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CAUCHY PROBLEM FOR A CLASS OF TOTALLY CHARACTERISTIC HYPERBOLIC OPERATORS WITH CHARACTERISTICS OF VARIABLE MULTIPLICITY IN GEVREY CLASSES①

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Abstract

This paper studies the Cauchy problem of totally characteristic hyperbolic operator (1. 1) in Gevrey classes, and obtains the following main result:

Under the conditions (I) — (VI), if $1 \leqslant s < \frac{\sigma}{\sigma-1}$ (σ is definded by (1 · 7)), then the Cauchy problem (1 · 1) is wellposed in $B([O, T], G^s_{L^2}(R^n))$; if $s = \frac{\sigma}{\sigma-1}$, then the Cauchy problem (1. 1) is wellposed in $B([0, \varepsilon], G^{\frac{\sigma}{\sigma-1}}_{L^2}(R^n))$ (where $\varepsilon > 0$, small enough).

1. Main Result

In this paper, we consider the Cauchy problem of totally characteristic hyperbolic operator with weight m-k in t, i. e.

$$\begin{cases} Pu = (t^k D_t^m + P_1(t, x; D_x) t^{k-1} D_t^{m-1} + \cdots + P_k(t, x; D_x) D_t^{m-k} + \cdots \\ \qquad + P_m(t, x; D_x)) u(t, x) = f(t, x), \quad (t, x) \in \Omega = [0, T] \times R^n \\ D_t^j u(t, x)|_{t=0} = u_j(x), \qquad 0 \leqslant j \leqslant m-k-1 \end{cases} \tag{1 · 1}$$

Problem (1 · 1) was discussed by [1], [2]; but in this paper our conditions are different from those in [1] or [2]. Suppose

(I). $k \in \mathbf{Z}_+$, $\quad 0 \leqslant k \leqslant m$

(II). Order $P_j(t, x; D_x) \leqslant j$, $\quad 1 \leqslant j \leqslant m$

(III). $P_j(t, x; D_x) = \sum_{|\beta| \leqslant j} a_{j\beta}(t, x) D_x^\beta$,

$$a_{j\beta}(t, x) \in B([0, T], G^s(R^n)) \qquad (s \geqslant 1, 1 \leqslant j \leqslant m)$$

(IV). The characteristic polynomial of P satisfies

$$\begin{aligned} &\tau^m + \sum_{j=1}^m [t^{\max(0, j-k)} \cdot \sum_{|\beta|=j} a_{j\beta}(t, x) \xi^\beta] \tau^{m-j} \\ &= \prod_{i=1}^{m_1} (\tau - \lambda_i(t, x; \xi)) \cdot \prod_{j=1}^{m_2} (\tau - t^q \mu_j(t, x; \xi)) \end{aligned} \tag{1 · 2}$$

where $m_1 + m_2 = m$, $m_2 \geqslant 2$; $q > 0$, a rational number; $\lambda_i(t, x; \xi)$, $\mu_j(t, x; \xi) \in B([0, T], S^1_{0,0})$ are all real valued functions on $\Omega \times R^n$; if $(t, x) \in \Omega$, $|\xi| = 1$, we have: $\lambda_i(t, x; \xi) \neq \lambda_j(t, x; \xi)$ $\quad (1 \leqslant i \neq j \leqslant m_1)$, $\mu_i(t, x; \xi) \neq \mu_j(t, x; \xi)$ $\quad (1 \leqslant i \neq j \leqslant m_2)$ and $\lambda_i(0, x; \xi) \neq 0$ $\quad (1 \leqslant i \leqslant m_1)$.

The indicial operator of P

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$$L(\lambda, x; D_x) = \lambda(\lambda-1)\cdots(\lambda-m+1) + P_1(0, x; D_x)\lambda(\lambda-1)\cdots(\lambda-m+2) + \cdots + P_k(0, x; D_x)\lambda(\lambda-1)\cdots(\lambda-m+k+1)$$

is a non-singular operator of order k with parameter λ, we assume

(V). $L(\lambda, x; D_x)$ is uniquely solvable in $G^s_{L^2}(R^n)$ for any $\lambda \in \mathbf{Z}$, such that $\lambda \geqslant m-k$.

Under the conditions above, Tahara [3] considered the H^∞ wellposed of Cauchy problem (1·1), but in [3], the lower order part of operator (1·1) was restricted. In this paper, in order to solve the problem (1.1) and improve the result of [3], we use successive approximation method in Gevrey classes, thus the restrictions in lower order terms of operator (1.1) are weakened. Let

$$P = \bar{P} + \bar{\bar{P}} \tag{1·3}$$

$\bar{P} = t^k D_t^m + \sum_{j=1}^{m} \sum_{|\beta|=j} a_{j\beta}(t, x) t^{\max(0, k-j)} D_t^{m-j} D_x^\beta + \sum_{j=1}^{m} a_{j0}(t, x) t^{\max(0, k-j)} D_t^{m-j}$ is the principal part of P; $\bar{\bar{P}} = \sum_{j=2}^{m} \sum_{1\leqslant|\beta|\leqslant j-1} a_{j\beta}(t, x) t^{\max(0, k-j)} D_t^{m-j} D_x^\beta$ is the lower order part of P.

Using successive approximation method we can get the formal solution series of Cauchy problem (1.1). Thus we have to impose some restrictions on coefficients of $\bar{\bar{P}}$ in order to ensure the convergence of the formal solution series, namely

(VI). $a_{j\beta}(t, x) = t^{w(j, \beta)} \hat{a}_{j\beta}(t, x)$, $\quad 1 \leqslant |\beta| \leqslant j-1, \quad 2 \leqslant j \leqslant m.$

where $w(j, \beta) \in \mathbf{Z}_+$, and $w(j, \beta) = 1$ if $1 \leqslant |\beta| \leqslant j-1$, $2 \leqslant j \leqslant k$.

In [9], the index of G^s-wellposed was introduced. Here we will see the index of G^s-wellposed of operator (1.1) depends on the order of degeneracy of principal part and the coefficients of lower order terms of operator (1.1). Set

$$d(m-j+|\beta|, \beta) = \begin{cases} w(j, \beta), & 1 \leqslant |\beta| \leqslant j-1, \ 2 \leqslant j \leqslant k \\ w(j, \beta) + j - k, & 1 \leqslant |\beta| \leqslant j-1, \ k+1 \leqslant j \leqslant m \end{cases} \tag{1·4}$$

then $d(m-j+|\beta|, \beta) \geqslant 1$ is a positive integer. Define

$$\sigma_i = \max_{1\leqslant|\beta|\leqslant i} \left(|\beta| - \frac{d(i, \beta)}{q}; 0\right), \quad (1 \leqslant i \leqslant m-1) \tag{1·5}$$

and for any positive integers $k_i \geqslant \sigma_i$, $(1 \leqslant k_i \leqslant m-1, 1 \leqslant i \leqslant m-1)$, suppose

$$\gamma = \max_{1\leqslant i\leqslant m-1} \left(\frac{\sigma_i}{k_i}\right), \quad (\in [0, 1]) \tag{1·6}$$

$$\sigma = \max_{1\leqslant i\leqslant m-1} \left(\frac{k_i\gamma + m - i}{m-i}\right) \tag{1·7}$$

then $\sigma \geqslant 1$, and $\frac{\sigma-1}{\sigma}$ is the index of G^s-wellposed of operator (1·1).

Our main result is as follows:

Theorem A: *Under the conditions (I—VI), for any $u_j(x) \in G^s_{L^2}(R^n)$ $(0 \leqslant j \leqslant m-k-1)$ and $f(t, x) \in B([0, T], G^s_{L^2}(R^n))$, if $1 \leqslant s < \frac{\sigma}{\sigma-1}$, the Cauchy problem (1·1) has a unique solution in $B([0, T], G^s_{L^2}(R^n))$; if $s = \frac{\sigma}{\sigma-1}$, then the Cauchy problem (1·1) has a unique local solution in t $u(t, x) \in B([0, \varepsilon], G^{\frac{\sigma}{\sigma-1}}_{L^2}(R^n))$ (where $\varepsilon > 0$, small enough).*

Similar to [1] and [2], by Borel's technique, theorem A can be deduced from the following result:

Theorem B. *Under the conditions* (I) — (IV) *and* (VI), *for any* $f(t, x) \in B([0, T], G^s_{L^2}(R^n))$, *satisfing* $D_t^i f(t, x)|_{t=0} = 0 (\forall i \geq 0)$; *if* $1 \leqslant s < \frac{\sigma}{\sigma - 1}$, *flat Cauchy problem*

$$\begin{cases} Pu(t, x) = f(t, x), & (t, x) \in \Omega \\ D_t^i u(t, x)|_{t=0} = 0, & \forall i \geq 0 \end{cases} \tag{1 • 8}$$

has a unique solution $u(t, x) \in B([0, T], G^s_{L^2} R^n))$, *if* $s = \frac{\sigma}{\sigma - 1}$, *then the flat Cauchy problem* (1 • 8) *has a unique local solution in t* $u(t, x) \in B([0, \varepsilon], G^{\frac{\sigma}{\sigma-1}}_{L^2}(R^n))$ (*where* $\varepsilon > 0$, *small enough*).

We know that the key problem is the proof of theorem B. By using successive approximation, to solve the flat Cauchy problem (1 • 8) is equivalent to solve

$$\begin{cases} \bar{P}u_0 = f(t, x), & (t, x) \in \Omega \\ D_t^i u_0(t, x)|_{t=0} = 0, & \forall i \geq 0 \end{cases} \tag{1 • 9}_0$$

and

$$\begin{cases} \bar{P}u_j = -\bar{P}u_{j-1}, & (t, x) \in \Omega, \quad j \geq 1 \\ D_t^i u_j|_{t=0} = 0, & \forall i \geq 0 \end{cases} \tag{1 • 9}_j$$

$(1.9)_0$ and $(1.9)_j$ have a sequence of solution $\{u_j\}_{j \geq 0} \subset B([O, T], G^s_{L^2}(R^n))$, and $\sum_{j=0}^{\infty} u_j$ is convergent in $B([O, T], G^s_{L^2}(R^n))$. Because of operation $\bar{P}$ satisfies the conditions in [3], thus we can obtain a sequence of solutions $\{u_j\}_{j \geq 0} \subset B([0, T], H^{\infty}(R^n))$ from $(1.9)_j$ $(j \geq 0)$. It only remains to prove

(H_1) $u_j(t, x) \in B([0, T], G^s_{L^2}(R^n))$, $\forall j \geq 0$

(H_2) $\sum_{j=0}^{\infty} u_j(t, x)$ is convergent in $B([0, T], G^s_{L^2}(R^n))$

It is obvious that (H_1) and (H_2) ensure the existence of solution in theorem B.

At first, several lemmas will be given in section 2; and then we will prove the consequence (H_1) by using energy estimates in section 3; in section 5 we will prove (H_2) and uniqueness of solution in theorem B.

Remark 1.1. From [1], Remark 1.2, condition (V) is reasonable.

Remark 1.2. If $\sigma = 1$, then theorem A implies that the Cauchy problem (1 • 1) is H^{∞} wellposed, this is the same as in [3]; if $m_1 = 0$, $m_2 = m$, then the main result of this paper is the same as in [4]; if $k = 0$ operator (1 • 1) is a non-characteristic operator, the main result of this paper is the same as in [5], [6], [7] and [8].

Remark 1.3. About Gevrey classes and Gevrey pseudodifferential operators, definitions and properties, see [6], [12], [13], [14], and [15].

2. Lemmas

Lemma 2.1. *Let* $p \geq 0$, $q \geq 0$, *and* $p + q = 1$; *denote* $\langle\xi\rangle = (1 + |\xi|^2)^{\frac{1}{2}}$, $\Lambda = Op(\langle\xi\rangle)$; *then for any* $r \geq 0$, *we have*

$$\| \Lambda^r u \| \leqslant \| \Lambda^{r+q} u \|^p \| \Lambda^{r-p} u \|^q, \quad (\textit{where } \| \cdot \| \textit{ is } L^2\textit{-norm}) \tag{2 • 1}$$

Proof. It is easy for us to prove the estimate (2.1) by Hölder inequality.

Let $N \in \mathbf{N}$, a natural mumber, $1 \leqslant j_i \leqslant m - 1$ $(1 \leqslant i \leqslant N)$ be positive integers, then

Lemma 2.2. *There exist constants* A_1, B_1, *such that*

$$N^{j_1 + \cdots + j_N} \leqslant A_1 B_1^N \cdot 1^{j_1} \cdot 2^{j_2} \cdot \cdots \cdot N^{j_N} \tag{2 • 2}$$

Proof. Set $H = N^{j_1 + \cdots + j_N} / 1^{j_1} 2^{j_2} \cdot \cdots \cdot N^{j_N}$, then

$$N! \geqslant ce^{-N} N^N \text{ (by Stirling formula)}$$

$$H \leqslant \left(\frac{N^N}{N\,!}\right)^{m-1} \leqslant \left(\frac{1}{c}\right)^{m-1} (e^{m-1})^N$$

Lemma 2.3. *Under the conditions above, let* $\gamma \in [0,1]$, *then there exist constants* A_2, $B_2 > 0$, *such that*

$$\{\gamma(j_1 + \cdots + j_N)\}\,! \leqslant A_2 B_2^N \cdot N^{(j_1+\cdots+j_N)\gamma} \qquad (2 \cdot 3)$$

Proof. For any $x \geqslant 0$, we have $x! \leqslant A_2 x^x$ $(x! = \Gamma(x+1))$; so

$$\{\gamma(j_1+\cdots+j_N)\}\,! \leqslant A_2\{\gamma(j_1+\cdots+j_N)\}^{(j_1+\cdots+j_N)\gamma}$$
$$\leqslant A_2[(m-1)^{m-1}]^N \cdot N^{(j_1+\cdots+j_N)\gamma}$$

Suppose $A > 0$ is large enough; for any non-negative integer l and real number $r \geqslant 0$, assume

$$Q_r(l,\ t,\ A) = A^r (r\,!)^s t^l \qquad (s \geqslant 1)$$

Then we have

Lemma 2.4. *Under the conditions above, there exist constants* A_3, $B_3 > 0$ *(independent of* r*), such that*

$$Q_{r+(j_1+\cdots+j_N)\gamma}(l,\ t,\ A) \leqslant A_3 B_3^N N^{(j_1+\cdots+j_N)s\gamma} Q_r(l,\ t,\ 2^s A) \qquad (2 \cdot 4)$$

Proof. Because of $[r + (j_1+\cdots+j_N)\gamma]\,! \leqslant 2^{r+(j_1+j_2+\cdots+j_N)\gamma}[(j_1+\cdots+j_N)\gamma]\,!\,r\,! \leqslant 2^{r+(m-1)N}[(j_1+\cdots+j_N)\gamma]\,!\,r\,!$. Hence by lemma 2.3

$$Q_{r+(j_1+\cdots+j_N)\gamma}(l,\ t,\ A) \leqslant A^{r+(m-1)N} 2^{(r+(m-1)N)s} A_2^s B_2^{Ns} (r\,!)^s t^l N^{(j_1+\cdots+j_N)s\gamma}$$
$$= A_2^s[2^{(m-1)s} B_2^s A^{(m-1)}]^N N^{(j_1+\cdots+j_N)s\gamma} Q_r(l,\ t,\ 2^s A)$$

3. Proof of (H_1)

In fact, it is sufficient to prove that for any $g(t,x) \in B([0,T],\ G^s_{L^2}(R^n))$, $D_t^i g(t,\ x)\,|_{t=0} = 0\ (\forall\ i \geqslant 0)$, the flat Cauchy problem

$$\begin{cases} \bar{P}u(t,\ x) = g(t,\ x), & (t,\ x) \in \Omega \\ D_t^i u(t,\ x)\,|_{t=0} = 0, & \forall\ i \geqslant 0 \end{cases} \qquad (3 \cdot 1)$$

has a solution $u(t,x) \in B([0,T],\ G^s_{L^2}(R^n))$.

Here we have known that the problem $(3 \cdot 1)$ has a solution in $B([0,T],\ H^\infty(R^n))$ (by [3]). Let the positive rational number $q = p/q_1$ (see condition (IV)), q_1 and p are positive integers. For $h \in R$, define

$$B^{\pm}_{q_1}([0,\ T],\ S^h_{G^s}) = \left\{a(t,\ x;\ \xi) = \sum_{i=1}^{N_1} t^{\pm l_i} a_i(t,\ x;\ \xi);\ l_i = \frac{p_i}{q_1},\ p_i \in \mathbf{Z}_+,\ N_1 \in \mathbf{N},\ a_i(t,\ x;\ \xi) \in B([0,\ T],\ S^h_{G^s})\right\} \qquad (3 \cdot 2)$$

It is obvious that $B([0,T],\ S^h_{G^s}) \subset B^+_{q_1}([0,T],\ S^h_{G^s}) \subset B^-_{q_1}([0,T],\ S^h_{G^s})$.

Let

$$\partial_i = tD_t - \lambda_i(t,\ x;\ D_x) \quad (1 \leqslant i \leqslant m_1)$$
$$\partial_{m_1+j} = tD_t - t^q \mu_j(t,\ x;\ D_x) \quad (1 \leqslant j \leqslant m_2)$$
$$\Pi_m = \partial_1\partial_2 \cdots \partial_{m_1}\partial_{m_1+1} \cdots \partial_{m_1+m_2}$$

we define the modules $W_\mu\ (0 \leqslant \mu \leqslant m)$ over the ring of PsDO of order zero:

$$W_m = \{c\Pi_m;\ c(t,\ x;\ \xi) \in B([0,\ T],\ S^0_{G^s})\};$$

$$W_{m-1} = \{\sum_{i=1}^{m} c_i\partial_1 \cdots \hat{\partial}_i \cdots \partial_m;\ c_i(t,\ x;\ \xi) \in B([0,\ T],\ S^0_{G^s})\};$$

$$\cdots$$

$$W_1 = \{\sum_{i=1}^{m} c_i\partial_i;\ c_i(t,\ x;\ \xi) \in B([0,\ T],\ S^0_{G^s})\};$$

$$W_0 = Op\{B([O,\ T],\ S^0_{G^s})\}$$

Similar to [1], section3 (or [4], section3), we can obtain easily.

Lemma 3.1. *For any i, j, there exist* PsDO $a_{ij}, b_{ij}, c_{ij} \in B^+_{q_1}([0, T], S^0_{\sigma})$ *such that*

$$[\partial_i, \partial_j] = a_{ij}\partial_i + b_{ij}\partial_j + c_{ij} \tag{3・3}$$

Lemma 3.2. *For any monomial $\omega^\alpha_\mu \in W_\mu$ $(0 \leqslant \mu \leqslant m-1)$, there exist ∂_i and $\omega^\beta_{\mu+1} \in W_{\mu+1}$, such that*

$$\partial_i \omega^\alpha_\mu = \omega^\beta_{\mu+1} + \sum_{j=1}^{\mu+1}\sum_{\gamma} c_{\gamma j}\omega^\gamma_{\mu+1-j} \tag{3・4}$$

where $c_{\gamma j}(t, x; \xi) \in B^+_{q_1}([0, T], S^0_{\sigma})$, $\omega^\gamma_{\mu+1-j} \in W_{\mu+1-j}$.

Let $\omega^\alpha_{m-j} \in W_{m-j}$, $u(t, x) \in C^\infty([0, T] \times R^n)$; define

$$W(t) = \sum_{j=1}^{m}\sum_{\alpha} \| \omega^\alpha_{m-j} u \| \tag{3・5}$$

Then we have the following energy estimate:

Lemma 3.3. *Under the assumptions above, we have*

$$t\frac{d}{dt}W(t) \leqslant \text{const} \cdot W(t) + \| \Pi_m u \| \tag{3・6}$$

It is well-known, flat Cauchy problem (3・1) is equivalent to

$$\begin{cases} \hat{P}u = t^{m-k}Pu = t^{m-k}g(t, x) = \hat{g}(t, x), & (t, x) \in \Omega \\ D^i_t u(t, x)|_{t=0} = 0, \ \forall i \geqslant 0 \end{cases} \tag{3・7}$$

If $u(t, x) \in B([0, T], H^\infty(R^n))$ is a solution of (3.7), let

$$E_0(t) = \sum_{j=0}^{m-1}\sum_{i=0}^{m-1-j} t^j \| \Lambda^i D^j_t u \|, \quad (t \in [0, T]) \tag{3・8}$$

Then we have

Lemma 3.4. *There exist constants c, $c_1 > 0$, such that*

$$E_0(t) \leqslant c_1 t^{-c} W(t), \quad (t \in [0, T]) \tag{3・9}$$

Proof. Using method in [16], the totally characteristic operator $t^j\Lambda^i D^j_t$ can be expressed in the form: $t^j\Lambda^i D^j_t = \sum_{i=1}^{m}\sum_{\alpha} c_{\alpha, i}\omega^\alpha_{m-i}$, $(c_{\alpha,i}(t, x; \xi) \in B^-_{q_1}([0, T], S^0_{\sigma})$, $\omega^\alpha_{m-i} \in W_{m-i})$; this means that there exists $c > 0$, such that $t^j \| \Lambda^i D^j_t u \| \leqslant \text{const} \cdot t^{-c}W(t)$. So the estimate (3・9) holds.

Lemma 3.5. *Under the assumptions above, there exist constants c, c_1, c_2, such that*

$$E_0(t) \leqslant c_1 t^{-c} \int_0^t t^{c_2}\tau^{-c_2-1} \| \hat{P}u \| d\tau, \quad (t \in [0, T]) \tag{3・10}$$

Proof. Using method in [16], we can factorize the totally characteristic operator $\hat{P}$ into

$$\hat{P} = \Pi_m + \sum_{j=1}^{m}\sum_{\alpha} c_{\alpha, j}\omega^\alpha_{m-j},$$

$$(c_{\alpha,j}(t, x; \xi) \in B^+_{q_1}([0, T], S^0_{\sigma}), \ \omega^\alpha_{m-j} \in W_{m-j}) \tag{3・11}$$

By estimate (3.6):

$t\frac{d}{dt}W(t) \leqslant \text{const} \cdot W(t) + \| (\Pi_m - \hat{P})u \| + \| \hat{P}u \| \leqslant c_2 W(t) + \| \hat{P}u \|$, so

$$\frac{d}{dt}(t^{-c_2}W(t)) \leqslant t^{-c_2-1} \| \hat{P}u \| \tag{3・12}$$

Since $u(t, x)$ is flat at $t = 0$, so we have

$$W(t) \leqslant \int_0^t t^{c_2}\tau^{-c_2-1} \| \hat{P}u \| d\tau \tag{3・13}$$

From estimate (3・9), the energy estimate (3・10) holds.

We know that the right hand side of equation (3・7) $\hat{g}(t, x) \in$

$B([0,T], G^s_{L^2}(R^n))$, and $\hat{g}(t,x)$ is flat at $t=0$, so we can assume

$$\| \Lambda^r \hat{g} \| \leqslant c' Q_r(l,t,A), \forall r \geqslant 0, \exists A > 0 \qquad (3 \cdot 14)$$

where $Q_r(l,t,A) = A^r r!^s t^l$, $l \in \mathbf{Z}_+$, be an arbitrary large number.

Let $u(t,x) \in B([0,T], H^\infty(R^n))$ be a solution of flat Cauchy problem (3.7), suppose

$$E_r(t) = \sum_{j=0}^{m-1} \sum_{i=0}^{m-1-j} t^j \| \Lambda^{r+i} D_t^j u \|, \qquad (\forall r \geqslant 0, \ t \in [0, T]) \qquad (3 \cdot 15)$$

Then the main result of this section is

Lemma 3.6. *For any $r \geqslant 0$, there exists a constant $l_0 > 0$. such that for $A, B > 0$, large enough, we have the following energy estimate*

$$E_r(t) \leqslant Bc' l_0^{-1} Q_r(l,t,A), \qquad (3 \cdot 16)$$

$(l \in \mathbf{Z}_+$, *be an arbitrarily large number*, $t \in [0, T])$

Proof. Because $u(0,x) = 0$ in $B([0,T], H^\infty(R^n))$, hence for any $\varepsilon > 0$, there exists $\delta = \delta(\varepsilon) > 0$, such that

$$E_r(t) < \varepsilon, \ t \in [0, \delta)$$

For any fixed ε and δ, define $\varphi(t) \in C^\infty(R)$, satisfying

$$0 \leqslant \varphi(t) \leqslant 1, \quad \text{and} \quad \varphi(t) = \begin{cases} 0, & t \leqslant \frac{1}{2}\delta \\ 1, & t \geqslant \frac{2}{3}\delta \end{cases}$$

Then it is obvious that

$$E_r(t) = \varphi(t) E_r(t) + (1 - \varphi(t)) E_r(t) < \varphi(t) E_r(t) + \varepsilon, \quad (t \in [0, T])$$

Similar to [1], lemma 3.5; since l is an arbitrarily large positive integer, so we can obtain by induction, starting from estimate (3 · 10), that there exists constant B_δ, which is dependent on δ; and $A > 0$ large enough; such that

$$\varphi(t) E_r(t) \leqslant \varphi(t) B_\delta c' l_0^{-1} Q_r(l,t,A), \qquad (t \in [0, T]) \qquad (3 \cdot 17)$$

This implies

$$E_r(t) < B_\delta c' l_0^{-1} Q_r(l,t,A) + \varepsilon, \qquad (t \in [0, T]) \qquad (3 \cdot 18)$$

Secondly we will prove that there exists a fixed $\delta_0 > 0$, such that

$$E_r(t) \leqslant B_{\delta_0} \cdot c' l_0^{-1} Q_r(l,t,A), \qquad (t \in [0, T]) \qquad (3 \cdot 19)$$

In fact, if estimate (3 · 19) were false, then for any $\delta > 0$, $E_r(t) > B_\delta c' l_0^{-1} Q_r(l,t,A)$, since $E_r(t)$ is independent of δ, this means there exists $\varepsilon_0 > 0$, such that

$$E_r(t) - B_\delta c' l_0^{-1} Q_r(l,t,A) \geqslant \varepsilon_0 > 0, \ (\forall \delta > 0)$$

That is

$$E_r(t) \geqslant B_\delta c' l_0^{-1} Q_r(l,t,A) + \varepsilon_0, \qquad (\forall \delta > 0) \qquad (3 \cdot 20)$$

This is contradictory to estimate (3 · 18).

Let $B \geqslant B_{\delta_0}$, the energy estimate (3 · 16) is proved.

Now the consequence (H_1) can be deduced from the estimate (3 · 16) easily:

Let $u(t,x) \in B([0,T], H^\infty(R^n))$ be a solution of flat Cauchy problem (3 · 7); for any $\alpha \in \mathbf{Z}^n_+$, from the energy estimate (3 · 16) we have

$$\| D^\alpha u \| \leqslant \| \Lambda^r u \| \leqslant E_r(t) \leqslant Bc' l_0^{-1} A^r r!^s t^l, \quad (r = |\alpha|) \qquad (3 \cdot 21)$$

(where B, c', l_0 and A are all independent of r). This means $u(t,x) \in B([0,T], G^s_{L^2}(R^n))$.

4. Estimate for Lower Order Part

Lemma 4.1. *Let $u(t,x) \in B([0,T], G^s_{L^2}(R^n))$ be a solution of flat Cauchy problem (3 · 7). Then for any $r \geqslant 0$, $0 \leqslant i+j \leqslant m-1$, there exist B, c', l_0 and A which are all independent of r, such that*

$$t^j \| \Lambda^{r+i} D_t^j u \| \leqslant Bc' l_0^{-(m-i-j)} Q_r(l,t,A), \quad (t \in [0, T]) \qquad (4 \cdot 1)$$

Proof. By energy estimate (3 · 16), we have

$$t^{m-1-i}\,\| \Lambda^{r+i} D_t^{m-1-i} u \| \leqslant Bc' \, l_0^{-1} Q_r(l,\ t,\ A) \tag{4・2}$$

Let $u_1(t, x) = \Lambda^{r+i} D_t^j u(t, x)$, $p = m-1-i-j \in \mathbf{Z}_+$, then

$$\| \Lambda^{r+i} D_t^j u \| = \| u_1 \| \leqslant \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_p} \| D_\tau^p u_1 \| \, d\tau dt_p \cdots dt_2$$

$$\leqslant Bc' \, l_0^{-1} \int_0^{t_1} \int_0^{t_2} \cdots \int_0^{t_p} \tau^{-(m-1-i)} Q_r(l,\ \tau,\ A) \, d\tau dt_p \cdots dt_2$$

$$(t_i \in (0,\ T],\ 2 \leqslant i \leqslant p;\ t_1 = t) \tag{4・3}$$

Since l is arbitrary, we can take $l - (m-1-i) \geqslant l_0 > 0$, $(0 \leqslant i \leqslant m-1-j)$, then

$$\| \Lambda^{r+i} D_t^j u \| \leqslant Bc' \, l_0^{-1} \left(\frac{1}{l_0} \right)^p t^{-j} Q_r(l,\ t,\ A)$$

Lemma 4. 2. *Let $u(t, x) \in B([0,\ T],\ G^s_{L^2}(\mathbf{R}^n))$ be a solution of (3・7). Then for any $r \geqslant 0$, and integers $1 \leqslant j \leqslant m-1$, $1 \leqslant i \leqslant j$, $0 \leqslant k \leqslant i$; there exist B, c', l_0 and A, such that*

$$t^{j-i}\,\| \Lambda^{r+i} D_t^{j-i} u \| \leqslant Bc' \, l_0^{-(m-j+k)} Q_{r+k}(l,\ t,\ A),\quad (t \in [0,\ T]) \tag{4・4}$$

Proof. Let $i_1 = i-k$, $j_1 = j-i$, $r_1 = r+k$; then using lemma 4. 1, we have

$$t^{j_1}\,\| \Lambda^{r_1+i_1} D_t^{j_1} u \| \leqslant Bc' \, l_0^{-(m-i_1-j_1)} Q_{r_1}(l,\ t,\ A)$$

Lemma 4. 3. *Let $u(t, x) \in B([0,\ T],\ G^s_{L^2}(\mathbf{R}^n))$ be a solution of (3・7). Then for any $r \geqslant 0$, integers $1 \leqslant j \leqslant m-1$, $1 \leqslant i \leqslant j$, $1 \leqslant k' \leqslant i$, and $\gamma \in [0,\ 1]$, there exist B, c', l_0 and A, such that*

$$t^{j-i}\,\| \Lambda^{r+1} D_t^{j-i} u \| \leqslant Bc' \, l_0^{-(m-j+k'\gamma)} Q_{r+k'\gamma}(l,\ t,\ A),\quad (t \in [0,\ T]) \tag{4・5}$$

Proof. Let $Z_r(t) = t^{j-i}\,\| \Lambda^{r+i} D_t^{j-i} u \|$. By lemma 4. 2, we have

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k')} Q_{r+k'}(l,\ t,\ A) \tag{4・6$_1$}$$

Let us replace k in lemma 4. 2 by $k' - h\,(h = 1,\ 2,\ \cdots,\ k')$ respectively, then

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k'-1)} Q_{r+k'-1}(l,\ t,\ A) \tag{4・6$_2$}$$

$$\cdots\cdots\cdots$$

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j)} Q_r(l,\ t,\ A) \tag{4.6$_{k'+1}$}$$

Let $p \in [0,\ 1]$, and $p + \gamma = 1$; by lemma 2. 1, we have

$$Z_r(t) \leqslant (t^{j-i})^\gamma \,\| \Lambda^{r-p+i} D_t^{j-i} u \|^\gamma \cdot (t^{j-i})^p \,\| \Lambda^{r+\gamma+i} D_t^{j-i} u \|^p = Z^\gamma_{r-p} \cdot Z^p_{r+\gamma} \tag{4・7}$$

By (4・6)$_1$, $Z^\gamma_{r-p} \leqslant [Bc' \, l_0^{-(m-j+k')} Q_{r+k'-p}(l, t, A)]^\gamma$; at the same time by (4・6)$_2$, $Z^p_{r+\gamma} \leqslant [Bc' \, l_0^{-(m-j+k'-1)} Q_{r+k'-(1-\gamma)}(l, t, A)]^p$; so estimate (4・7) gives

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k'-p)} Q_{r+k'-p}(l,\ t,\ A) \tag{4・8$_1$}$$

Secondly, from (4・6)$_i$ and (4・6)$_{i+1}$ $(i = 2,\ 3,\ \cdots,\ k')$, and (4.7), we can obtain

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k'-1-p)} Q_{r+k'-1-p}(l,\ t,\ A) \tag{4・8$_2$}$$

$$\cdots\cdots$$

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+1-p)} Q_{r+1-p}(l,\ t,\ A) \tag{4・8$_{k'}$}$$

Next we repeat the above process to estimate (4・8)$_1$, $\cdots$, (4・8)$_{k'}$; then we have

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k'-2p)} Q_{r+k'-2p}(l,\ t,\ A) \tag{4・9$_1$}$$

$$\cdots\cdots$$

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+2-2p)} Q_{r+2-2p}(l,\ t,\ A) \tag{4・9$_{k'-1}$}$$

Hence we finally arrive at the final estimate as follows

$$Z_r(t) \leqslant Bc' \, l_0^{-(m-j+k'-k'p)} Q_{r+k'-k'p}(l,\ t,\ A) = Bc' \, l_0^{-(m-j+k'\gamma)} Q_{r+k'\gamma}(l,\ t,\ A) \tag{4・10}$$

Let $\widehat{P} = t^{m-k} P$, hence by condition (VI):

$$\widehat{P} = \sum_{j=2}^{m} \sum_{|\beta|=1}^{j-1} \hat{a}_{j\beta}(t,\ x)\, t^{\omega(j,\beta)+m-j+\max(0;\,j-k)} \cdot D_t^{m-j} D_x^\beta$$

$$=\sum_{j=2}^{m}\sum_{|\beta|=1}^{j-1}\hat{a}_{j\beta}t^{d(m-j+|\beta|,\beta)+m-j}D_t^{m-j}D_x^{\beta}$$

Let us replace $m-j+|\beta|$ by j, then we can get

$$\widehat{P}=\sum_{j=1}^{m-1}\sum_{|\beta|=1}^{j}\hat{a}_{m+|\beta|-j,\beta}(t,x)\,t^{d(j,\beta)+j-|\beta|}D_t^{j-|\beta|}D_x^{\beta} \qquad (4\cdot 11)$$

Since $d(j,\beta)\geqslant 1(1\leqslant|\beta|\leqslant j, 1\leqslant j\leqslant m-1)$ is a positive integer; hence we can choose several positive integers $1\leqslant k_j\leqslant|\beta|(1\leqslant j\leqslant m-1)$, such that $k_j\geqslant\sigma_j$ Thus γ defined by $(1\cdot 6)$ belongs to $[0,1]$. Using lemma 4. 3, the main result of this section is

Lemma 4. 4. *Let $u(t,x)\in B([0,T], G^s_{L^2}(R^n))$ be a solution of* $(3\cdot 7)$. *Then for any $r\geqslant 0$, there exist B, c', l_0, A and A_1 which are all independent of r, such that*

$$\|\Lambda^r\widehat{P}u\|\leqslant Bc'A_1\sum_{j=1}^{m-1}l_0^{-k_j\theta\gamma}Q_{r+k_j\gamma}(l+1,t,A),\ \left(\theta=\frac{\sigma}{\sigma-1}>1,\ t\in[0,T]\right). \qquad (4\cdot 12)$$

Proof. By $(4\cdot 11)$, we have

$$\|\Lambda^r\widehat{P}u\|\leqslant c\sum_{j=1}^{m-1}\sum_{|\beta|=1}^{j}t^{d(j,\beta)+j-|\beta|}\,\|\Lambda^rD_t^{j-|\beta|}D_x^{\beta}u\|. \qquad (4\cdot 13)$$

By lemma 4. 3

$$t^{d(j,\beta)+j-|\beta|}\,\|\Lambda^rD_t^{j-|\beta|}D_x^{\beta}u\|\leqslant Bc'\,l_0^{-(m-j+k_j\gamma)}Q_{r+k_j\gamma}(l+d(j,\beta),t,A)$$
$$\leqslant Bc'\,T^{d(j,\beta)-1}l_0^{-(m-j+k_j\gamma)}Q_{r+k_j\gamma}(l+1,t,A).$$

Let $A_1=c(m-1)T^{\max(d(j,\beta))-1}$, and we know $\sigma\geqslant\frac{1}{m-j}(m-j+k_j\gamma)$, $(1\leqslant j\leqslant m-1)$; hence $-k_j\theta\gamma\geqslant-(m-j+k_j\gamma)$ and estimate $(4\cdot 13)$ becomes

$$\|\Lambda^r\widehat{P}u\|\leqslant Bc'A_1\sum_{j=1}^{m-1}l_0^{-k_j\theta\gamma}Q_{r+k_j\gamma}(l+1,t,A). \qquad (4\cdot 14)$$

5. Proof of (H_2) and Uniqueness of Solution

We know that the flat Cauchy problem $(1\cdot 9)_j(j\geqslant 0)$ is equivalent to

$$\begin{cases}\widehat{P}u_0=t^{m-k}f=\hat{f}(t,x),\ (t,x)\in\Omega.\\ D_t^iu_0(t,x)|_{t=0}=0,\ \forall\, i\geqslant 0.\end{cases} \qquad (5\cdot 1)_0$$

$$\begin{cases}\widehat{P}u_j=-\widehat{P}u_{j-1},\ (t,x)\in\Omega,\ j\geqslant 1.\\ D_t^iu_j(t,x)|_{t=0}=0,\ \forall\, i\geqslant 0.\end{cases} \qquad (5\cdot 1)_j$$

By consequence (H_1) (see section 3), the flat Cauchy problem $(5.1)_j(j\geqslant 0)$ has a solution $u_j(t,x)\in B([0,T],G^s_{L^2}(R^n))$; i. e.

$$\sum_{j=0}^{\infty}u_j(t,x),\ (u_j(t,x)\in B([0,T],G^s_{L^2}(R^n)),\ \textit{be a solution of}\ (5\cdot 1)_j) \qquad (5\cdot 2)$$

is a formal series solution of flat Cauchy problem $(1\cdot 8)$. Next let us consider the convergence of the formal series solution $(5\cdot 2)$:

Lemma 5. 1. *For any $r\geqslant 0$, $N\in\mathbf{Z}_+$; let $u_N(t,x)\in B([0,T],G^s_{L^2}(R^n))$ be a solution of $(5.1)_N$. Then there exist sufficiently large constants A, $\overline{B}$ and $\overline{C}$ which are all independent of r, such that*

$$\|\Lambda^ru_N\|\leqslant\overline{C}\,\overline{B}^NN^{N(s-\theta)\gamma}Q_r(l+N,t,2^sA),\ \left(\theta=\frac{\sigma}{\sigma-1}>1,\ t\in[0,T]\right) \qquad (5\cdot 3)$$

Proof. Let $\|\Lambda^r\hat{f}\|\leqslant c'\,Q_r(l,t,A)$. Using lemma 4. 1 (let $i=j=0$), we can get from equation $(5.1)_0$ that

$$\|\Lambda^ru_0\|\leqslant Bc'\,l_0^{-m}Q_r(l,t,A)\leq Bc'\,Q_r(l,t,A).\ (l_0\geqslant 1) \qquad (5\cdot 4)$$

By lemma 4. 4, we have

$$\| \Lambda^r \widehat{P} u_0 \| \leqslant Bc' A_1 \sum_{j=1}^{m-1} l_0^{-k_j \theta \gamma} Q_{r+k_j \gamma}(l+1, t, A) \quad (5 \cdot 5)$$

Hence we can repeat the above process to $(5 \cdot 1)_1$ to get

$$\| \Lambda^r u_1 \| \leqslant B^2 c' A_1 \sum_{j=1}^{m-1} l_0^{-k_j \theta \gamma} Q_{r+k_j \gamma}(l+1, t, A) \quad (5 \cdot 6)$$

and

$$\| \Lambda^r \widehat{P} u_1 \| \leqslant B^2 c' A_1^2 \sum_{i=1}^{m-1} \sum_{j=1}^{m-1} (l_0+1)^{-k_i \theta \gamma} l_0^{-k_j \theta \gamma} Q_{r+k_j \gamma + k_i \gamma}(l+2, t, A) \quad (5 \cdot 7)$$

Repeating these steps, we can obtain inductively

$$\| \Lambda^r u_N \| \leqslant B^{N+1} c' A_1^N \sum_{i_1=1}^{m-1} \sum_{i_2=1}^{m-1} \cdots \sum_{i_N=1}^{m-1} E_{k_{i_1} k_{i_2} \cdots k_{i_N}} Q_{r+(k_{i_1}+\cdots+k_{i_N})\gamma}(l+N, t, A) \quad (5 \cdot 8)$$

where $E_{k_{i_1} \cdots k_{i_N}} = [l_0 + (N-1)]^{-k_{i_N} \theta \gamma} \cdot [l_0 + (N-2)]^{-k_{i_{N-1}} \theta \gamma} \cdots l_0^{-k_{i_1} \theta \gamma} \leqslant N^{-k_{i_N} \theta \gamma} (N-1)^{-k_{i_{N-1}} \theta \gamma} \cdots 1^{-k_{i_1} \theta \gamma} (l_0 \geqslant 1)$. Applying lemma 2. 2, we have

$$E_{k_{i_1} k_{i_2} \cdots k_{i_N}} \leqslant A_2 B_1^N N^{-(k_{i_1}+k_{i_2} \cdots + k_{i_N}) \theta \gamma} \quad (5 \cdot 9)$$

By lemma 2. 4, we have

$$Q_{r+(k_{i_1}+k_{i_2}+\cdots+k_{i_N})\gamma}(l+N, t, A) \leqslant A_3 B_2^N N^{(k_{i_1}+k_{i_2}+\cdots+k_{i_N}) s \gamma} Q_r(l+N, t, 2^s A) \quad (5 \cdot 10)$$

Hence from $(5 \cdot 9)$, $(5 \cdot 10)$ and estimate $(5 \cdot 8)$, we obtain

$$\| \Lambda^r u_N \| \leqslant c' B^{N+1} A_1^N A_2 B_1^N A_3 B_2^N \{ \sum_{i_1=1}^{m-1} \cdots \sum_{i_N=1}^{m-1} N^{(k_{i_1}+\cdots+k_{i_N})(s-\theta)\gamma} Q_r(l+N, t, 2^s A) \quad (5 \cdot 11)$$

Because $s \leqslant \theta$, $1 \leqslant k_{i_j} \leqslant m-1, (1 \leqslant j \leqslant N)$; so let $\overline{B} = BA_1 B_1 B_2 (m-1)$, $\overline{C} = c' B A_2 A_3$, we have

$$\| \Lambda^r u_N \| \leqslant \overline{C} \overline{B}^N N^{N(s-\theta)\gamma} Q_r(l+N, t, 2^s A)$$

The energy estimate $(5 \cdot 3)$ tells us, for any $r \geqslant 0$, we have

$$\| \Lambda^r u_N \| \leqslant \overline{C} \overline{A}^r r!^s t^l (\overline{B}^N N^{N(s-\theta)\gamma} t^N), \; (\overline{A} = 2^s A, \forall N \in \mathbf{Z}_+) \quad (5 \cdot 12)$$

Thus, the consequence (H_2) can be deduced from the energy estimate (5. 12) immediately, namely

Proposition 5. 2. *Under the conditions of theorem B, if* $1 \leqslant s < \theta = \frac{\sigma}{\sigma - 1}$, *then the formal series solution* $(5 \cdot 2)$ *is convergent in* $B([0, T], G^s_{L^2}(R^n))$; *if* $s = \theta = \frac{\sigma}{\sigma-1}$, *then there exists* $\varepsilon \in \left(0, \frac{1}{\overline{B}}\right)$, *such that the formal series solution* $(5 \cdot 2)$ *is convergent in* $B([0, \varepsilon], G_{L^2}^{\frac{\sigma}{\sigma-1}}(R^n))$.

Secondly, let us prove the uniqueness of solution in theorem B:

Proposition 5. 3. *Under the conditions of theorem B, let* $u(t, x)$ *be a solution of the homogeneous flat Cauchy problem*

$$\begin{cases} \widehat{P} u = t^{m-k} P u = 0, & (t, x) \in \Omega \\ D_t^i u(t, x) |_{t=0} = 0, & \forall i \geqslant 0 \end{cases} \quad (5 \cdot 13)$$

Then $u(t,x)$ *vanishes identically.*

Proof: Let $u_j(t,x) = u(t,x) (\forall j \in \mathbf{Z}_+)$, then (5. 13) is equivalent to

$$\begin{cases} \widehat{P} u_j = - \widehat{P} u_{j-1}, \; (u_{-1} = 0), \; (t, x) \in \Omega, \; j \geqslant 0 \\ D_t^i u_j(t, x) |_{t=0} = 0, \; \forall i \geqslant 0 \end{cases} \quad (5 \cdot 14)$$

By the estimate (5. 12), for any $r \geqslant 0$, there exist constants $\overline{A}, \overline{B}, \overline{C}$, sufficiently large, such that

$$\| \Lambda^r u \| = \| \Lambda^r u_N \| \leqslant \bar{C}\bar{A}^r r!^s t^l [(\bar{B}t)^N N^{N(s-\theta)\gamma}], \quad (\forall\ N \in \mathbf{Z}_+) \quad (5 \cdot 15)$$

If $1 \leqslant s < \theta = \dfrac{\sigma}{\sigma - 1}$, let $N \to \infty$, then we can get from the above estimate $(5 \cdot 15)$ that $u(t,x) \equiv 0$ in $B([0,T], G^s_{L^2}(R^n))$. If $s = \theta = \dfrac{\sigma}{\sigma-1}$, let $t \in [0,\varepsilon], \varepsilon \in \left(0, \dfrac{1}{B}\right)$, and $N \to \infty$, then we also have $u(t,x) \equiv 0$ in $B([0, \varepsilon], G^{\frac{\sigma}{\sigma-1}}_{L^2}(R^n))$.

Finally, let us give two examples:

Example 1. Let $m_1 = 0, m_2 = m = 2, k = 0, (t,x) \in [0,T] \times R$.

$$P = D_t^2 - t^{2q} D_x^2 + a(t,x) D_t + b(t,x) t^p D_x + c(t,x), \quad (q,\ p \in \mathbf{Z}_+) \quad (5 \cdot 16)$$

Then
$$\sigma_1 = \max(1 - \frac{p+2}{q+1}; 0) = \begin{cases} 0, & p \geqslant q-1 \\ \dfrac{q-1-p}{q+1}, & 0 \leqslant p < q-1 \end{cases};$$

$$\sigma = \sigma_1 + 1 = \begin{cases} 1, & p \geqslant q-1 \\ \dfrac{2q-p}{q+1}, & 0 \leqslant p < q-1 \end{cases}$$

By theorem A, if $p \geqslant q-1$, then for any $s \geqslant 1$, the Cauchy problem for operator $(5 \cdot 16)$ is wellposed in $B([0, T], G^s_{L^2}(R))$; if $0 \leqslant p < q-1$, and $1 \leqslant s < \dfrac{2q-p}{q-1-p}$, then the preceding Cauchy problem is wellposed in $B([0, T], G^s_{L^2}(R))$; if $s = \dfrac{2q-p}{q-1-p}$, then there exists $\varepsilon > 0$, such that the preceding Cauchy problem has a unique solution $u(t,x) \in B([0, \varepsilon], G^{\frac{2q-p}{q-1-p}}_{L^2}(R))$.

Historical Notes: The operator $(5 \cdot 16)$ was first studied by Chi Min-you [10] in 1958. Next, many works on this operator by several authors appeared. For instance Uryu [11] proved the Cauchy problem for operator $(5 \cdot 16)$ is C^∞ wellposed iff $p \geqslant q-1$; if $0 \leqslant p < q-1$, Ivrii [5] showed the Cauchy problem for operator $(5 \cdot 16)$ is G^s wellposed iff $1 \leqslant s \leqslant \dfrac{2q-p}{q-1-p}$. This result is the same as theorem A. Thus it can be seen that it is significant to study the Cauchy problem $(1 \cdot 1)$ in Gevrey category. On the other hand, the preceding process also tells us the index of G^s-wellposed defined by $(1 \cdot 7)$ is precise.

Example 2. Let $m_1 = 1, m_2 = 2, k = 2, (t,x) \in [0,T] \times R$.

$$P = t^2 \partial_t^3 + P_1 t \partial_t^2 + P_2 \partial_t + P_3 \quad (5 \cdot 17)$$

where

$$P_1 = a(t,x) - \partial_x + 2;$$
$$P_2 = tb(t,x)\partial_x - t^{2q}\partial_x^2 - a(t,x)\partial_x + t\frac{\partial a(t,x)}{\partial t} + a(t,x) + c(t,x) - \frac{\partial a(t,x)}{\partial x};$$
$$P_3 = t^{2q-1}\partial_x^3 - (b(t,x) + 2qt^{2q-1})\partial_x^2 + \left(b(t,x) - \frac{c(t,x)}{t} + t\frac{\partial b(t,x)}{\partial t} - \frac{\partial b(t,x)}{\partial x}\right)\partial_x + \frac{\partial c(t,x)}{\partial t} - \frac{1}{t}\frac{\partial c(t,x)}{\partial x}; \quad (c(t,x) \in O(t),\ 2q-1 \geqslant 0).$$

Then the characteristic roots of operator $(5 \cdot 17)$ are

$$\lambda_1 = \xi,\ \lambda_2 = t^q \xi,\ \lambda_3 = -t^q \xi$$

If $a(t,x) \in O(t)$, we know that the inditial operator of $(5 \cdot 17)$ is a simple ordinary differential operator

$$\frac{d}{dx} - \lambda \quad (5 \cdot 18)$$

Hence conditions (I)——(VI) are all satisfied. By definition, $\sigma=\begin{cases}1 & , q=\frac{1}{2} \\ \frac{3q-1}{q}, & q>\frac{1}{2}\end{cases}$

Then theorem A ensures the Cauchy problem

$$\begin{cases}Pu=f(t,x) , & (t,x)\in [0,T]\times R \\ u(0,x)=u_0(x)\end{cases} \tag{5·19}$$

is wellposed in $B([0,T], G^s_{L^2}(R))$ for any $s\geqslant 1$ if $q=\frac{1}{2}$; if $q>\frac{1}{2}$, the Cauchy problem (5.19) is wellposed in $B([0,T], G^s_{L^2}(R))$ for $1\leqslant s<\frac{3q-1}{2q-1}$, and it is wellposed in $B([0,\varepsilon], G^s_{L^2}(R))$ for $s=\frac{3q-1}{2q-1}$, where $\varepsilon>0$ is small enough.

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