COMMUNICATIONS IN COMPUTATIONAL PHYSICS
Vol. 5, No. 5, pp. 1045-1053

Commun. Comput. Phys.
May 2009

Continuous Opinion Dynamics in Complex Networks

L. Guo* and X. Cai

Complexity Science Center and Institute of Particle Physics, Huazhong (Central China) Normal University, Wuhan 430079, China.

Received 12 July 2008; Accepted (in revised version) 3 September 2008

Communicated by Dietrich Stauffer

Available online 30 October 2008

Abstract. Many realistic social networks share some universal characteristic properties, such as the small-world effects and the heterogeneous distribution of connectivity degree, which affect the dynamics in society system, especially the opinion dynamics in society. To see this, we study the opinion dynamics of the Improved Deffuant Model (IDM) in complex networks. When the two opinions differ by less than the confidence parameter ϵ $(0<\epsilon<1)$, each opinion moves partly in the direction of the other with the convergence parameter μ, which is a function of the opposite's degree k; otherwise, the two refuse to discuss and no opinion is changed. We analyze the evolution of the steady opinion s_* as a function of the confidence parameter ϵ, the relation between the minority steady opinion s_*^{min} and the individual connectivity k, and find some interesting results that show the dependence of the opinion dynamics on the confidence parameter and on the system topology. This study provides a new perspective and tools to understand the effects of complex system topology on opinion dynamics.

PACS: 05.45.-a, 87.23.Ge, 89.75.-k

Key words: Opinion dynamics, complex networks, bifurcation phenomena.

1 Introduction

Our local society, which can be well modelled as complex network, has its own structure depending on the geography, culture and history. Recently it has also been realized that many real social networks arising in society, such as networks of sexual relationships [1], collaborations between actors [2, 3] and scientists [4, 5], web-based social networks [6], P2P social network [7], and the BBS networks [8] all share some universal characteristics such as the small-world effect and the power-law degree distribution. Those features

*Corresponding author. *Email addresses:* `longkuo0314@gmail.com` (L. Guo), `xcai@mail.ccnu.edu.cn` (X. Cai)

affect the dynamics in society systems, especially the opinion dynamics in complex network. Many natural and man-made networks have been successfully studied as a framework of several celebrated opinion models. Some relevant results have been summarized in a recent review article by Toral and Tessone [9]. Nevertheless, the understanding of the opinion dynamics in complex networks remains a challenge.

Social impact theory founded by Latané [10, 11], was developed as a metatheoretical framework for modelling situations where beliefs, attributes or behaviors of an individual are influenced by those of others around him/her. Based on the social impact theory, there are two celebrated opinion models proposed in recent years. One celebrated model is the binary opinion model that is proposed by K. Sznajd-Weron and J. Sznajd (S model) [12] to describe a simple mechanism of making up decisions in a closed community. In this model, the opinion of individual is a binary variable assuming the value +1 and -1 that referring to two opposite opinions on a particular issue. The updating rules follow the principle of "united we stand, divided we fall". The other is the continuous model proposed by Deffuant et al (D model) [13]. In D model, the opinion s of individual varies continuously between zero and one. Each agent selects randomly one of the other agents and checks first if an exchange of opinions makes sense. If the two opinions differ by less than ϵ $(0<\epsilon<1)$, each opinion moves partly in the direction of the other, by amount $\mu\Delta s$, where Δs is the opinion difference and μ the convergence parameter $(0<\mu\leq 0.5)$; otherwise, the two refuse to discuss and no opinion is changed. The parameter ϵ is called confidence bound or confidence parameter. In a society, people typically have continuous opinions and always change their opinions due to the influence of acquaintances or other external factors.

Since the D model was introduced, the model has been paid much attention [14, 15]. Many previous works about D model have considered the convergence parameters μ between pairwise agents are uniform on regular lattices and complex networks [13, 16–18]. For instance, G. Weisbuch [18] had studied the D model on regular lattice and scale-free network and discussed the influence of possible social networks topologies in the opinion dynamics of D model. Furthermore, D. Stauffer et al. [19] studied the discrete opinion dynamics of D model on scale-free network with single layer and multi-layer and introduced noise and advertising. They found that the simulation of the D model with discrete opinion could be simplified and made less ambiguous. And, noise and a more realistic network with stronger clustering do not change the results much in the discrete model. An ageing model with several layers representing different age groups gave results not much different from those of one single layer, also if advertising is included. On the other hand, our society is not the homogeneous one, i.e., each individual has his/her confidence parameter ϵ and convergence parameter μ (i.e., the influence of individual). G. Deffuant et al. [20] studied the opinion dynamics of D model with different confidence parameter and analyzed the role of the extremists and got many fruitful results. However, in our society, we often change our opinion as the one of individual who is a famous expert about the particular issue according to the celebrity effect. In our present work, we assume that the larger the agent's connectivity is, the more famous expert the agent will be. Hence, the convergence parameters μ between pairwise agents are different, which is a function of the opposite's connectivity k.

The main goal of this paper is to study how does the opinion dynamics of IDM depend on the complex system topology. Generically, the system reaches a steady state as the updating time $t \to \infty$, i.e., all the agents' opinions do not change any more, and we call the opinion unchanged as the steady opinion, noted by s_*. Then, we focus on the evolution of the s_* as a function of the confidence parameter ϵ in the regular lattice (RL), the small-world network (SWN) and the scale-free network (SFN). We find that there exists a bifurcation diagram of s_* as a function of ϵ in SWN and SFN and not in RL, which indicates the effect of shortcuts of complex networks on the opinion dynamics. Second, we analyze the relation between the minority steady opinion s_*^{min} and the individual connectivity degree k in SWN and SFN and show that in the two complex networks, the processes of the opinion dynamics follow different paths. Our observations indicate the dependence of opinion dynamics of IDM on the confidence parameter ϵ between individuals and the complex system topology.

2 Improved Deffuant model

Many real society systems can be well mapped to complex networks, which is a set of distinguishable nodes $i = 1,2,\cdots,N$, connected by a fixed number of $l = 1,2,\cdots,L$ indistinguishable edges. Those edges represent the different relationships among agents in society, such as friendship, collaboration, business, sexual and other interactions [21]. The network is represented by its adjacency matrix A, where $a_{ij} = 1$, if an edge connects nodes i and j and $a_{ij} = 0$, otherwise. There are no self-connections or multiple edges.

To realize our model simulation, we employ the celebrated small-world network (SWN) proposed by Watts and Strogatz [2] and scale-free network (SFN) proposed by Barabási and Albert [3] to study the opinion dynamics. The SWN is defined on a lattice consisting of N nodes arranged in a ring. Initially each node is connected to all of its neighbors up to some fixed range k to make the network with average coordination number $z = 2k$, randomness is then introduced by rewiring edges between two randomly chosen nodes with rewiring probability ϕ. The random rewiring process introduces ϕNk long ranges which connect nodes that otherwise would be part of different neighborhoods. By varying ϕ one can closely monitor the transition between order $(\phi = 0)$ and randomness $(\phi = 1)$ [22]. In order to show the role of the shortcuts in complex networks, we also analyze the opinion dynamics in regular lattice (RL), which is built following the above mentioned model rules with the rewiring probability $\phi = 0$. And the scale-free network is built following the principle of growing and preferential attachment. The SFN of size N is generated starting from a randomly connected core of m_0 nodes and a set $U(0)$ of $(N - m_0)$unconnected nodes. At each time step, a new node is chosen from $U(0)$ and linked to $m(m < m_0)$ other nodes with the probability of Π that a new node will be connected to node i depends on the degree k_i of node i, i.e, $\Pi(k_i) = k_i / \sum_j k_j$. Numerical simulations indicated that this network evolves into a scale-invariant state with the probability that a node has k edges following a power law with an exponent $\gamma=3$ [22].

In our society, each agent has his/her own influence of persuading others to agree with him/her, and on the other hand, each agent also has his/her own ability to keep his/her opinion from changing. In our present work, we study the dynamics of continuous opinion of improved Deffuant model (IDM) with heterogeneous convergence parameters μ in complex networks.

We choose a pairwise agents i and j randomly at each time step. If the two opinions differ more than a fixed threshold parameter $\epsilon(0<\epsilon<1)$, which is called the confidence parameter, both opinions refuse to discuss and no opinion is changed. If, instead, $|s_i(t)-s_j(t)|<\epsilon$, then each opinion moves partly in the direction of the other as:

$$\begin{cases} s_i(t+1)=s_i(t)+\mu_j[s_j(t)-s_i(t)], \text{ with prob.} p_i; \\ s_j(t+1)=s_j(t)+\mu_i[s_i(t)-s_j(t)], \text{ with prob.} p_j, \end{cases} \tag{2.1}$$

where $\mu_j=k_j/(2*k_{max})$ is the convergence parameter $(0<\mu\leq 1/2)$ that agent j interacts other agents and k_{max} is the largest connectivity degree in social complex system. The probability $p_j(=1-\mu_j)$ is the probability that agent j is persuaded to change his/her opinion, since each agent has the ability to keep his/her opinion from changing. The famous expert changes his/her opinion with smaller probability.

3 Numerical results

We simulate the opinion dynamics of IDM, Eq. (2.1) in RL and SWN and SFN of size $N=1000$. The initial opinion of agent varies continuously from zero to one randomly. All the results have been calculated from at least 100 configurations, with each running lasting for at least 2×10^4 updating steps. The society is so active that nearly all agents change their opinion by interacting others at each updating step.

Generally, the system of opinion dynamics reaches a steady state as the updating time $t\rightarrow\infty$, namely, all agents' opinions do not change any more as time elapses when the system reaches the steady state, which also been found in many previous works about D model [13,23] and the pictures of the evolution of opinions as a function of time steps t are not shown in our present work. We pay most of our attention to the evolution of the steady opinion s_* as a function of the confidence parameter ϵ and to the effects of complex system topology on opinion dynamics.

In Fig. 1 we represent the evolution of the steady opinion s_* as a function of the confidence parameter ϵ in RL ($*$) and SWN ($\times$) and SFN (+). We find that there exists bifurcation phenomena of the steady opinion s_* as the ϵ increases, from the plurality state to the polarization state and then to the consensus state, in SWN and SFN and not in RL. Here, the polarization state is defined as that the individuals can be divided into two or more camps according to their opinions. Each camp has its opinion that different from others obviously. The consensus state is defined as that all the individuals share the same opinion. Comparing with the topology of RL and SWN, we find that the main reason of the bifurcation phenomena is the existence of shortcuts in SWN. Interestingly, the opinions do reach the steady state with the $s_* = 0.50(8)$ when $\epsilon > 0.40(5)$ in RL, which is called the pseudo-consensus state, because of the compromise factor in our model and the regular lattice topology. On the other hand, we find that the steady opinion s_* of consensus state is also around 0.5 in SWN and SFN. However, the fluctuation of steady opinion s_* in SFN is larger than that in SWN, which is due to the topology structure of complex networks. A detailed finite size scaling analysis performed for both complex networks shows that the critical value of polarization and the critical value of consensus, (ϵ_p, ϵ_c), corresponds in SFN to (0.21(4), 0.48(2) [24]), and in SWN to (0.15(5), 0.40(3)), accordingly, as shown in Fig. 1. From the bifurcation diagram of steady opinion s_* as the function of confidence parameter ϵ, the ability of polarization and consensus of SWN is much stronger than that of SFN. Namely, the ability of polarization and consensus depends on the heterogeneous property of complex networks, the more heterogeneous the complex network is, the weaker the ability of polarization and consensus of complex network will be.

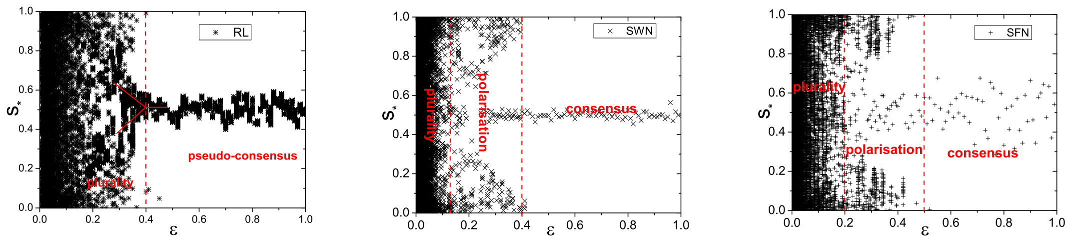

Figure 1: (color online) Bifurcation diagram for the steady opinion s_* as a function of the confidence parameter ϵ in regular lattice ($*$ RL) and in small-world network ($\times$ SWN) and in scale-free network ($+$ SFN). The parameters of the three complex networks are: $N = 1000$, $\phi = 0.05$, $z = 18$, $m_0 = 10$, $m = 6$.

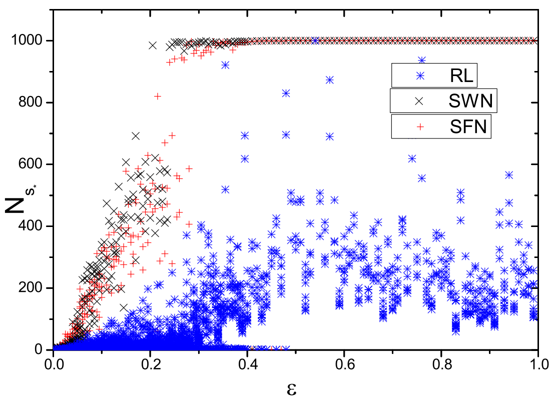

Figure 2: (color online) Plots for the evolution of the N_{s_*} as a function of the confidence parameter ϵ in SFN ($+$) and in SWN ($\times$) and in RL ($*$). The network parameters are as in Fig. 1.

The above results show the effects of shortcuts in complex systems on the opinion dynamics and the bifurcation diagram of s_* as a function of ϵ in the both celebrated complex networks. Additionally, the opinion dynamics reaches the polarization state when $\epsilon_p^{SFN} < \epsilon < \epsilon_c^{SFN}$ in SFN and $\epsilon_p^{SWN} < \epsilon < \epsilon_c^{SWN}$ in SWN. Further light can be shed on the polarization region in the both complex networks. In order to do this, we define the physical quantity N_{s_*} as the number of agents with the same steady opinion s_* and study the evolution of N_{s_*} as a function of ϵ.

In Fig. 2 we represent the evolution of the N_{s_*} as a function of the confidence parameter ϵ in RL and SWN and SFN respectively. We find that the system reaches a state that consists of a finite set of distinct opinion clusters (or "parties") in RL when $\epsilon > 0.5$, which is not found in SWN and SFN that shows the effects of shortcuts on opinion dynamics further. On the other hand, we focus on the evolution of N_{s_*} in SWN and SFN. In the polarization region, there exists the largest number of agents sharing the same steady opinion s_*, which is called the majority steady opinion s_*^{max}; otherwise, the steady opinion that a few agents share is called the minority steady opinion, noted by s_*^{min}. Interestingly, there also exists the second-largest and the third-largest number of agents sharing the same steady opinion s_*, see the middle part of the picture in Fig. 2. The evolution of the largest and the second-largest cluster as a function of confidence parameter ϵ of D model on adaptive networks has been analyzed by Balazs Kozma and Alain Barrat [25]. Here, we pay most of our attention to the minority steady opinion s_*^{min} and study the relationship between the s_*^{min} and the connectivity degree k of the SWN and SFN.

As known, the obvious difference of SWN and SFN is the connectivity degree distribution, the bell-form distribution to SWN and the power-law distribution to SFN accordingly. In order to analyze the relation between s_*^{min} and the connectivity degree k, we define the relative connectivity degree λ as follows,

$$\lambda = \frac{k}{k_{max}}, \tag{3.1}$$

where k_{max} is the largest connectivity degree in complex network. The larger is the relative connectivity degree λ of one agent, the more famous is the agent as an expert, who plays the more important role in affecting others.

In Fig. 3 we represent the relative degree λ and the minority steady opinion s_*^{min} as a function of ϵ. From Fig. 3(b), we find that there exists a bifurcation phenomena of the minority steady opinion s_*^{min} as ϵ increases, which is called the second-bifurcation here, in SWN and SFN. The second-bifurcation phenomena indicates that the agents with minority steady opinions may pay their less or most attention to the particular issue in the both complex networks, i.e., their opinions are smaller than 0.2 or larger than 0.8, which is due to the compromise factor in our model. Interestingly, we find that there exists a huge difference between the relative degree of agents with the minority steady opinions in SWN and SFN, see Fig. 3(a). Agents with the minority steady opinion s_*^{min} have smaller connectivity degree ($\lambda < 0.18$) in SFN and higher connectivity degree ($\lambda > 0.65$) in SWN. These results indicate that the process of the polarization and consensus in SFN is much different from that in SWN. In SFN, the agents with higher connectivity are always connected with the agents with smaller connectivity and the difference between those agents'

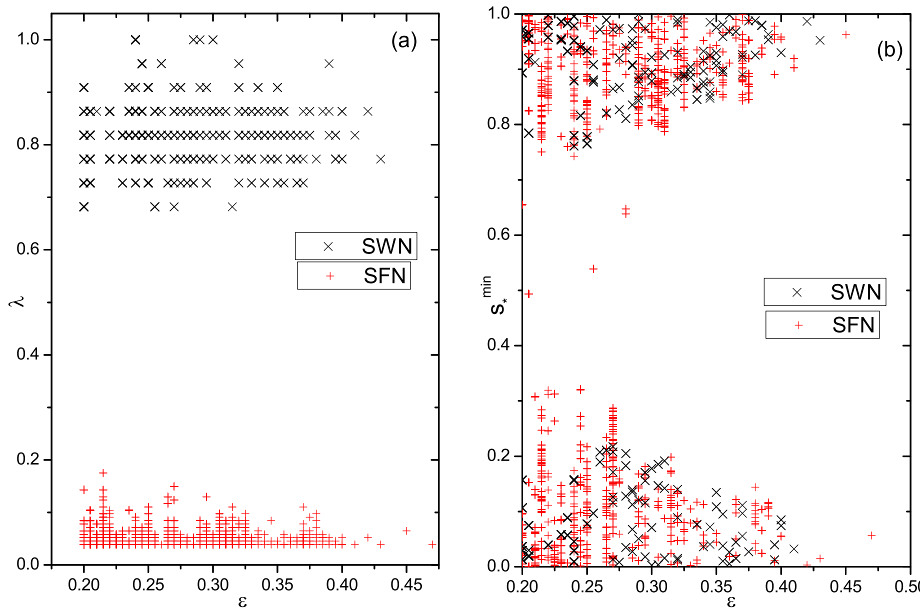

Figure 3: (color online)Plots for the evolution of (a), the relative degree λ and (b) the minority steady opinion s_*^{min} as a function of confidence parameter ϵ when $N_{s_*} < 10$ and $0.20 < \epsilon < 0.50$, i.e., in the polarization region, in SFN (+) and in SWN (×) respectively. The network parameters are as in Fig. 1.

connectivity is so much. Agents with smaller connectivity are always persuaded to move their opinions enough in the direction of their nearest neighbor who has higher connectivity, namely, the common people always change their opinions following the famous experts according to the celebrity effect. Noted that the process of the polarization and consensus starts from the agents with highest connectivity and then to his nearest neighbors and last to all the agents in SFN, even though there exists several clusters of agents with the same steady opinion. The agents with smaller connectivity away from all the agents who have higher connectivity will be separated as the minority with larger probability. On the other hand, the difference between agents' connectivity is much smaller because of the bell-form connectivity degree distribution in SWN. Hence, each pairwise agents with almost the same connectivity reaches the consensus opinion easily. Although agents changes their opinions according to the celebrity effect in our model, the agents who have higher connectivity degrees will be separated as the minority with larger probability. For SWN, many different clusters of agents with the same steady opinions merge together to form a larger one as the confidence parameter ϵ increases. Hence, the process of the polarization and consensus in SWN is different from that in SFN. The picture is confirmed in Fig. 4, where we represent the evolution of the clusters of agents with the same s_* in SWN and SFN for several values of ϵ. The ultimate reason behind these two different processes to complete consensus state is the heterogeneous character of the SFN and the role played by the hubs, which also found in the synchronization of complex networks [26].

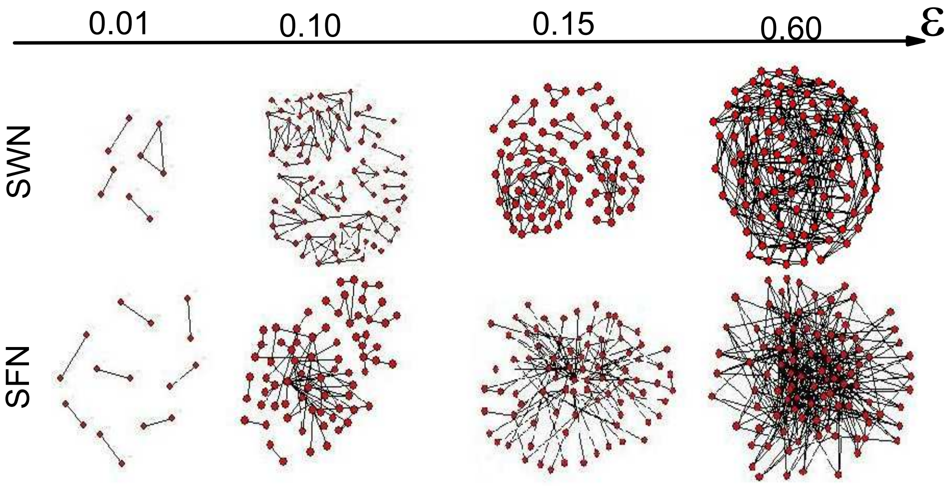

Figure 4: (color online) Consensus clusters for several values of ϵ for the two celebrated complex networks (SWN and SFN) have been studied. Those networks are made up of 100 nodes, in order to have a vivid process picture of the opinion dynamics. Although the evolutions of the polarization and consensus are always agglomerative, the paths of the polarization and consensus of the opinion dynamics in SWN and SFN are different.

4 Conclusions

In this paper, we propose an improved Deffuant model (IDM) of opinion dynamics in complex networks, where the convergence parameter μ is a function of the opposite's connectivity k according to the celebrity effect, and show that the opinion dynamics depends on the confidence parameter ϵ. Generically, the system reaches the steady state, such as the plurality state, the polarization state and the consensus state, which is related to the confidence parameter ϵ among agents. We find that the steady opinion s_* undergoes a bifurcation phenomena as the confidence parameter ϵ increases in SWN and SFN and not in RL, which indicates the effects of shortcuts in complex network on opinion dynamics. In polarization region, although there exists the largest number and the second-largest number of agents sharing the same steady opinion s_*, we pay most of our attention to the property of the agents with the same minority steady opinion s_*^{min}. We find that there exists a second-bifurcation phenomena of the s_*^{min} as ϵ increases in SFN and SWN, which is due to the compromise factor in our model. Further, we find that a few agents who has smaller connectivity degree are persuaded easily as the minority with larger probability in SFN; otherwise, a few agents who has higher connectivity degree are persuaded easily as the minority with larger probability in SWN. All those results show that the process of the polarization and consensus of opinion dynamics in SFN is different from that in SWN, because of the heterogeneous property of complex networks. Our present work opens new paths to show the process of opinion dynamics in complex networks and new tools to analyze the effects of complex system topology on opinion dynamics.

Acknowledgments

L. Guo would like to thank Prof. W. Li for helpful suggestions and comments. This work was supported by the National Natural Science Foundation of China under Grants 70571027, 10635020, and by the Ministry of Education in China under Grant 306022.

References

[1] F. Liljeros, C. R. Edling, L. A. N. Amaral, H. E. Stanley and Y. Aberg, Nature 411 (2001) 907.
[2] D. J. Watts and S. H. Strogatz, Nature 393 (1998) 440.
[3] A.-L. Barabási and R. Albert, Science 286 (1999) 509.
[4] M. E. J. Newman, Proc. Natl. Acad. Sci. U.S.A. 98 (2001) 404.
[5] A.-L. Barabási, H. Jeong, Z. Néda, E. Ravasz, A. Schubert and T. Vicsek, Physica A 311 (2002) 590.
[6] G. Csányi and B. Szendrői, Phys. Rev. E 69 (2004) 036131.
[7] F. Wang, Y. Moreno and Y. Sun, Phys. Rev. E 73 (2006) 036123.
[8] K.-I. Goh, Y.-H. Eom, H. Jeong, B. Kahng and D. Kim, Phys. Rev. E 73 (2006) 066123.
[9] R. Toral and C. J. Tessone, Commun. Comput. Phys. 2 (2007) 177.
[10] B. Latané, American Psychologist 36 (1981) 343.
[11] B. Latané, Psychological Review 88 (1981) 438.
[12] K. Sznajd-Weron and J. Sznajd, Int. J. Mod. Phys. C 11 (2000) 1157.
[13] G. Deffuant, D. Neau, F. Amblard and G. Weisbuch, Adv. Complex Systems 3 (2000) 87.
[14] C. Castellano, S. Fortunato and V. Loreto, arXiv:0710.3256.
[15] J. Lorentz, Int. J. Mod. Phys. C 18 (2007) 1819.
[16] M. Porfiri, E. M. Bollt and D. J. Stilwell, Eur. Phys. J. B 57 (2007) 481.
[17] G. Weisbuch, G. Deffuant and F. Amblard, Physica A 353 (2005) 555.
[18] G. Weisbuch, Eur. Phys. J. B 38 (2004) 339.
[19] D. Stauffer, A. O. Sousa and C. Schulze, Journal of Artificial Societies and Social Simulation 7 (3), paper 7 (`http://jasss.soc.surrey.ac.uk/7/3/7.html`)
[20] G. Deffuant, F. Amblard, G. Weisbuch and T. Faure, Journal of Artificial Societies and Social Simulation 5 (4), paper 1 (`http://jasss.soc.surrey.ac.uk/5/4/1.html`)
[21] S. Boccaletti, V. Latora, Y. Moreno, M. Chavez and D.-U. Hwang, Phys. Rep. 424 (2006) 175.
[22] R. Albert and A.-L. Barabási, Rev. Mod. Phys. 74 (2002) 47.
[23] F. Bagnoli, T. Carletti, D. Fanelli, A. Guarino and A. Guazzini, Phys. Rev. E 76(2007) 066105.
[24] S. Fortunato, Int. J. Mod. Phys. C 15 (2004) 1301.
[25] B. Kozma and A. Barrat, Phys. Rev. E 77 (2008) 016102.
[26] J. Gomez-Gardeñes, Y. Moreno and A. Arenas, Phys. Rev. Lett. 98 (2007) 034101.