
	$-E^{b}$	2.175 23				21.420 75				60.668 65
	$-E^{c}$	2.175 28								
	$-E^{exp}$	2.175 24								
$1s3s$	$-E^{pw}$	2.068 69	4.753 58	8.549 59	13.456 71	19.474 93	26.604 27	34.844 72	44.196 28	54.658 95
	$-E^{a}$	2.067 11	4.751 17	8.546 34	13.452 62	19.470 01	26.598 51	34.838 12	44.188 85	54.650 68
	$-E^{b}$	2.068 69				19.470 40				54.650 91
$1s4s$	$-E^{pw}$	2.031 40	4.631 77	8.294 65	13.020 02	18.807 90	25.658 27	33.571 15	42.546 52	52.584 40
	$-E^{a}$	2.035 74	4.636 70	8.300 15	13.026 11	18.814 57	25.665 52	33.578 98	42.554 94	52.593 39
	$-E^{c}$	2.036 51								
	$-E^{exp}$	2.036 54								

^{pw} Present work.
^{a} Ivanov and Safronova [7].
^{b} Arias de Saavedra *et al.* [8].
^{c} Tang *et al.* [6].
^{exp} Experiment [19].

Table 2: Comparison of total energy of $(1snp)^3P^o$, $n\leq 4$ of He-like systems up to $Z=10$ with some experimental and theoretical values. All energies are reported in a.u.

States		Z								
		2	3	4	5	6	7	8	9	10
$1s2p$	$-E^{pw}$	2.133 17	5.017 14	9.151 10	14.535 07	21.169 04	29.053 01	38.186 97	48.570 94	60.204 91
	$-E^{a}$	2.121 55	5.020 82	9.170 09	14.569 36	21.218 63	29.117 91	38.267 18	48.666 45	60.315 72
	$-E^{b}$	2.133 19								
	$-E^{exp}$	2.133 17								
$1s3p$	$-E^{pw}$	2.058 08	4.728 18	8.509 39	13.401 71	19.405 14	26.519 68	34.745 34	44.082 10	54.529 98
	$-E^{a}$	2.055 51	4.728 99	8.513 59	13.409 29	19.405 14	26.534 04	34.763 08	44.103 23	54.554 49
	$-E^{b}$	2.058 13								
	$-E^{exp}$	2.058 08								
$1s4p$	$-E^{pw}$	2.031 80	4.627 04	8.284 79	13.005 03	18.787 78	25.633 02	33.540 77	42.511 01	52.543 76
	$-E^{a}$	2.031 34	4.627 91	8.286 99	13.008 57	18.792 65	25.639 23	33.548 30	42.519 88	52.533 96
	$-E^{c}$	2.032 39								
	$-E^{exp}$	2.032 35								

^{pw} Present work.
^{a} Ivanov and Safronova [7].
^{b} Tang *et al.* [6].
^{exp} Experiment [19].

Table 3: Comparison of total energy of $(1snd)^3D^e$, $n \leq 5$ of He-like systems up to $Z=10$ with some experimental and theoretical values. All energies are reported in a.u.

States		Z								
		2	3	4	5	6	7	8	9	10
$1s3d$	$-E^{pw}$	2.055 66	4.722 43	8.500 31	13.389 31	19.389 41	26.500 63	34.722 95	44.056 39	54.500 94
	$-E^{a}$	2.055 30	4.722 31	8.500 42	13.389 64	19.389 98	26.501 42	34.723 98	44.057 65	54.502 43
	$-E^{b}$	2.055 69								
	$-E^{exp}$	2.055 66								
$1s4d$	$-E^{pw}$	2.031 31	4.625 12	8.281 43	13.000 23	18.781 54	25.625 35	33.531 66	42.500 47	52.531 78
	$-E^{a}$	2.031 08	4.625 02	8.281 45	13.000 38	18.781 81	25.625 74	33.532 17	42.501 11	52.532 54
	$-E^{b}$	2.031 33								
	$-E^{exp}$	2.031 32								
$1s5d$	$-E^{pw}$	2.020 04	4.580 08	8.180 11	12.820 15	18.500 19	25.220 23	32.980 26	41.780 30	51.620 34

^{pw} Present work.
^{a} Ivanov and Safronova [7].
^{b} Tang *et al.* [6].
^{exp} Experiment [19].

Table 4: Resonance energies for ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of the helium isoelectronic sequence up to $Z=10$. The results are expressed in atomic units: 1 a.u = 27.211396 eV

	Z								
	2	3	4	5	6	7	8	9	10
${}_2(1,0)_3{}^{+}$	0.589 83	1.415 30	2.601 89	4.149 59	6.058 40	8.328 32	10.959 35	13.951 50	17.304 75
${}_2(1,0)_4{}^{+}$	0.544 26	1.278 87	2.325 98	3.685 59	5.357 70	7.342 31	9.639 42	12.249 03	15.171 14
${}_2(1,0)_5{}^{+}$	0.526 82	1.222 06	2.207 30	3.482 53	5.047 77	6.903 01	9.048 24	11.483 48	14.208 72
${}_2(1,0)_6{}^{+}$	0.517 93	1.192 11	2.144 07	3.373 81	4.881 32	6.666 61	8.729 68	11.070 53	13.689 16
${}_2(1,0)_7{}^{+}$	0.512 27	1.173 44	2.105 02	3.307 02	4.779 41	6.522 22	8.535 44	10.819 06	13.373 10
${}_2(1,0)_8{}^{+}$	0.507 94	1.160 16	2.078 00	3.261 47	4.710 57	6.425 28	8.405 63	10.651 60	13.163 19
${}_2(1,0)_9{}^{+}$	0.504 15	1.149 64	2.057 48	3.227 66	4.660 18	6.355 05	8.312 27	10.531 83	13.013 74
${}_2(1,0)_{10}{}^{+}$	0.500 52	1.140 57	2.040 63	3.200 68	4.620 74	6.300 79	8.240 85	10.440 90	12.900 96
	Z								
	2	3	4	5	6	7	8	9	10
${}_2(1,0)_3{}^{-}$	0.603 33	1.439 94	2.637 68	4.196 52	6.166 47	8.397 53	11.039 71	14.043 00	17.407 39
${}_2(1,0)_4{}^{-}$	0.547 61	1.286 33	2.337 55	3.701 27	5.377 49	7.366 21	9.667 42	12.281 14	15.207 36
${}_2(1,0)_5{}^{-}$	0.528 27	1.225 13	2.211 99	3.488 85	5.055 71	6.912 57	9.059 43	11.496 29	14.233 15
${}_2(1,0)_6{}^{-}$	0.518 43	1.192 87	2.145 09	3.375 09	4.882 86	6.668 41	8.731 74	11.072 85	13.691 74
${}_2(1,0)_7{}^{-}$	0.512 65	1.173 61	2.104 98	3.306 76	4.778 94	6.521 54	8.534 54	10.817 95	13.371 77
${}_2(1,0)_8{}^{-}$	0.508 95	1.161 17	2.079 01	3.262 48	4.711 58	6.426 30	8.406 64	10.652 61	13.164 21
${}_2(1,0)_9{}^{-}$	0.506 45	1.152 68	2.061 26	3.232 18	4.665 45	6.361 06	8.319 02	10.539 33	13.021 98
${}_2(1,0)_{10}{}^{-}$	0.504 69	1.146 64	2.048 60	3.210 55	4.632 51	6.314 46	8.256 41	10.458 37	12.920 32

Table 5: Comparison of the present modified atomic orbital theory (MAOT) with other theoretical energy resonances of ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of the helium atom: SPCR, saddle-point complex-rotation method [11]; CC, Close-coupling method [12]; TD, truncated- diagonalization method [10]; DT, discretization technique [13]. All results are expressed in atomic units: 1 a.u.=27.211396 eV.

	$-E$				
	Present MAOT	SPCR	CC	TD	DT
${}_2(1,0)_3{}^+$	0.589 83	0.589 83	0.589 86	0.588 14	0.589 92
${}_2(1,0)_4{}^+$	0.544 26	0.544 88	0.544 87	0.544 02	0.544 88
${}_2(1,0)_5{}^+$	0.526 82	0.526 68	0.526 67	0.526 24	0.526 67
${}_2(1,0)_6{}^+$	0.517 93	0.517 64	0.517 63	0.517 38	0.517 26
${}_2(1,0)_7{}^+$	0.512 27	0.512 45	0.512 45		
${}_2(1,0)_3{}^-$	0.603 33	0.602 58	0.602 58	0.602 10	0.602 59
${}_2(1,0)_4{}^-$	0.547 61	0.548 84	0.548 84	0.548 54	0.548 84
${}_2(1,0)_5{}^-$	0.528 27	0.528 41	0.528 41	0.528 24	0.528 34
${}_2(1,0)_6{}^-$	0.518 43	0.518 55			0.518 34
${}_2(1,0)_7{}^-$	0.512 65	0.513 04			

Table 6: Comparison of the present modified atomic orbital theory (MAOT) for excitation energies of ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of Li^+ with the complex -coordinate rotational (CCR) results of Chung and Lin [14]. Here and in Ref [14], the energy of each state is measured from the ground state of Li^+ equals to -7.280521 a.u with 1 a.u. (^{7}Li)=27.20927 eV, which includes the reduced mass correction. All results are expressed in eV.

		E(eV)	
State	K, T, A	Present(MAOT)	Chung and Lin(CCR)
$2s3s^1S^e$	1, 0, +	159.577 6	159.579 6
$2s4s^1S^e$	1, 0, +	163.290 2	163.305 0
$2s5s^1S^e$	1, 0, +	164.836 1	164.914 2
$2s6s^1S^e$	1, 0, +	165.651 0	165.748 0
$2s7s^1S^e$	1, 0, +	166.159 0	166.234 2
$2s8s^1S^e$	1, 0, +	166.520 4	166.542 4
$2s9s^1S^e$	1, 0, +	166.806 6	166.750 8
$2s3s^1S^e$	1, 0, -	158.907 1	158.906 7
$2s4s^1S^e$	1, 0, -	163.087 1	163.087 0
$2s5s^1S^e$	1, 0, -	164.752 6	164.816 2
$2s6s^1S^e$	1, 0, -	165.630 2	165.695 1
$2s7s^1S^e$	1, 0, -	166.154 4	166.201 9
$2s8s^1S^e$	1, 0, -	166.492 9	166.520 4
$2s9s^1S^e$	1, 0, -	166.723 9	166.733 5
$2s10s^1S^e$	1, 0, -	166.888 2	166.833 1

Table 7: Comparison of the present modified atomic orbital calculations for total energies $(-E)$ of ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ autoionizing states of the helium isoelectronic series up to $Z=10$ with other theoretical values: CCR, complex-coordinate rotation method Ho [15]; CDS, computing double sum method [7]; DFT, density functional theory [17]; DT, discretization technique [13]. All results are expressed in eV: 1 a.u=2 Ryd= 27.211396 eV.

		Z								
		2	3	4	5	6	7	8	9	10
${}_2(1,0)_3{}^+$	$E^{Present}$	0.589 8	1.415 3	2.601 9	4.149 6	6.058 4	8.328 3	10.959 3	13.951 5	17.304 7
	E^{CCR}	0.589 9	1.415 6	2.602 0	4.149 5	6.058 0	8.327 5	10.958 2	13.949 9	17.302 7
	E^{CDS}	0.573 6	1.384 8	2.577 1	4.090 5	5.985 0	8.240 6	10.857 0	13.835 0	17.174 0
	E^{DFT}	0.576 9	1.391 2	2.567 2	1.104 4					
${}_2(1,0)_4{}^+$	$E^{Present}$	0.544 3	1.278 9	2.326 0	3.685 6	5.357 7	7.342 3	9.639 4	12.249 0	15.171 1
	E^{CDS}	0.535 3	1.262 8	2.302 8	3.655 3	5.320 3	7.297 8	9.587 8	12.190 3	15.105 4
		Z								
		2	3	4	5	6	7	8	9	10
${}_2(1,0)_3{}^-$	$E^{Present}$	0.603 3	1.439 9	2.637 7	4.196 5	6.116 5	8.397 5	11.039 7	14.043 0	17.407 4
	E^{DT}	0.602 6	1.440 2	2.639 0	4.198 9					
	E^{CDS}	0.585 6	1.411 6	2.599 0	4.147 4	6.056 84	8.327 42	10.959 12	13.951 92	17.305 84
	E^{DFT}	0.593 5	1.423 7	2.615 1	4.167 5					
${}_2(1,0)_4{}^-$	$E^{Present}$	0.547 6	1.286 3	2.337 6	3.701 3	5.377 5	7.366 2	9.667 4	12.281 1	15.207 4
	E^{CDS}	0.541 2	1.273 5	2.318 3	3.675 6	9.622 4	12.299 7	15.149 5		

Table 8: Comparison of the present modified atomic orbital calculations for excitation energies (E) of ${}_2(1,0)_3{}^{+1}S^e$ and ${}_2(1,0)_3{}^{-3}S^e$ autoionizing states of the helium atom with other theoretical results. Our excitation energies and those of Roy *et al.* are obtained with respect to the accurate ground-state energies of Frankowski and Pekeris [16]. The other results are quoted in Roy *et al.* [17]. Ground-state energies of Frankowski and Pekeris [16] are (in au): -2.90372(HeI),-7.27991(LiII),-13.65556(BeIII) and -22.03097(BIV). Energy is expressed in atomic units.

		E			
		2	3	4	5
${}_2(1,0)_3{}^{+1}S^e$	Present	2.313 9	5.864 6	11.053 7	17.881 4
	Roy *et al.*	2.326 7	5.888 7	11.088 4	17.926 6
	Others	2.313 8[a]	5.864 3[b]	11.053 5[b]	17.881 5[b]
		2.319 4[c]	5.868 2[d]	11.072 1[e]	17.913 1[e]
${}_2(1,0)_3{}^{-3}S^e$	Present	2.300 4	5.840 0	11.017 9	17.834 5
	Roy *et al.*	2.310 2	5.856 2	11.040 4	17.863 4
	Others	2.301 1[f]	5.839 7[f]	11.016 6[f]	17.832 0[f]
		2.301 1[g]	5.840 6[d]	11.019 3[h]	17.837 9[h]
		2.318 8[i]	5.869 5[i]	11.059 3[i]	17.885 5[i]

[a] Burgers *et al.* (1995) [22]; [b] Ho (1981) [15];
[c] Koyama *et al.* (1986) [23]; [d] Conneely and Lipsky (1978) [10];
[e] Ray and Mukherjee (1991) [24]; [f] Macias and Riera (1986) [13];
[g] Lindroth (1994) [25]; [h] Lipsky *et al.* (1977) [26];
[i] Das *et al.* (1994) [27].

Table 9: Energy resonances of $1sns\ ^1S^e$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u.=2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
1s2s	$-E^p$	73.226 62	87.374 47	102.772 32	119.420 17	137.318 02	156.465 87	176.863 72	198.511 57	221.409 42
	$-E^a$	73.189 45	87.332 62	102.725 80	119.368 97	137.262 15	156.405 32	176.798 50	198.441 67	221.334 85
1s3s	$-E^p$	66.136 43	78.811 45	92.597 58	107.494 82	123.503 17	140.622 63	158.853 21	178.194 89	198.647 69
	$-E^a$	66.113 54	78.786 07	92.569 70	107.464 44	123.470 30	140.587 27	158.815 34	178.154 53	198.604 83
1s4s	$-E^p$	63.654 87	75.814 39	89.036 42	103.320 95	118.667 97	135.077 50	152.549 53	171.084 05	190.681 08
	$-E^a$	63.649 75	75.808 53	89.029 81	103.313 58	118.659 86	135.068 64	152.539 92	171.073 69	190.669 97
1s5s	$-E^p$	62.506 26	74.427 18	87.388 11	101.389 04	116.429 97	132.510 90	149.631 82	167.792 75	186.993 68
		Z								
		20	21	22	23	24	25	26	27	28
1s2s	$-E^p$	245.557 27	270.955 12	297.602 97	325.500 82	354.648 67	385.046 52	416.694 37	449.592 23	483.740 08
	$-E^a$	245.478 03	270.871 20	297.514 38	325.407 55	354.550 73	384.943 90	416.587 08	449.480 25	483.623 43
1s3s	$-E^p$	220.211 60	242.886 61	266.672 74	291.569 98	317.578 33	344.697 80	372.928 37	402.270 06	432.722 85
	$-E^a$	63.649 75	75.808 53	89.029 81	103.313 58	118.659 86	135.068 64	152.539 92	171.073 69	190.669 97
1s4s	$-E^p$	211.340 61	233.062 63	255.847 16	279.694 19	304.603 72	330.575 74	357.610 27	385.707 30	414.866 82
	$-E^a$	211.328 75	233.062 63	255.833 80	279.680 08	304.588 86	330.560 13	357.593 91	385.690 19	414.848 96
1s5s	$-E^p$	207.234 61	228.515 54	250.836 46	274.197 39	298.598 32	324.039 25	350.520 18	378.041 10	406.602 03
		Z								
		29	30	31	32	33	34	35	36	237
1s2s	$-E^p$	519.137 93	555.785 78	593.683 63	632.831 48	673.229 33	714.877 18	757.775 03	801.922 88	847.320 73
	$-E^a$	519.016 61	555.659 78	593.552 96	632.696 13	673.089 31	714.732 48	757.625 66	801.768 84	847.162 01
1s3s	$-E^p$	464.286 76	496.961 78	530.747 91	565.645 15	601.653 50	638.772 96	677.003 53	716.345 22	756.798 01
	$-E^a$	464.218 95	496.891 47	530.675 11	565.569 85	601.575 71	638.692 67	676.920 75	716.259 94	756.710 24
1s4s	$-E^p$	445.088 85	476.373 38	508.720 40	542.129 93	576.601 96	612.136 48	648.733 51	686.393 04	725.115 06
	$-E^a$	445.070 24	476.354 02	508.700 29	542.109 07	576.580 35	612.114 13	648.710 40	686.369 18	725.090 46
1s5s	$-E^p$	436.202 96	466.843 89	498.524 82	531.245 74	565.006 67	599.807 60	635.648 53	672.529 46	710.450 38
		Z								
		38	39	40	41	42	43	44	45	46
1s2s	$-E^p$	893.968 58	941.866 43	991.014 28	1041.412 1	1093.059 9	1145.957 8	1200.105 6	1255.503 5	1312.151 3
	$-E^a$	893.805 19	941.698 36	990.841 54	1041.234 7	1092.877 8	1145.771 0	1199.914 2	1255.307 4	1311.950 5
1s3s	$-E^p$	798.361 92	841.036 94	884.823 07	929.720 31	975.728 66	1022.848 1	1071.078 7	1120.420 3	1170.873 1
	$-E^a$	798.271 65	840.944 18	884.727 81	929.622 56	975.628 41	1022.745 3	1070.973 4	1120.312 6	1170.762 9
1s4s	$-E^p$	764.899 59	805.746 62	847.656 14	890.628 17	934.662 70	979.759 72	1025.919 2	1073.141 2	1121.425 8
	$-E^a$	764.874 23	805.720 51	847.629 29	890.600 57	934.634 34	979.730 62	1025.889 4	1073.110 6	1121.394 4
1s5s	$-E^p$	749.411 31	789.412 24	830.453 17	872.534 10	915.655 02	959.815 95	1005.016 8	1051.257 8	1098.538 7
		Z								
		47	48	49	50	51	52	53	54	55
1s2s	$-E^p$	1370.149 2	1429.197 0	1489.594 9	1551.242 7	1614.140 6	1678.288 4	1743.686 3	1810.334 1	1878.232 0
	$-E^a$	1369.843 7	1428.986 9	1489.380 1	1551.023 2	1613.916 4	1678.059 6	1743.452 8	1810.096 0	1877.989 1
1s3s	$-E^p$	1222.437 0	1275.112 1	1328.898 2	1383.795 4	1439.803 8	1496.923 2	1555.153 8	1614.495 5	1674 948 3
	$-E^a$	1222.324 3	1274.996 8	1328.780 5	1383.675 2	1439.681 1	1496.798 0	1555.026 1	1614.365 3	1674.815 6
1s4s	$-E^p$	1170.772 8	1221.182 3	1272.654 3	1325.188 9	1378.785 9	1433.445 4	1489.167 4	1545.952 0	1603.799 0
	$-E^a$	1170.740 7	1221.149 5	1272.620 7	1325.154 5	1378.750 8	1433.409 6	1489.130 8	1545.914 6	1603.760 9
1s5s	$-E^p$	1146.859 6	1196.220 5	1246.621 5	1298.062 4	1350.543 3	1404.064 3	1458.625 2	1514.226 1	1570.867 0
		Z								
		56	57	58	60	70	80	90	91	92
1s2s	$-E^p$	1947.379 8	2017.777 7	2089.425 5	2162.323 4	2236.471 2	3046.699 7	3981.928 2	5042.156 7	5269.202 4
	$-E^a$	1947.132 3	2017.525 5	2089.168 7	2162.061 8	2236.205 0	3046.386 8	3981.568 5	5041.750 3	5268.786 6
1s3s	$-E^p$	1736.512 2	1799.187 2	1862.973 3	1927.870 6	1993.878 9	2715.073 6	3547.379 3	4490.796 1	4692.812 9
	$-E^a$	1736.377 0	1799.049 5	1862.833 2	1927.727 9	1993.733 8	2714.903 4	3547.184 2	4490.576 1	4692.587 8
1s4s	$-E^p$	1662.708 5	1722.680 6	1783.715 1	1845.812 1	1908.971 6	2599.004 4	3395.287 2	4297.819 9	4491.076 5
	$-E^a$	1662.669 7	1722.641 0	1783.674 7	1845.771 0	1908.929 8	2598.955 1	3395.230 3	4297.755 6	4491.010 7
1s5s	$-E^p$	1628.548 0	1687.268 9	1747.029 8	1807.830 8	1869.671 7	2545.281 0	3324.890 2	4208.499 5	4397.701 4

[p] present work.
[a] Ivanov and Safronova (1993) [7].

Table 10: Energy resonances of $1sns\ ^3S^e$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u.=2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
1s2s	$-E^p$	73.512 63	87.689 01	103.115 39	119.791 76	137.718 14	156.894 52	177.320 89	198.997 27	221.923 65
	$-E^a$	73.605 20	87.792 27	103.229 34	119.916 41	137.853 48	157.040 55	177.477 62	199.164 70	222.101 77
1s3s	$-E^p$	66.232 74	78.917 63	92.713 64	107.620 75	123.638 98	140.768 32	159.008 77	178.360 33	198.823 00
	$-E^a$	66.223 64	78.907 70	92.702 86	107.609 14	123.626 54	140.755 04	158.994 65	178.345 38	198.807 21
1s4s	$-E^p$	63.684 77	75.847 65	89.073 02	103.360 90	118.711 27	135.124 15	152.599 52	171.137 40	190.737 77
	$-E^a$	63.694 35	75.857 81	89.083 76	103.372 22	118.723 17	135.136 63	152.612 59	171.151 04	190.752 00
1s5s	$-E^p$	62.505 43	74.426 69	87.387 94	101.389 19	116.430 45	132.511 70	149.632 96	167.794 21	186.995 47
		Z								
		20	21	22	23	24	25	26	27	28
1s2s	$-E^p$	246.100 03	271.526 40	298.202 78	326.129 16	355.305 53	385.731 91	417.408 29	450.334 66	484.511 04
	$-E^a$	246.288 84	271.725 91	298.412 98	326.350 05	355.537 12	385.974 20	417.661 27	450.598 34	484.785 41
1s3s	$-E^p$	220.396 78	243.081 68	266.877 68	291.784 80	317.803 03	344.932 37	373.172 82	402.524 38	432.987 05
	$-E^a$	220.380 16	243.064 22	266.859 39	291.765 67	317.783 06	344.911 56	373.151 18	402.501 90	432.963 74
1s4s	$-E^p$	211.400 65	233.126 02	255.913 90	279.764 27	304.677 15	330.652 52	357.690 40	385.790 78	414.953 65
	$-E^a$	211.328 75	233.062 63	255.833 80	279.680 08	304.588 86	330.560 13	357.593 91	385.690 19	414.848 96
1s5s	$-E^p$	207.236 72	228.517 98	250.839 23	274.200 49	298.601 74	324.043 00	350.524 25	378.045 51	406.606 76
		Z								
		29	30	31	32	33	34	35	36	37
1s2s	$-E^p$	519.937 42	556.613 80	594.540 17	633.716 55	674.142 93	715.819 30	758.745 68	802.922 06	848.348 43
	$-E^a$	520.222 48	556.909 55	594.540 17	634.033 69	674.470 77	716.157 84	759.094 91	803.281 98	848.719 05
1s3s	$-E^p$	464.560 83	497.245 73	531.041 73	565.948 85	601.967 07	639.096 41	677.336 86	716.688 42	757.151 10
	$-E^a$	464.536 69	497.220 74	531.015 91	565.922 19	601.939 59	639.068 09	677.307 70	716.658 43	757.120 26
1s4s	$-E^p$	445.179 03	476.466 90	508.817 28	542.230 15	576.705 53	612.243 40	648.843 78	686.506 65	725.232 03
	$-E^a$	445.199 06	476.487 52	508.838 48	542.251 93	576.727 89	612.266 35	648.867 30	686.530 76	725.256 71
1s5s	$-E^p$	436.208 02	466.849 27	498.530 53	531.251 78	565.013 04	599.814 29	635.655 55	672.536 80	710.458 06
		Z								
		38	39	40	41	42	43	44	45	46
1s2s	$-E^p$	895.024 81	942.951 19	992.127 57	1042.553 9	1094.230 3	1147.156 7	1201.333 0	1256.759 4	1313.435 8
	$-E^a$	895.406 12	943.343 19	992.530 27	1042.967 3	1094.654 4	1147.591 4	1201.778 5	1257.215 6	1313.902 6
1s3s	$-E^p$	798.724 88	841.409 77	885.205 78	930.112 89	976.131 12	1023.260 4	1071.500 9	1120.852 4	1171.315 1
	$-E^a$	798.693 21	841.377 27	885.172 44	930.078 72	976.096 11	4023.224 6	1071.464 2	1120.814 9	1171.276 7
1s4s	$-E^p$	765.019 90	805.870 28	847.783 15	890.758 53	934.796 40	979.896 78	1026.059 6	1073.285 0	1121.572 9
	$-E^a$	765.045 17	805.896 13	847.809 58	890.785 54	934.824 00	979.924 95	1026.088 4	1073.314 3	1121.602 8
1s5s	$-E^p$	749.419 31	789.420 57	830.461 82	872.543 08	915.664 33	959.825 59	1005.026 8	1051.268 1	1098.549 3
		Z								
		47	48	49	50	51	52	53	54	55
1s2s	$-E^p$	1371.362 2	1430.538 5	1490.964 9	1552.641 3	1615.567 7	1679.744 0	1745.170 4	1811.846 8	1879.773 2
	$-E^a$	1371.839 7	1431.026 8	1491.463 9	1553.150 9	1616.088 0	1680.275 1	1745.712 1	1812.399 2	1880.336 3
1s3s	$-E^p$	1222.888 9	1275.573 8	1329.369 8	1384.276 9	1440.295 1	1497.424 5	1555.664 9	1615.016 5	1675.479 1
	$-E^a$	1222.849 7	1275.533 7	1329.328 9	1384.235 2	1440.252 6	1497.381 1	1555.620 7	1614.971 4	1675.433 3
1s4s	$-E^p$	1170.923 2	1221.336 1	1272.811 5	1325.349 4	1378.949 7	1433.612 6	1489.338 0	1546.125 9	1603.976 2
	$-E^a$	1170.953 7	1221.367 2	1272.843 1	1325.381 6	1378.982 6	1433.646 0	1489.372 0	1546.160 4	1604.011 4
1s5s	$-E^p$	1146.870 6	1196.231 8	1246.633 1	1298.074 3	1350.555 6	1404.076 8	1458.638 1	1514.239 3	1570.880 6
		Z								
		56	57	58	60	70	80	90	91	92
1s2s	$-E^p$	1948.949 6	2019.375 9	2091.052 3	2163.978 7	2238.155 1	3048.668 8	3984.182 6	5044.696 4	5271.799 1
	$-E^a$	1949.523 4	2019.960 4	2091.647 5	2164.584 6	2238.771 6	3049.392 4	3985.013 1	5045.633 8	5272.757 9
1s3s	$-E^p$	1737.052 9	1799.737 8	1863.533 8	1928.440 9	1994.459 2	2715.752 6	3548.157 1	4491.672 7	4693.709 1
	$-E^a$	1737.006 2	1799.690 3	1863.485 4	1928.391 7	1994.409 1	2715.694 1	3548.090 3	4491.597 5	4693.632 3
1s4s	$-E^p$	1662.889 1	1722.864 5	1783.902 4	1846.002 7	1909.165 6	2599.231 9	3395.548 1	4298.114 4	4491.377 6
	$-E^a$	1662.924 8	1722.900 8	1783.939 3	1846.040 2	1909.203 7	2599.275 7	3395.597 8	4298.169 9	4491.434 3
1s5s	$-E^p$	1628.561 9	1687.283 1	1747.044 4	1807.845 6	1869.686 9	2545.299 4	3324.912 0	4208.524 5	4397.727 0

[p] present work.
[a] Ivanov and Safronova (1993) [7].

Table 11: Energy resonances of $1snp\ ^1P^o$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u.=2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
1s2p	$-E^p$	72.940 00	87.058 99	102.427 99	119.046 98	136.915 97	156.034 96	176.403 96	198.022 95	220.891 94
	$-E^a$	72.923 43	87.038 57	102.403 70	119.018 83	136.883 96	155.999 09	176.364 22	197.979 35	220.844 48
1s3s	$-E^p$	66.028 89	78.692 89	92.467 99	107.354 21	123.351 54	140.459 98	158.679 54	178.010 20	198.451 97
	$-E^a$	66.059 02	78.725 52	92.503 14	107.391 87	123.391 71	140.502 66	158.724 73	178.057 90	198.502 19
1s4s	$-E^p$	63.610 00	75.764 75	88.982 00	103.261 74	118.603 99	135.008 74	152.475 99	171.005 74	190.597 99
	$-E^a$	63.617 31	75.772 70	88.990 59	103.270 98	118.613 87	135.019 26	152.487 15	171.017 54	190.610 43
1s5s	$-E^p$	62.490 40	74.409 44	87.368 48	101.367 52	116.406 56	132.485 59	149.604 63	167.763 67	186.962 71
		Z								
		20	21	22	23	24	25	26	27	28
1s2s	$-E^p$	245.010 94	270.379 93	296.998 92	324.867 91	353.986 91	384.355 90	415.974 89	448.843 88	482.962 88
	$-E^a$	244.959 61	270.324 75	296.939 88	324.805 01	353.920 14	384.285 27	415.900 40	448.765 53	482.880 66
1s3s	$-E^p$	220.004 86	242.668 86	266.443 96	291.330 18	317.327 51	344.435 96	372.655 51	401.986 17	432.427 95
	$-E^a$	220.057 58	242.724 09	266.501 71	291.390 44	317.390 28	344.501 23	372.723 30	402.056 47	432.500 76
1s4s	$-E^p$	211.252 73	232.969 98	255.749 73	279.591 98	304.496 73	330.463 97	357.493 72	385.585 97	414.740 72
	$-E^a$	211.265 82	232.983 71	255.764 10	279.606 99	304.512 38	330.480 26	357.510 65	385.603 54	414.758 93
1s5s	$-E^p$	207.201 75	228.480 79	250.799 83	274.158 87	298.557 90	323.996 94	350.475 98	377.995 02	406.554 06
		Z								
		29	30	31	32	33	34	35	36	37
1s2s	$-E^p$	518.331 87	554.950 86	592.819 86	631.938 85	672.307 84	713.926 83	756.795 83	800.914 82	846.283 81
	$-E^a$	518.245 79	554.860 92	592.726 06	631.841 19	672.206 32	713.821 45	756.686 58	800.801 71	846.166 84
1s3s	$-E^p$	463.980 83	496.644 83	530.419 94	565.306 15	601.303 48	638.411 93	676.631 48	715.962 14	756.403 92
	$-E^a$	464.056 15	496.722 66	530.500 28	565.389 01	601.388 85	638.499 80	676.721 87	716.055 04	756.499 32
1s4s	$-E^p$	444.957 97	476.237 72	508.579 96	541.984 71	576.451 96	611.981 71	648.573 96	686.228 70	724.945 95
	$-E^a$	444.976 82	476.257 21	508.600 10	542.005 49	576.473 38	612.033 77	648.596 66	686.252 05	724.969 94
1s5s	$-E^p$	436.153 10	466.792 14	498.471 18	531.190 22	564.949 25	599.748 29	635.587 33	672.466 37	710.385 41
		Z								
		38	39	40	41	42	43	44	45	46
1s2s	$-E^p$	892.902 81	940.771 80	989.890 79	1040.259 7	1091.878 7	1144.747 7	1198.866 7	1254.235 7	1310.854 7
	$-E^a$	892.781 97	940.647 10	989.762 24	1040.127 3	1091.742 5	1144.607 6	1198.722 7	1254.087 8	1310.703 0
1s3s	$-E^p$	797.956 80	840.620 80	884.395 91	929.282 13	975.279 46	1022.387 9	1070.607 4	1119.938 1	1170.379 8
	$-E^a$	798.054 72	840.721 23	884.498 85	929.387 58	975.387 42	1022.498 3	1070.720 4	1120.053 6	1170.497 8
1s4s	$-E^p$	764.725 70	805.567 95	847.472 70	890.439 95	934.469 69	979.561 94	1025.716 6	1072.933 9	1121.213 6
	$-E^a$	764.750 33	805.593 22	847.498 61	890.466 50	934.496 89	979.589 78	1025.745 1	1072.963 0	1121.243 4
1s5s	$-E^p$	749.344 45	789.343 49	830.382 53	872.461 57	915.580 60	959.739 64	1004.938 6	1051.177 7	1098.456 7
		Z								
		47	48	49	50	51	52	53	54	55
1s2s	$-E^p$	1368.723 7	1427.842 7	1488.211 7	1549.830 7	1612.699 7	1676.818 7	1742.187 7	1808.806 6	1876.675 6
	$-E^a$	1368.568 1	1427.683 2	1488.048 4	1549.663 5	1612.528 6	1676.643 8	1742.008 9	1808.624 0	1876.489 2
1s3s	$-E^p$	1221.932 7	1274.596 7	1328.371 8	1383.258 1	1439.255 4	1496.363 8	1554.583 4	1613.914 0	1674.355 8
	$-E^a$	1222.053 2	1274.719 8	1328.497 4	1383.386 1	1439.385 9	1496.496 9	1554.719 0	1614.052 1	1674.496 4
1s4s	$-E^p$	1170.555 9	1220.960 6	1272.427 9	1324.957 6	1378.549 9	1433.204 6	1488.921 9	1545.701 6	1603.543 9
	$-E^a$	1170.586 3	1220.991 7	1272.459 6	1324.990 0	1378.582 8	1433.238 2	1488.956 1	1545.736 5	1603.579 4
1s5s	$-E^p$	1146.775 8	1196.134 8	1246.533 8	1297.972 9	1350.451 9	1403.970 9	1458.530 0	1514.129 0	1570.768 1
		Z								
		56	57	58	60	70	80	90	91	92
1s2s	$-E^p$	1948.949 6	2019.375 9	2091.052 3	2163.978 7	2238.155 1	3048.668 8	3984.182 6	5044.696 4	5271.799 1
	$-E^a$	1949.523 4	2019.960 4	2091.647 5	2164.584 6	2238.771 6	3049.392 4	3985.013 1	5045.633 8	5272.757 9
1s3s	$-E^p$	1735.908 7	1798.572 7	1862.347 8	1927.234 0	1993.231 4	2714.315 8	3546.511 3	4489.817 9	4691.812 6
	$-E^a$	1736.051 8	1798.718 3	1862.495 9	1927.384 7	1993.384 5	2714.494 0	3546.714 7	4490.046 4	4692.046 1
1s4s	$-E^p$	1662.448 6	1722.415 9	1783.445 6	1845.537 9	1908.692 6	2598.677 6	3394.912 6	4490.046 4	4490.644 6
	$-E^a$	1662.484 8	1722.452 7	1783.483 1	1845.576 0	1908.731 4	2598.722 7	3394.964 1	4297.455 5	4490.703 8
1s5s	$-E^p$	1628.447 1	1687.166 1	1746.925 2	1807.724 2	1869.563 3	2545.153 6	3324.744 0	4208.334 4	4397.532 5

[p] present work.
[a] Ivanov and Safronova (1993) [13].

Table 12: Energy resonances of $1snp\ ^3P^o$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u.=2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
$1s2p$	$-E^p$	73.088 87	87.222 84	102.606 81	119.240 78	137.124 74	156.258 71	176.642 68	198.276 65	221.160 61
	$-E^a$	74.191 79	88.433 98	103.926 17	120.668 36	138.660 54	157.902 73	178.394 92	200.137 11	223.129 29
$1s3s$	$-E^p$	66.088 97	78.759 07	92.540 28	107.432 60	123.436 03	140.550 57	158.776 23	178.112 99	198.560 87
	$-E^a$	66.116 86	78.790 35	92.574 94	107.470 65	123.477 46	140.595 39	158.824 43	178.164 58	198.615 84
$1s4s$	$-E^p$	63.639 00	75.796 74	89.016 99	103.299 73	118.644 98	135.052 72	152.522 97	171.055 71	190.650 96
	$-E^a$	63.650 54	75.809 62	89.031 19	103.315 27	118.661 85	135.070 93	152.542 51	171.076 58	190.673 16
$1s5s$	$-E^p$	62.505 02	74.425 61	87.386 21	101.386 81	116.427 41	132.508 00	149.628 60	167.789 20	186.989 80
		Z								
		20	21	22	23	24	25	26	27	28
$1s2s$	$-E^p$	245.294 58	270.678 55	297.312 51	325.196 48	354.330 45	384.714 42	416.348 38	449.232 35	483.366 32
	$-E^a$	247.371 48	272.863 67	299.605 86	327.598 04	356.840 23	387.332 42	419.074 61	452.066 79	486.308 98
$1s3s$	$-E^p$	220.119 86	242.789 95	266.571 16	291.463 48	317.466 92	344.581 46	372.807 11	402.143 88	432.591 76
	$-E^a$	220.178 22	242.851 70	266.636 30	291.532 00	317.538 82	344.656 75	372.885 79	402.225 94	432.677 20
$1s4s$	$-E^p$	211.308 70	233.028 95	255.811 69	279.656 94	304.564 68	330.534 92	357.567 67	385.662 91	414.820 66
	$-E^a$	211.332 24	233.053 82	255.837 89	279.684 47	304.593 55	330.565 13	357.599 21	385.695 78	414.854 86
$1s5s$	$-E^p$	207.230 40	228.510 99	250.831 59	274.192 19	298.592 79	324.033 39	350.513 98	378.034 58	406.595 18
		Z								
		29	30	31	32	33	34	35	36	37
$1s2s$	$-E^p$	218.750 28	555.384 25	593.268 22	632.402 19	672.786 15	714.420 12	757.304 09	801.438 06	846.822 02
	$-E^a$	521.801 17	558.543 36	596.535 54	635.777 73	676.269 92	718.012 11	761.004 29	805.246 48	850.738 67
$1s3s$	$-E^p$	464.150 74	496.821 84	530.602 05	565.494 37	601.497 80	638.612 35	676.838 00	716.174 77	756.622 64
	$-E^a$	464.239 57	496.913 06	530.697 65	565.593 36	601.600 18	638.718 10	676.947 14	716.287 29	756.738 56
$1s4s$	$-E^p$	445.040 90	476.323 65	508.668 89	542.076 64	576.546 88	612.079 63	648.674 87	686.332 62	725.052 86
	$-E^a$	445.076 44	476.360 52	508.707 10	542.116 17	576.587 75	612.121 83	648.718 41	686.377 49	725.099 06
$1s5s$	$-E^p$	436.195 78	466.836 38	498.516 97	531.237 57	564.998 17	599.798 77	635.639 36	672.519 96	710.440 56
		Z								
		38	39	40	41	42	43	44	45	46
$1s2s$	$-E^p$	893.455 99	941.339 96	990.473 92	1040.857 8	1092.491 8	1145.375 8	1199.509 7	1254.893 7	1311.527 7
	$-E^a$	897.480 86	945.473 04	994.715 23	1045.207 4	1096.949 6	1149.941 7	1204.183 9	1259.676 1	1316.418 3
$1s3s$	$-E^p$	798.181 63	840.851 73	884.632 94	929.525 26	975.528 69	1022.643 2	1070.868 8	1120.205 6	1170.653 5
	$-E^a$	798.300 93	840.974 41	884.759 01	929.654 71	975.661 53	1022.779 4	1071.008 5	1120.348 6	1170.799 9
$1s4s$	$-E^p$	764.835 60	805.680 85	847.588 59	890.558 84	934.591 58	979.686 83	1025.844 5	1073.064 8	1121.347 5
	$-E^a$	764.883 14	805.729 72	847.638 80	890.610 37	934.644 45	979.741 03	1025.900 1	1073.121 6	1121.405 7
$1s5s$	$-E^p$	749.401 16	789.401 76	830.442 35	872.522 95	915.643 55	959.804 15	1005.004 7	1051.245 3	1098.525 9
		Z								
		47	48	49	50	51	52	53	54	55
$1s2s$	$-E^p$	1369.411 6	1428.545 6	1488.929 6	1550.563 6	1613.447 5	1677.581 5	1742.965 5	1809.599 4	1877.483 4
	$-E^a$	1374.410 5	1433.652 7	1494.144 9	1555.887 1	1618.879 2	1683.121 4	1748.613 6	1815.355 8	1883.348 0
$1s3s$	$-E^p$	1222.212 5	1274.882 6	1328.663 8	1383.556 1	1439.559 5	1496.674 1	1554.899 7	1614.236 5	1674.684 4
	$-E^a$	1222.362 2	1275.035 7	1328.820 3	1383.716 0	1439.722 8	1496.840 8	1555.069 8	1614.410 0	1674.861 2
$1s4s$	$-E^p$	1170.692 8	1221.100 5	1272.570 8	1325.103 5	1378.698 7	1433.356 5	1489.076 7	1545.859 5	1603.704 7
	$-E^a$	1170.752 3	1221.161 4	1272.633 0	1325.167 0	1378.763 6	1433.422 7	1489.144 3	1545.928 3	1603.774 9
$1s5s$	$-E^p$	1146.846 5	1196.207 1	1246.607 7	1298.048 3	1350.528 9	1404.049 5	1458.610 1	1514.210 7	1570.851 3
		Z								
		56	57	58	60	70	80	90	91	92
$1s2s$	$-E^p$	1946.617 4	2017.001 3	2088.635 3	2161.519 3	2235.653 2	3045.742 9	3980.832 6	5040.922 2	5267.940 2
	$-E^a$	1952.590 2	2023.082 4	2094.824 6	2167.816 7	2242.058 9	3053.230 8	3989.402 7	5050.574 6	5277.808 9
$1s3s$	$-E^p$	1736.243 4	1798.913 5	1862.694 7	1927.587 0	1993.590 4	2714.735 9	3546.992 4	4490.360 0	4692.366 9
	$-E^a$	1736.423 6	1799.097 1	1862.881 7	1927.777 4	1993.784 2	2714.963 5	3547.253 9	4490.655 4	4692.669 0
$1s4s$	$-E^p$	1662.612 5	1722.582 7	1783.615 5	1845.710 7	1908.868 4	2598.883 4	3395.148 3	4297.663 3	4490.916 3
	$-E^a$	1662.684 0	1722.655 6	1783.689 7	1845.786 2	1908.945 3	2598.973 6	3395.251 9	4297.780 1	4491.035 8
$1s5s$	$-E^p$	1628.531 9	1687.252 5	1747.013 1	1807.813 7	1869.654 3	2545.260 2	3324.866 2	4208.472 2	4397.673 4

[p] present work.
[a] Ivanov and Safronova (1993) [13].

Table 13: Energy resonances of $1snd\ ^1D^e$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u.=2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
$1s3d$	$-E^p$	66.056 62	78.723 40	92.501 28	107.390 27	123.390 38	140.501 60	128.723 93	178.057 37	198.501 92
	$-E^a$	66.055 75	78.722 26	92.499 88	107.388 60	123.388 45	140.499 40	158.721 46	178.054 64	198.498 92
$1s4d$	$-E^p$	63.625 60	75.781 91	89.000 72	103.282 03	118.625 84	135.032 15	152.500 96	171.032 27	190.626 08
	$-E^a$	63.625 05	75.781 22	88.999 89	103.281 06	118.624 72	135.030 89	152.499 56	171.030 73	190.624 40
$1s5d$	$-E^p$	62.500 38	74.420 42	87.380 46	101.380 50	116.420 54	132.500 58	149.620 61	167.780 65	186.980 69
$1s6d$	$-E^p$	61.889 16	73.680 85	86.500 32	100.347 57	115.222 60	131.125 40	148.055 98	166.014 34	185.000 48
		Z								
		20	21	22	23	24	25	26	27	28
$1s3d$	$-E^p$	220.057 58	242.724 35	266.502 24	291.391 23	317.391 34	344.502 56	372.724 89	402.058 33	432.502 88
	$-E^a$	220.054 32	242.720 83	266.498 44	291.387 17	317.387 01	344.497 97	372.720 03	402.053 20	432.497 49
$1s4d$	$-E^p$	211.282 39	233.001 20	255.782 51	279.626 32	304.532 63	330.501 44	357.532 75	385.626 56	414.782 87
	$-E^a$	211.280 56	232.999 23	255.780 40	179.624 07	304.530 24	330.498 90	357.530 07	385.623 74	414.779 91
$1s5d$	$-E^p$	207.220 73	228.500 77	250.820 81	274.180 84	298.580 88	234.020 92	350.500 96	378.021 00	406.581 04
$1s6d$	$-E^p$	205.014 40	226.056 09	248.125 56	271.222 81	295.347 84	320.500 64	346.681 22	373.889 58	402.125 72
		Z								
		29	30	31	32	33	34	35	36	37
$1s3d$	$-E^p$	464.058 54	496.725 31	530.503 20	565.392 19	601.392 30	638.503 52	676.725 85	716.059 29	756.503 84
	$-E^a$	464.052 89	496.719 39	530.497 01	565.385 74	601.385 58	638.496 54	676.718 60	716.051 77	756.496 06
$1s4d$	$-E^p$	445.001 68	476.282 99	508.626 80	542.033 11	576.501 92	612.033 23	648.627 04	686.283 35	725.002 16
	$-E^a$	444.998 58	476.279 74	508.623 41	542.029 58	576.498 25	612.029 42	648.623 08	686.279 25	724.997 92
$1s5d$	$-E^p$	436.181 07	466.821 11	498.501 15	531.221 19	564.981 23	599.781 27	635.621 30	672.501 34	710.421 38
$1s6d$	$-E^p$	431.389 64	461.681 33	493.000 80	525.348 05	558.723 08	593.125 88	628.556 46	665.014 82	702.500 96
		Z								
		38	39	40	41	42	43	44	45	46
$1s3d$	$-E^p$	798.059 50	840.726 27	884.504 16	929.939 15	975.393 26	1022.504 4	1070.726 8	1120.060 2	1170.504 8
	$-E^a$	798.051 45	840.717 96	884.495 58	929.384 31	975.384 15	1022.495 1	1070.717 1	1120.050 3	1170.494 6
$1s4d$	$-E^p$	764.783 47	805.627 28	847.533 59	890.502 40	934.533 71	979.627 52	1025.783 8	1073.002 6	1121.283 9
	$-E^a$	764.779 09	805.622 76	847.528 92	890.497 59	934.528 76	979.622 43	1025.778 6	1072.997 2	1121.278 4
$1s5d$	$-E^p$	749.381 42	789.381 46	830.421 50	872.501 54	915.621 57	959.781 61	1004.981 6	1051.221 6	1098.501 7
$1s6d$	$-E^p$	741.014 88	780.556 57	821.126 04	862.723 29	905.348 32	949.001 12	993.681 70	1039.390 0	1086.126 2
		Z								
		47	48	49	50	51	52	53	54	55
$1s3d$	$-E^p$	1222.060 4	1274.727 2	1328.505 1	1383.394 1	1439.394 2	1496.505 4	1554.727 7	1614.061 2	1674.505 7
	$-E^a$	1222.050 0	1274.716 5	1328.494 1	1383.382 8	1439.382 7	1496.493 6	1554.715 7	1614.048 9	1674.493 2
$1s4d$	$-E^p$	1170.627 7	1221.034 0	1727.502 8	1325.034 1	1378.628 0	1433.284 3	1489.003 1	1545.784 4	1603.628 2
	$-E^a$	1170.622 1	1221.028 2	1272.496 9	1325.028 1	1378.621 7	1433.277 9	1488.996 6	1545.777 7	1603.621 4
$1s5d$	$-E^p$	1146.821 7	1196.181 8	1246.581 8	1298.021 8	1350.501 9	1404.021 9	1458.582 0	1514.182 0	1570.822 0
$1s6d$	$-E^p$	1133.890 1	1182.681 8	1232.501 2	1283.348 5	1335.223 5	1388.126 3	1442.056 9	1497.015 3	1553.001 4
		Z								
		56	57	58	60	70	80	90	91	92
$1s3d$	$-E^p$	1736.061 4	1798.728 1	1862.506 0	1927.395 0	1993.395 1	2714.507 3	3546.730 6	4490.065 0	4692.065 2
	$-E^a$	1736.048 5	1798.715 1	1862.492 7	1927.381 4	1993.381 2	2714.490 8	3546.711 4	4490.043 1	4692.042 8
$1s4d$	$-E^p$	1662.534 5	1722.503 3	1783.534 6	1845.628 4	1908.784 7	2598.785 3	3395.035 9	4297.536 5	4490.786 7
	$-E^a$	1662.527 6	1722.496 2	1783.527 4	1845.621 1	1908.777 2	2598.776 4	3395.025 6	4297.524 8	4490.774 6
$1s5d$	$-E^p$	1628.502 1	1687.222 1	1746.982 1	1807.782 2	1869.622 2	2545.776 4	3324.823 0	4208.423 4	4397.623 4
$1s5s$	$-E^p$	1610.015 3	1668.057 0	1727.126 5	1787.223 7	1848.348 7	2516.126 8	3286.682 6	4106.016 2	4347.016 3

[p] present work.
[a] Ivanov and Safronova (1993) [13].

Table 14: Energy resonances of $1snd\ ^3D^e$ singly excited states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u. = 2 Ryd).

		Z								
		11	12	13	14	15	16	17	18	19
1s3d	$-E^p$	66.056 60	78.723 37	92.501 25	107.390 25	123.390 35	140.501 57	158.723 89	178.057 33	198.501 88
	$-E^a$	66.058 32	78.725 32	92.503 44	107.392 66	123.393 00	140.504 44	158.727 00	178.060 67	198.505 45
1s4d	$-E^p$	63.625 59	75.781 90	89.000 70	103.282 01	118.625 82	135.032 13	152.500 94	171.032 25	190.626 06
	$-E^a$	63.626 47	75.782 90	89.001 83	103.283 26	118.627 20	135.033 63	152.502 56	171.033 99	190.627 92
1s5d	$-E^p$	62.500 38	74.420 41	87.380 45	101.380 49	116.420 53	132.500 56	149.620 60	167.780 64	186.980 68
1s6d	$-E^p$	61.889 15	73.680 84	86.500 31	100.347 56	115.222 59	131.125 39	148.055 97	166.014 33	185.000 47
		Z								
		20	21	22	23	24	25	26	27	28
1s3d	$-E^p$	220.057 54	242.714 31	266.502 19	291.391 19	317.391 29	344.502 51	372.724 83	402.058 27	432.502 82
	$-E^a$	220.061 34	242.724 34	266.506 45	291.395 68	317.396 01	344.507 46	372.730 02	402.063 69	432.508 47
1s4d	$-E^p$	211.282 37	233.001 17	255.782 48	279.626 29	304.532 60	330.501 41	357.532 72	385.626 53	414.782 84
	$-E^a$	211.284 35	233.003 29	255.784 72	279.628 65	304.535 08	330.504 01	357.535 44	385.629 38	414.785 81
1s5d	$-E^p$	207.220 71	228.500 75	250.820 79	274.180 83	298.580 86	324.020 90	350.500 94	378.020 98	406.581 01
1s6d	$-E^p$	205.014 38	226.056 08	248.125 55	271.222 80	295.347 82	320.500 63	346.681 21	373.889 57	504.125 70
		Z								
		29	30	31	32	33	34	35	36	37
1s3d	$-E^p$	464.058 48	496.725 25	530.503 13	565.392 13	601.392 23	638.503 45	979.725 77	716.059 21	756.503 76
	$-E^a$	464.064 36	496.731 36	530.509 47	565.398 70	601.399 03	638.510 48	676.733 04	716.066 71	756.511 48
1s4d	$-E^p$	445.001 64	476.282 95	508.626 76	542.033 07	576.501 88	612.033 19	648.627 00	686.283 31	725.002 11
	$-E^a$	445.004 74	476.286 17	508.630 10	542.036 53	576.505 47	612.036 90	648.630 83	686.287 26	725.006 19
1s5d	$-E^p$	436.181 05	466.821 09	498.501 13	531.221 17	564.981 20	599.781 24	635.621 28	672.501 32	710.421 35
1s6d	$-E^p$	431.389 62	461.681 31	493.000 78	525.348 03	558.723 06	593.125 86	628.556 44	665.014 80	702.500 94
		Z								
		38	39	40	41	42	43	44	45	46
1s3d	$-E^p$	798.059 42	840.726 19	884.504 07	929.393 06	975.393 17	1022.504 3	1070.726 7	1120.060 1	1170.504 7
	$-E^a$	764.787 63	805.631 56	847.537 99	890.506 92	934.538 35	979.632 28	1025.788 7	1073.007 6	1121.289 0
1s4d	$-E^p$	749.381 39	789.381 43	830.421 47	872.501 50	915.621 54	959.781 58	1004.981 6	1051.221 6	1098.501 6
	$-E^a$	741.014 85	780.556 55	821.126 02	862.723 27	905.348 29	949.001 10	993.681 68	1039.390 0	1086.126 1
1s5d	$-E^p$	749.381 39	789.381 43	830.421 47	872.501 50	915.621 54	959.781 58	1004.981 6	1051.221 6	1098.501 6
1s6d	$-E^p$	741.014 85	780.556 55	821.126 02	862.723 27	905.348 29	949.001 102	993.681 68	1039.390 0	1086.126 1
		Z								
		47	48	49	50	51	52	53	54	55
1s3d	$-E^p$	1222.060 3	1274.727 1	1328.505 0	1383.394 0	1439.394 1	1496.505 3	1554.727 6	1614.061 0	1674.505 6
	$-E^a$	1222.070 3	1274.737 4	1328.515 5	1383.404 7	1439.405 0	1496.516 5	1554.739 0	1614.072 7	1674.517 5
1s4d	$-E^p$	1170.627 7	1221.034 0	1272.502 8	1325.034 1	1378.627 9	1433.284 2	1489.003 0	1545.784 3	1603.628 1
	$-E^a$	1170.633 0	1221.039 4	1272.508 3	1325.039 8	1378.633 7	1433.290 1	1489.009 1	1545.790 5	1603.634 4
1s5d	$-E^p$	1146.821 7	1196.181 7	1246.581 8	1298.021 8	1350.501 8	1404.021 9	1458.581 9	1514.181 9	1570.822 0
1s6d	$-E^p$	1133.890 0	1182.681 7	1232.501 2	1283.348 5	1335.223 5	1388.126 3	1442.056 9	1497.015 2	1553.001 4
		Z								
		56	57	58	60	70	80	90	91	92
1s3d	$-E^p$	1736.061 3	1798.728 0	1862.505 9	1927.394 9	1993.395 0	2714.507 2	3546.730 4	4490.064 8	4692.065 0
	$-E^a$	1736.073 4	1798.740 4	1862.518 5	1927.407 7	1993.408 0	2714.490 8	3546.748 1	4490.084 8	4692.085 4
1s4d	$-E^p$	1662.534 4	1722.503 2	1783.534 6	1845.628 4	1908.784 7	2598.785 3	3395.035 8	4297.536 4	4490.786 5
	$-E^a$	1662.540 9	1722.509 8	1783.541 2	1845.635 1	1908.791 6	2598.793 4	3395.045 2	4297.547 0	4490.797 4
1s5d	$-E^p$	1628.502 0	1687.222 1	1746.982 1	1807.782 1	1869.622 2	2545.222 5	3324.822 9	4208.423 3	4397.623 4
1s5s	$-E^p$	1610.015 3	1668.057 0	1727.126 4	1787.223 7	1848.348 7	2516.126 8	3286.682 6	4106.016 2	4347.016 2

[p] present work.
[a] Ivanov and Safronova (1993) [13].

Table 15: Energy resonances of $_2(1,0)_n\ ^{+1}S^e$ autoionizing states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u. = 2 Ryd).

	Z								
	11	12	13	14	15	16	17	18	19
$_2(1,0)_3\ -E^p$	21.019 11	25.094 59	29.531 18	34.328 88	39.487 69	45.007 61	50.888 64	57.130 78	63.734 04
$-E^a$	20.874 28	24.935 47	29.357 77	34.141 18	39.285 70	44.791 33	50.658 07	56.885 93	63.474 89
$_2(1,0)_4\ -E^p$	18.405 75	21.952 86	25.812 47	29.984 58	34.469 19	39.266 30	44.375 91	49.798 02	55.532 63
$-E^a$	18.332 86	21.872 87	25.725 37	29.890 38	34.367 88	39.157 89	44.260 39	49.675 40	55.402 90
$_2(1,0)_5\ -E^p$	17.223 96	20.529 19	24.124 43	28.009 67	32.184 90	36.650 14	41.405 38	46.450 61	51.785 85
$_2(1,0)_6\ -E^p$	16.585 56	19.759 74	23.211 70	26.941 44	30.948 95	35.234 24	39.797 31	44.638 16	49.756 79
$_2(1,0)_7\ -E^p$	16.197 54	19.292 38	22.657 64	26.293 31	30.199 38	34.375 86	38.822 75	43.540 05	48.527 75

	Z								
	20	21	22	23	24	25	26	27	28
$_2(1,0)_3\ -E^p$	70.698 40	78.023 88	85.710 47	93.758 17	102.166 98	110.936 90	120.067 93	129.560 07	139.413 33
$-E^a$	70.424 97	77.736 16	85.408 46	93.441 87	101.836 39	110.592 02	119.708 77	129.186 62	139.025 59
$_2(1,0)_4\ -E^p$	61.579 74	67.939 35	74.611 46	81.596 07	88.893 18	96.502 79	104.424 90	112.659 51	121.206 62
$-E^a$	61.442 91	67.795 41	74.460 42	81.437 92	88.727 93	96.330 43	104.245 44	112.472 94	121.012 95
$_2(1,0)_5\ -E^p$	57.411 09	63.326 33	69.531 56	76.026 80	82.812 04	89.887 27	97.252 51	104.907 75	112.852 99
$_2(1,0)_6\ -E^p$	55.153 19	60.827 37	66.779 33	73.009 07	79.516 58	86.301 87	93.364 94	100.705 79	108.324 41
$_2(1,0)_7\ -E^p$	53.785 87	59.314 39	65.113 32	71.182 66	77.522 40	84.132 56	91.013 12	98.164 09	105.585 47

	Z								
	29	30	31	32	33	34	35	36	37
$_2(1,0)_3\ -E^p$	149.627 69	160.203 17	171.139 76	182.437 45	194.096 26	206.116 19	218.497 22	231.239 36	244.342 61
$-E^a$	149.255 66	159.786 85	170.709 15	181.992 56	193.637 08	205.642 71	218.009 46	230.737 31	243.826 28
$_2(1,0)_4\ -E^p$	130.066 23	139.238 34	148.722 95	158.520 06	168.629 67	179.051 78	189.786 39	200.833 50	212.193 11
$-E^a$	129.865 45	139.030 46	148.507 96	158.297 97	168.400 47	178.815 48	189.542 98	200.582 99	211.935 49
$_2(1,0)_5\ -E^p$	121.088 22	129.613 46	138.428 70	147.533 93	156.929 17	166.614 41	176.589 65	186.854 88	197.410 12
$_2(1,0)_6\ -E^p$	116.220 82	124.395 00	132.846 96	141.576 69	150.584 21	159.869 50	169.432 57	179.273 42	189.392 04
$_2(1,0)_7\ -E^p$	113.277 26	121.239 45	129.472 06	137.975 07	146.748 49	155.792 32	165.106 55	174.691 20	184.546 25

	Z								
	38	39	40	41	42	43	44	45	46
$_2(1,0)_3\ -E^p$	257.806 98	271.632 46	285.819 04	300.366 74	315.275 55	330.545 47	346.176 51	362.168 65	378.521 90
$-E^a$	257.276 35	271.087 54	285.259 84	299.793 25	314.687 77	329.943 40	345.560 15	361.538 00	377.876 97
$_2(1,0)_4\ -E^p$	223.865 22	235.849 83	248.146 94	260.756 55	273.678 66	286.913 28	300.460 39	314.320 00	328.492 11
$-E^a$	223.600 50	235.578 00	247.868 01	260.470 51	273.385 52	286.613 02	300.153 03	314.005 53	328.170 54
$_2(1,0)_5\ -E^p$	208.255 36	219.390 59	230.815 83	242.531 07	254.536 31	266.831 54	279.416 78	292.292 02	305.457 25
$_2(1,0)_6\ -E^p$	199.788 45	210.462 63	221.414 59	232.644 32	244.151 84	255.937 13	268.000 20	280.341 05	292.959 67
$_2(1,0)_7\ -E^p$	113.277 26	121.239 45	129.472 06	137.975 07	146.748 49	155.792 32	165.106 55	174.691 20	184.546 25

	Z								
	47	48	49	50	51	52	53	54	55
$_2(1,0)_3\ -E^p$	395.236 27	412.311 75	429.748 33	477.645 03	465.704 84	484.224 76	503.105 79	522.347 94	541.951 19
$-E^a$	394.577 04	411.638 23	429.060 53	446.843 94	464.988 46	483.494 09	502.360 84	521.588 69	541.177 66
$_2(1,0)_4\ -E^p$	342.976 72	235.849 83	372.883 44	388.305 55	404.040 16	420.087 27	436.446 88	453.118 99	470.103 60
$-E^a$	342.648 04	357.438 05	372.540 55	387.955 56	403.683 06	419.723 07	436.075 58	452.740 58	469.718 09
$_2(1,0)_5\ -E^p$	318.912 49	332.657 73	346.692 97	361.018 20	375.633 44	390.538 68	405.733 91	421.219 15	436.994 39
$_2(1,0)_6\ -E^p$	305.856 08	319.030 26	332.482 22	346.211 95	360.219 47	374.504 76	389.067 83	403.908 68	419.027 30
$_2(1,0)_7\ -E^p$	297.969 23	310.798 77	323.898 72	337.269 08	350.909 85	364.821 02	379.002 60	393.454 60	408.177 00

	Z								
	56	57	58	60	70	80	90	91	92
$_2(1,0)_3\ -E^p$	561.915 56	582.241 03	602.927 62	623.975 32	645.384 13	879.333 34	1149.393 6	1455.565 0	1521.132 7
$-E^a$	561.127 73	581.438 92	602.111 22	623.144 63	644.539 15	878.345 47	1148.262 9	1454.291 4	1519.830 5
$_2(1,0)_4\ -E^p$	487.400 71	505.010 32	522.932 43	541.167 04	559.714 15	762.372 75	996.281 35	1261.439 9	1318.221 6
$-E^a$	487.008 09	504.610 60	522.525 60	540.753 11	559.293 11	761.880 66	995.718 21	1260.805 7	1317.573 2
$_2(1,0)_5\ -E^p$	453.059 62	469.414 86	486.060 10	502.995 34	520.220 57	708.422 94	925.625 32	1171.827 6	1224.548 1
$_2(1,0)_6\ -E^p$	434.423 71	450.097 89	466.049 85	482.279 58	498.787 10	679.140 02	887.270 72	1123.179 2	1173.694 2
$_2(1,0)_7\ -E^p$	423.169 80	438.433 02	453.966 64	469.770 68	485.845 12	661.461 97	864.119 64	1093.818 1	1143.002 7

[p] present work.
[a] Ivanov and Safronova (1993) [13].

Table 16: Energy resonances of $_2(1,0)_n\ ^{-3}S^e$ autoionizing states in the helium isoelectronic series with $11 \leq Z \leq 58$ and $Z = 60$, 70, 80, 90, 91 and 92. All energies are expressed in atomic units (1 a.u. = 2 Ryd).

	Z								
	11	12	13	14	15	16	17	18	19
$_2(1,0)_3\ -E^p$	21.132 90	25.219 52	29.667 25	34.476 09	39.646 05	45.177 11	51.069 29	57.322 57	63.936 97
$-E^a$	21.020 87	25.097 01	29.534.26	34.332.62	39.492.10	45.012 68	50.894 38	57.137 18	63.741 10
$_2(1,0)_4\ -E^p$	18.446.08	21.997 30	25.861 02	30.037 60	32.207 46	39.674 32	41.431 18	46.478 04	51.814 90
$-E^a$	18.381 82	21.926 61	25.783 90	29.953 70	34.435 99	39.230 78	44.338 07	49.757 87	55.490 16
$_2(1,0)_5\ -E^p$	17.240 01	20.546 88	24.143 74	28.030 60	32.207 46	36.674 32	41.431 18	46.478 04	51.814 90
$_2(1,0)_6\ -E^p$	16.588 40	19.762 84	23.215 06	26.945 05	30.952 83	35.238 38	39.801 71	44.642 82	49.761 70
$_2(1,0)_7\ -E^p$	16.196 00	19.290 63	22.655 68	26.291 13	30.196 99	34.373 25	38.819 93	43.537 01	48.524 51
	Z								
	20	21	22	23	24	25	26	27	28
$_2(1,0)_3\ -E^p$	70.912 48	78.249 10	85.946 83	94.005 67	102.425 62	111.206 69	120.348 86	129.852 15	139.716 55
$-E^a$	70.706 13	78.032 27	85.719 52	93.767 88	102.177 35	110.947 94	120.079 63	129.572 44	139.426 36
$_2(1,0)_4\ -E^p$	61.657 05	68.020 76	74.696 98	81.685 70	88.986 92	96.600 64	104.526 86	112.765 57	121.316 79
$-E^a$	61.534 95	67.892 24	74.562 03	81.544 33	88.839 12	96.446 41	104.366 20	112.598 49	121.143 29
$_2(1,0)_5\ -E^p$	57.441 76	63.358 62	69.565 49	76.062 35	82.849 21	89.926 07	97.292 93	104.949 79	112.896 65
$_2(1,0)_6\ -E^p$	55.158 36	60.832 80	66.785 02	73.015 02	79.522 79	86.308 34	93.371 67	100.712 78	108.331 67
$_2(1,0)_7\ -E^p$	53.782 41	59.310 72	65.109 43	71.178 56	77.518 09	84.128 03	91.008 38	98.159 14	105.580 30
	Z								
	29	30	31	32	33	34	35	36	37
$_2(1,0)_3\ -E^p$	149.942 06	160.528 68	171.476 41	182.785 25	194.455 20	206.486 27	218.878 44	231.631 73	244.746 12
$-E^a$	149.641 39	160.217 53	171.154 78	182.453 14	194.112 61	206.133 20	218.514 89	231.257 70	244.361 62
$_2(1,0)_4\ -E^p$	130.180 51	139.356 73	148.845 45	158.646 67	168.760 38	179.186 60	189.925 32	200.976 54	212.340 26
$-E^a$	130.000 58	139.170 37	148.652 66	158.447 45	168.554 75	178.974 54	189.706 83	200.751 62	212.108 91
$_2(1,0)_5\ -E^p$	121.133 51	129.660 37	138.477 24	147.584 10	156.980 96	166.667 82	176.644 68	186.911 54	197.468 40
$_2(1,0)_6\ -E^p$	116.228 33	124.402 77	132.854 99	141.584 98	150.592 76	159.878 31	169.441 64	179.282 75	189.401 63
$_2(1,0)_7\ -E^p$	113.271 88	121.233 86	129.466 25	137.969 05	146.742 26	155.785 87	165.099 89	174.684 32	184.539 16
	Z								
	38	39	40	41	42	43	44	45	46
$_2(1,0)_3\ -E^p$	258.221 63	272.058 25	286.255 98	300.814 83	315.734 78	331.015 84	346.658 02	362.661 30	379.025 70
$-E^a$	257.826 65	271.652 79	285.840 04	300.388 40	315.297 87	330.568 46	346.200 15	361.192 96	378.546 88
$_2(1,0)_4\ -E^p$	224.016 48	236.005 19	248.306 41	260.920 13	273.846 35	287.085 07	300.636 29	314.500 01	328.676 22
$-E^a$	223.778 71	235.761 00	248.306 41	260.663 08	273.582 87	286.815 17	300.359 96	314.217 25	328.387 04
$_2(1,0)_5\ -E^p$	208.315 26	219.452 12	230.878 98	242.595 85	254.602 71	266.899 57	279.486 43	292.363 29	305.530 15
$_2(1,0)_6\ -E^p$	199.798 29	210.472 73	221.424 95	232.654 95	244.162 72	255.948 27	268.011 60	280.352 71	292.971 60
$_2(1,0)_7\ -E^p$	194.664 41	205.060 07	215.726 13	226.662 60	237.869 48	249.346 77	261.094 47	273.112 57	285.401 08
	Z								
	47	48	49	50	51	52	53	54	55
$_2(1,0)_3\ -E^p$	395.751 21	412.837 83	430.285 56	448.094 40	466.264 36	484.795 42	503.687 60	522.940 88	542.555 28
$-E^a$	395.261 90	412.338 04	429.775 30	447.573 66	465.733 13	484.253 72	503.135 41	522.378 22	541.982 13
$_2(1,0)_4\ -E^p$	343.164 94	357.966 16	373.079 88	388.506 10	404.244 82	420.296 03	436.659 75	453.335 97	470.314 69
$-E^a$	342.869 34	357.664 13	372.771 42	388.191 21	403.923 50	419.968 30	436.325 59	452.995 38	469.977 67
$_2(1,0)_5\ -E^p$	318.987 01	332.733 87	346.770 73	361.097 60	375.714 46	390.621 32	405.818 18	421.305 04	437.081 90
$_2(1,0)_6\ -E^p$	305.868 26	319.042 70	332.494 92	346.224 91	360.232 69	374.518 24	389.081 57	403.922 68	419.041 56
$_2(1,0)_7\ -E^p$	297.960 00	310.789 33	323.889 07	337.259 22	350.899 77	364.810 73	378.992 10	393.443 88	408.166 06
	Z								
	56	57	58	60	70	80	90	91	92
$_2(1,0)_3\ -E^p$	562.230 79	582.867 41	603.565 14	624.623 98	646.043 93	880.104 57	1150.276 3	1456.559 1	1522.149 1
$-E^a$	561.947 16	582.273 30	602.960 55	624.008 92	645.418 39	879.374 23	1149.441 1	1455.619 2	1521.188 2
$_2(1,0)_4\ -E^p$	487.625 91	505.239 63	523.165 84	541.404 56	559.955 78	762.655 46	996.605 15	1261.804 8	1318.594 7
$-E^a$	487.272 46	504.879 76	522.799.55	541.031 84	559.576 63	762.212 05	996.097 47	1261.232 8	1318.009 9
$_2(1,0)_5\ -E^p$	453.148 76	469.505.62	486.152 48	503.089 34	520.316 21	708.534 82	925.753 43	1171.972 0	1224.695 7
$_2(1,0)_6\ -E^p$	434.438 22	450.112 66	466.064 88	482.294 88	498.802 65	679.158 17	887.291 46	1123.202 5	1173.718 0
$_2(1,0)_7\ -E^p$	423.158 66	438.421 66	453.955 07	469.758 89	485.833 12	661.447 84	864.103 37	1093.799 7	1142.983 8

[p] present work.
[a] Ivanov and Safronova (1993) [13].

4 Summary

A host of results in total and excitation energies for high-lying $(1s,ns)^{1,3}S^e$, $(1s,np)^{1,3}P^o$, $(1s,nd)^{1,3}D^e$, ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ excited states of the helium- like ions have been tabulated using our modified atomic orbital theory in the framework of a semi-empirical procedure. The results up to $Z=10$ compared with experimental and theoretical available results have indicated a good agreement. It has been also demonstrated the possibilities to apply the MAOT formalism in the calculation of energy resonances in very high Z-helium - like ions. A multitude of results listed may be of interest for future experimental and theoretical investigations in high-lying ${}^1(1s,ns)^{1,3}S^e$, $(1s,np)^{1,3}P^o$, $(1s,nd)^{1,3}D^e$, ${}_2(1,0)_n{}^{+1}S^e$ and ${}_2(1,0)_n{}^{-3}S^e$ excited states as well as in lower than in very high Z-He isoelectronic series bearing in mind the simplicity of the present formalism.

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