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ON THE GENERALIZED SYSTEM OF FERRO-MAGNETIC CHAIN WITH GILBERT DAMPING TERM

Tan Shaobin
(Institute of Applied Physics and Computational Mathematics,
P. O. Box 8009, Beijing)
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Abstract In this paper we have established the existence of global weak solutions and blow-up properties for the generalized system of ferro-magnetic chain with Gilbert damping term by means of Galerkin method and concavity argument. In addition, the convergence as $\alpha \to 0$ and $\varepsilon \to 0$ have also been discussed.

Key Words existence; blow-up; asymptotic behavior.
Classifications 35Q10; 35B40.

0. Introduction

The evolution of spin fields in continuum ferromagnets is described by Landau-Lifshitz equations

$$M_t = -\alpha M \times (M \times H^e) + \beta M \times H^e \tag{1}$$

which bears a fundamental role in the understanding of nonequilibrium magnetism, where the magnetic field $H^e = H + \gamma \triangle M$, α, β, γ are constants with $\alpha > 0$. The first term in the right hand side of (1) is called Landau-Lifshitz-Gilbert or simply Gilbert damping term.

Let $\Omega \subset R^3$ be a bounded open domain with boundary $\partial\Omega \in C^2$. The generalized system of ferromagnetic chain

$$Z_t = -\alpha Z \times (Z \times \triangle Z) + \beta Z \times \triangle Z + f(x,t,Z) \tag{2}$$

is obviously a nonlinear degenerate parabolic system of 3-dimensional vector value function $Z = (u, v, w)$, where $f(x,t,Z)$ is a given 3-dimensional vector valued function with $t \in R^+$ and $x, Z \in R^3$.

If $\alpha = 0, \beta = 1$, i. e. for the following system

$$Z_t = Z \times \triangle Z + f(x,t,Z) \tag{3}$$

there are some works e. g. [2—8] concerning the global existence of weak solutions for various boundary value problems and initial value problem.

In Part I of the paper we consider the homogeneous boundary value problem

$$(P_\varepsilon)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z) + \beta Z\times\triangle Z + f(x,t,Z) & (4)\\ Z|_{\partial\Omega} = 0 & (5)\\ Z(x,0) = \varphi(x) & (6)\end{cases}$$

where $\varepsilon\geqslant 0$ is a constant. We have established the existence of golbal weak solutions for the problem by means of Galerkin method. In addition, the convergence, as $\varepsilon\rightarrow 0$ and $\alpha\rightarrow 0$, of the weak solutions have also been discussed in this part. Part Ⅱ is devoted to the blow-up properties for the following problem

$$(\overline{P}_\varepsilon)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z) + \beta Z\times\triangle Z + |Z|^p Z\\ Z|_{\partial\Omega} = 0\\ Z(x,0) = Z_0(x)\end{cases}$$

where the constants $p>0, \varepsilon>o$.

Part I. Global Existence for the Problem (P_ε)

We shall employ Galerkin' s method to show the existence of weak solutions for the problem (P_ε). For this purpose, we first deal with an auxiliary problem

$$(P_\varepsilon^*)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z) + \beta Z\times\triangle Z + F(x,t,Z) & (4^*)\\ (5),(6)\end{cases}$$

where the function $F(x,t,Z)$ is made as following:

$$F(x,t,Z) = \eta(Z)f(x,t,Z) \tag{7}$$

where $\eta(Z)$ is C^1 cut-off function such that $0\leqslant\eta(Z)\leqslant 1$ for any $Z\in R^3$, and

$$\eta(Z) = \begin{cases} 1, & \text{if } |Z| < M_0\\ 0, & \text{if } |Z| \geqslant 2M_0\end{cases}$$

where M_0 is a positive constant to be determined in Section 3.

Let $W_n(x)$ be the eigenfunctions of the problem

$$\triangle W + \lambda_n W = 0,\quad W|_{\partial\Omega} = 0$$

and λ_n be the relevant eigenvalues. We look for an approximate solution $Z_k(x,t)$ of (P_ε^*) in the form

$$Z_k(x,t)=\sum_{n=1}^{k}a_{kn}(t)W_n(x),\quad (n=1,2,\cdots,k;k=1,2,\cdots)$$

where the unknown 3-dimensional vector valued functions a_{kn} are determined by the following system of ordinary differential equations

$$\begin{aligned}(Z_{kt},W_s(x))=\varepsilon(\triangle Z_k,W_s(x))-\alpha(Z_k\times(Z_k\times\triangle Z_k),W_s(x))\\+\beta(Z_k\times\triangle Z_k,W_s(x))+(F(x,t,Z_k),W_s(x))\end{aligned}\qquad(8)$$

with the initial condition

$$(Z_k(x,0),W_s(x))=(\varphi(x),W_s(x)),\quad s=1,2,\cdots,k\qquad(9)$$

where $(p,q)=\int_\Omega pq dx$.

We suppose that $f(x,t,Z)$ and $\varphi(x)$ satisfy the following conditions:

(Ⅰ) $f(x,t,Z)$ is continuously differentiable with respect to x and Z such that

$$f(x,t,0)|_{\partial\Omega}=0$$

The 3×3 Jacobi derivative matrix $\frac{\partial f}{\partial Z}$ is semibounded with a constant b, i. e. $\xi\cdot\frac{\partial f}{\partial Z}\xi\leqslant b|\xi|^2$ for any $\xi\in R^3$, and $(x,t,Z)\in R^3\times R^+\times R^3$, where " $\cdot$ " denotes the scalar product of two 3-dimensional vectors.

(Ⅱ) The initial data $\varphi(x)$ belongs to $H_0^1(\Omega)\cap L^\infty(\Omega)$.

1. Basic Lemmas and a Priori Estimates

Lemma 1 *Suppose that $g,g_t\in L^p(0,T;X)$, $(1\leqslant p\leqslant\infty)$. Then with the exception of a zero measure subset, the mapping $g:[0,T]\to X$ is a continuous mapping.*

Lemma 2 *Let X_0,X,X_1 be three Banach spaces such that $X_0\subset X\subset X_1$; where the injections are continuous, and X_i is reflexive, $i=0,1$; the injection $X_0\to X$ is compact. Then the injection of $Y=\{v\in L^{\alpha_0}(0,T;X_0),\quad v_t\in L^{\alpha_1}(0,T;X_1)\}$ into $L^{\alpha_0}(0,T;X)$ is compact, where the constants $\alpha_0,\alpha_1>1$.*

It is well-known that there exists a local smooth solution $a_{kn}(t)$ for the initial value problem (8)(9).

Lemma 3 *Let $Z_k(x,t)$ be a solution of the problem (8) (9). Then under the conditions (Ⅰ)(Ⅱ), there exists a constant C_1 independent of ε,α and k but M_0, such that*

$$\sup_{0\leqslant t\leqslant T}\|Z_k(\cdot,t)\|_{L^2(\Omega)}\leqslant C_1\qquad(10)$$

Proof Multiplying the s-th equation in (8) by $a_{ks}(t)$, and summing over s from 1 to k, we can obtain

$$\| Z_k(\cdot t) \|^2_{L^2(\Omega)} + \varepsilon \int_0^t \| \nabla Z_k \|^2_{L^2(\Omega)} dt \leqslant C(M_0) + \int_0^t \| Z_k \|^2_{L^2(\Omega)} dt$$

By using Gronwall's inequality, it follows that

$$\| Z_k(\cdot, t) \|^2_{L^2(\Omega)} \leqslant C_1$$

Which implies (10).

Lemma 4 *Under the conditions of Lemma 3, we have*

$$\sup_{0 \leqslant t \leqslant T} \| \nabla Z_k(\cdot, t) \|_{L^2(\Omega)} + \sqrt{a} \| Z_k \times \triangle Z_k \|_{L^2(Q_T)} + \sqrt{\varepsilon} \| \triangle Z_k \|_{L^2(Q_T)} \leqslant C_2 \tag{11}$$

where $Q_T = \Omega \times [0, T]$, C_2 is a constant independent of ε, a and k but M_0.

Proof Multiplying the s-th equation in (8) by $-\lambda_s a_{ks}(t)$, summing over s from 1 to k, we get

$$(Z_{kt}, \triangle Z_k) = \varepsilon(\triangle Z_k, \triangle Z_k) - a(Z_k \times (Z_k \times \triangle Z_k), \triangle Z_k) + (F(x, t, Z_k), \triangle Z_k) \tag{12}$$

Since

$$(Z_k \times (Z_k \times \triangle Z_k), \triangle Z_k) = -\int_\Omega |Z_k \times \triangle Z_k|^2 dx$$

and

$$|(F(x,t,Z_k), \triangle Z_k| = \left| \int_\Omega \eta(Z_k) f(x,t,Z_k) \cdot \triangle Z_k dx \right|$$
$$= \left| \sum_{i=1}^{3} \int_\Omega Z_{kx_i} \cdot (\eta_z \cdot Z_{kx_i} f + \eta f_{x_i} + \eta f_z Z_{kx_i}) \right|$$
$$\leqslant C' + C'' \int_\Omega |\nabla Z_k(x,t)|^2 dx$$

where we have used the condition (I), C', C'' are constants, thus from (12) we have

$$\frac{d}{dt} \| \nabla Z_k(\cdot, t) \|^2_{L^2(\Omega)} + 2\varepsilon \| \triangle Z_k \|^2_{L^2(\Omega)} + 2a \| Z_k \times \triangle Z_k \|^2_{L^2(\Omega)}$$
$$\leqslant 2C' + 2C'' \| \nabla Z_k(\cdot, t) \|^2_{L^2(\Omega)}$$

By using Gronwall's inequality, which gives the estimate of the Lemma.

Corollary 1 *We have*

$$\sup_{0 \leqslant t \leqslant T} \| Z_k(\cdot, t) \|_{L^4(\Omega)} + \sup_{0 \leqslant t \leqslant T} \| Z_k(\cdot, t) \times \nabla Z_k(\cdot, t) \|_{L^{3/2}(\Omega)} \leqslant C_3 \tag{13}$$

where the constant C_3 is independent of ε, k.

Lemma 5 *Under the conditions of Lemma 3, we have*

$$\| Z_{kt} \|_{L^{3/2}(0,T;H^{-1}(\Omega))} \leqslant C_4 \tag{14}$$

where the constant C_4 is independent of ε, k.

Proof For any 3-dimensional vector valued test function $V(x,t) \in C^1(Q_T)$, the relevant Galerkin approximation

$$V_m(x,t) = \sum_{n=1}^{m} \beta_n(t) W_n(x), \quad m = 1,2,\cdots$$

converge strongly to $V(x,t)$, in $L^\infty(0,T;H^1(\Omega))$ as $m \to \infty$, where

$$\beta_n(t) = \int_\Omega V(x,t) W_n(x) dx, \quad n = 1,2,\cdots,m$$

Similarly, by using the condition (I) and the estimates (10), (11), we can obtain easily that

$$\iint_{Q_T} Z_{kt} \cdot V dx dt = \iint_{Q_T} Z_{kt} \cdot V_k dx dt \leqslant C_4 \| V_k \|_{L^3(Q_T)}$$

Since

$$\| V_k \|_{L^3(Q_T)} \leqslant C \| V_k \|_{L^3(0,T;H^1)} \leqslant C \| V \|_{L^3(0,T;H^1)}$$

thus we obtain

$$\iint_{Q_T} Z_{kt} \cdot V dx dt \leqslant C_4 \| V \|_{L^3(0,T;H^1(\Omega))}$$

Which implies the estimate (14).

Lemma 6 *Under the conditions of Lemma 3, for any small $\delta > 0$, there exists a function $Z(x,t)$, and a subsequence $\{Z_{k'}(x,t)\}$ of $\{Z_k(x,t)\}$, such that $Z_{k'}(x,t) \to Z(x,t)$ in $L^{6-\delta}(Q_T)$ strongly.*

Proof From the estimates (11)(14), we have

$$Z_k(x,t) \in L^\infty(0,T;H^1(\Omega))$$
$$Z_{kt}(x,t) \in L^{3/2}(0,T;H^{-1}(\Omega)) \tag{15}$$

Since the injection of $H^1(\Omega)$ into $L^{6-\delta}(\Omega)$ is compact, by use of Lemma 2 and (15), we see the assertion of the Lemma.

2. Global Existence for the Problem (P_ε^*)

For convenience' sake, in this section we denote the solution of problem (P_ε^*) by $Z^{(\varepsilon)}(x,t)$, and the solution of (P_0^*) by $Z^{(0)}(x,t)$ or simply by $Z(x,t)$.

Definition *The 3-dimensional vector valued function*

$$Z(x,t) \in L^\infty(0,T;H_0^1(\Omega)) \cap W^{1,3/2}(0,T;H^{-1}(\Omega))$$

is called the weak solution of the problem $(P_\varepsilon^*)(\varepsilon \geqslant 0)$, if for any function

$$\psi(x,t) \in L^\infty(Q_T) \cap L^\infty(0,T;H_0^1(\Omega))$$

we have the following integral relation

$$\iint_{Q_T} Z_t \cdot \psi dxdt + \varepsilon \iint_{Q_T} \sum_{i=1}^{3} Z_{x_i} \cdot \psi_{x_i} dxdt$$

$$- \alpha \iint_{Q_T} \sum_{i=1}^{3} (Z \times Z_{x_i})_{x_i} \cdot (Z \times \psi) dxdt + \beta \iint_{Q_T} \sum_{i=1}^{3} (Z \times Z_{x_i})_{x_i} \cdot \psi dxdt$$

$$= \iint_{Q_T} F(x,t,Z) \cdot \psi dxdt$$

with the initial condition $Z(x,0) = \varphi(x)$, *a. e. in* Ω

Theorem 1 *Suppose that the conditions* (I) (II) *are satisfied. Then the problem $(P_\varepsilon^*)(\varepsilon \geqslant 0)$ has at least one global weak solution $Z^{(\varepsilon)}(x,t)(\varepsilon \geqslant 0)$ and*

$$Z^{(\varepsilon)}(x,t) \in L^\infty(0,T;H_0^1(\Omega)) \cap C^{(0,1/3)}(0,T;H^{-1}(\Omega))$$

Proof For any function $\psi(x,t) \in L^\infty(0,T;H_0^1(\Omega)) \cap L^\infty(Q_T)$, the relevant Galerkin approximation $\psi_k(x,t) = \sum_{s=1}^{k} \beta_s(t) W_s(x)$ converge to $\psi(x,t)$ in $L^\infty(0,T;H_0^1(\Omega)) \cap L^\infty(Q_T)$ strongly, where

$$\beta_s(t) = \int_\Omega \psi(x,t) W_s(x) dx$$

From the a priori estimates (10)(11)(14), we see that the function $Z_k(x,t)$ is bounded uniformly with respect to $k=1,2,\cdots$ in the following Banach space

$$B = L^\infty(0,T;H_0^1(\Omega)) \cap W^{1,3/2}(0,T;H^{-1}(\Omega))$$

Accordingly, we can choose a subsequence from $Z_k(x,t)$ (write simply as $\{Z_k(x,t)\}$) and there exists a function $Z^{(\varepsilon)}(x,t)\in B$, such that

$$Z_k(x,t)\to Z^{(\varepsilon)}(x,t) \text{ in } L^\infty(0,T;H_0^1(\Omega)) \text{ weakly star}$$
$$Z_{kt}(x,t)\to Z_t^{(\varepsilon)}(x,t) \text{ in } L^{3/2}(0,T;H^{-1}(\Omega)) \text{ weakly}$$

Multiplying the n-th equation in (8) by $\beta_n(t)$, summing over n from 1 to k, and integrating with respect to t, we get

$$\iint_{Q_T} Z_{kt}\cdot\psi_k dxdt+\varepsilon\iint_{Q_T}\sum_{i=1}^{3}Z_{kx_i}\cdot\psi_{kx_i}dxdt-\alpha\iint_{Q_T}\sum_{i=1}^{3}(Z_k\times Z_{kx_i})_{x_i}\cdot(Z_k\times\psi_k)dxdt$$
$$-\beta\iint_{Q_T}\sum_{i=1}^{3}(Z_k\times Z_{kx_i})_{x_i}\cdot\psi_k dxdt$$
$$=\iint_{Q_T}F(x,t,Z_k)\cdot\psi_k dxdt\qquad(\varepsilon\geqslant 0)\tag{16}$$

For the purpose of passing to the limit in both sides of the last equality, we need the following auxiliary results.

Lemma 7 *There exists a subsequence of $\{Z_k(x,t)\}$ such that*

(i) $\quad Z_k\times Z_{kx_i}\to Z^{(\varepsilon)}\times Z^{(\varepsilon)}_{x_i}$ *in $L^\infty(0,T;L^{3/2}(\Omega))$ weakly star*

(ii) $\quad \sum_{i=1}^{3}(Z_k\times Z_{kx_i})_{x_i}\to\sum_{i=1}^{3}(Z^{(\varepsilon)}\times Z^{(\varepsilon)}_{x_i})_{x_i}$ *in $L^2(Q_T)$ weakly*

Proof By using the estimate (11), we can prove the assertion (i) without any difficulty.

Now we show the second assertion (ii).

Since $\sum_i(Z_k\times Z_{kx_i})_{x_i}$ is bounded uniformly in $L^2(Q_T)$, therefore there exists a function $v(x,t)\in L^2(Q_T)$ such that

$$\iint_{Q_T}\sum_{i=1}^{3}(Z_k\times Z_{kx_i})_{x_i}\cdot\Phi dxdt\to\iint_{Q_T}v(x,t)\cdot\Phi dxdt\quad\text{as } k\to\infty\tag{$*$}$$

for any function $\Phi(x,t)\in C^1(Q_T)$.

On the other hand, by using the assertion (i), we get

$$\iint_{Q_T}\sum_{i=1}^{3}(Z_k\times Z_{kx_i})_{x_i}\cdot\Phi dxdt=-\iint_{Q_T}\sum_{i=1}^{3}(Z_k\times Z_{kx_i})\cdot\Phi_{x_i}dxdt$$
$$\to-\iint_{Q_T}\sum_{i=1}^{3}(Z^{(\varepsilon)}\times Z^{(\varepsilon)}_{x_i})\cdot\Phi_{x_i}dxdt\tag{$**$}$$

Combining ($*$) with ($**$), we have

$$\iint_{Q_T} \sum_{i=1}^{3} (Z^{(\varepsilon)} \times Z_{x_i}^{(\varepsilon)}) \cdot \Phi_{x_i} dxdt = - \iint_{Q_T} v(x,t) \cdot \Phi dxdt$$

Which means, by the definition of weak derivative, that the function $Z^{(\varepsilon)} \times Z_{x_i}^{(\varepsilon)}$ is differentiable weakly with respect to $x_i (i=1,2,3)$, and

$$\sum_{i=1}^{3} (Z^{(\varepsilon)} \times Z_{x_i}^{(\varepsilon)})_{x_i} = v(x,t) \in L^2(Q_T)$$

The lemma is now proved.

Now we go on with the proof of Theorem 1.

From the previous discussion, it can be derived that (as $k \to \infty$)

$$\iint_{Q_T} Z_{kt} \cdot \psi_k dxdt \to \iint_{Q_T} Z_t^{(\varepsilon)} \cdot \psi dxdt$$

$$\iint_{Q_T} \sum_{i=1}^{3} Z_{kx_i} \cdot \psi_{kx_i} dxdt \to \iint_{Q_T} \sum_{i=1}^{3} Z_{x_i}^{(\varepsilon)} \cdot \psi_{x_i} dxdt$$

$$\iint_{Q_T} \sum_{i=1}^{3} (Z_k \times Z_{kx_i})_{x_i} \cdot \psi_k dxdt \to \iint_{Q_T} \sum_{i=1}^{3} (Z^{(\varepsilon)} \times Z_{x_i}^{(\varepsilon)})_{x_i} \cdot \psi dxdt$$

$$\iint_{Q_T} \sum_{i=1}^{3} (Z_k \times Z_{kx_i})_{x_i} \cdot (Z_k \times \psi_k) dxdt \to \iint_{Q_T} \sum_{i=1}^{3} (Z^{(\varepsilon)} \times Z_{x_i}^{(\varepsilon)})_{x_i} \cdot (Z^{(\varepsilon)} \times \psi) dxdt$$

$$\iint_{Q_T} F(x,t,Z_k) \cdot \psi_k dxdt \to \iint_{Q_T} (F(x,t,Z^{(\varepsilon)}) \cdot \psi dxdt$$

Therefore, taking the limiting process $k \to \infty$ in (16), we infer that the function $Z^{(\varepsilon)}(x,t)$ satisfies the integral relation in Definition. It now remains to show that the limiting function $Z^{(\varepsilon)}(x,t)$ satisfies the initial condition (6).

In fact, since $Z_k(x,t)$ and $Z_{kt}(x,t)$ converge weakly to $Z^{(\varepsilon)}(x,t)$ and $Z_t^{(\varepsilon)}(x,t)$ in $L^{3/2}(0,T;H^{-1}(\Omega))$, thus by Lemma 1 we can directly obtain the following assertion

$$Z_k(x,0) \to Z^{(\varepsilon)}(x,0) \text{ in } H^{-1}(\Omega) \text{ weakly}$$

On the other hand, from (9) we have $Z_k(x,0) \to \varphi(x)$ in $L^2(\Omega)$ strongly. Combining the last two limiting process, we get $Z^{(\varepsilon)}(x,0) = \varphi(x)$, a. e. in Ω. Furthermore

$$\| Z^{(\varepsilon)}(\cdot,t_1) - Z^{(\varepsilon)}(\cdot,t_2) \|_{H^{-1}(\Omega)} = \left\| \int_{t_1}^{t_2} Z_t^{(\varepsilon)}(\cdot,t)dt \right\|_{H^{-1}(\Omega)} \leqslant C_4 |t_1 - t_2|^{1/3}$$

where the constant C_4 is independent of ε, t_1 and t_2.

The proof of Theorem 1 is now completed.

Corollary 2 *Under the conditions of Theorem* 1, *then the weak solution* $Z^{(\varepsilon)}(x,t)(\varepsilon \geqslant 0)$ *of problem* (P_ε^*) *such that* $Z_t^{(\varepsilon)}(x,t) \in L^{3/2}(Q_T)$

Proof Since $Z^{(\varepsilon)}(x,t)$ satisfies the integral relation of Definition, with the a priori estimates that obtained in last section, we can easily obtain

$$\iint_{Q_T} Z_t^{(\varepsilon)} \cdot \psi dx dt \leqslant C \| \psi \|_{L^3(Q_T)}$$

for any $\psi(x,t) \in C^1(Q_T)$, where C is a constant independent of ε.

Which means the result of the Corollary.

3. Global Existence for the Problem (P_ε)

In the last section we have obtained a weak solution $Z^{(\varepsilon)}(x,t)$ of (P_ε^*). Now we show that, if the constant M_0 is sufficiently large, the function $Z^{(\varepsilon)}(x,t)$ is also a solution of the problem $(P_\varepsilon)(\varepsilon \geqslant 0)$.

First of all, we shall establish a maximum-norm estimate for the function $Z^{(\varepsilon)}(x,t)$.

Lemma 8 *Let* $Z^{(\varepsilon)}(x,t)(\varepsilon \geqslant 0)$ *be a solution of the problem* (P_ε^*) *obtained in Theorem* 1. *Then there exists a constant* M_0 *independent of* ε, α, *such that*

$$\sup_{Q_T} |Z^{(\varepsilon)}(x,t)| \leqslant M_0 \tag{17}$$

Proof We divide the proof into two parts.

Case 1 $(\varepsilon > 0)$. Since

$$Z_t^{(\varepsilon)} = \varepsilon \triangle Z - \alpha Z^{(\varepsilon)} \times (Z^{(\varepsilon)} \times \triangle Z^{(\varepsilon)}) + \beta Z^{(\varepsilon)} \times \triangle Z^{(\varepsilon)} + F(x,t,Z^{(\varepsilon)}) \tag{4'}$$

pointwise almost everywhere in Q_T, then we take the $L^2(\Omega)$-inner product of (4^ε) with $|Z^{(\varepsilon)}|^p Z^{(\varepsilon)}, (p>0)$ and obtain

$$\frac{1}{p+2}\frac{d}{dt}\int_\Omega |Z^{(\varepsilon)}|^{p+2}dx$$
$$=\int_0^1 ds\int_\Omega \eta(Z^{(\varepsilon)})|Z^{(\varepsilon)}|^p Z^{(\varepsilon)}\cdot\frac{\partial f(x,t,sZ^{(\varepsilon)})}{\partial Z}Z^{(\varepsilon)}dx$$
$$-\varepsilon(p+1)\int_\Omega |Z^{(\varepsilon)}|^p|\nabla Z^{(\varepsilon)}|^2 dx$$
$$+\int_\Omega \eta(Z^{(\varepsilon)})|Z^{(\varepsilon)}|^p Z^{(\varepsilon)}\cdot f(x,t,0)dx \tag{18}$$

Applying the condition (I) and the boundedness of $\eta(Z)$, from which we infer that

$$\frac{d}{dt}\| Z^{(\varepsilon)}(\cdot,t)\|_{L^{p+2}(\Omega)}\leqslant |b|\,\| Z^{(\varepsilon)}(\cdot,t)\|_{L^{p+2}(\Omega)}+\| f(x,t,0)\|_{L^{p+2}(\Omega)}$$

Using Gronwall's inequality, we get

$$\| Z^{(\varepsilon)}(\cdot,t)\|_{L^{p+2}(\Omega)}\leqslant\left(\|\varphi\|_{L^{p+2}(\Omega)}+\int_0^T\| f(\cdot,s,0)\|_{L^{p+2}(\Omega)}ds\right)\exp(|b|T)$$

Which implies the estimate (17).

Case 2 ($\varepsilon=0$). For any test function

$$\psi(x,t)\in L^\infty(0\ T;H_0^1(\Omega))\cap L^\infty(Q_T)$$

the function $Z^{(0)}(x,t)$ satisfies the integral relation in Definition. We now construct a special test function ψ_0 in the form

$$\psi_0(x,t)=\frac{Z^{(0)}(x,t)}{1+|Z^{(0)}|}h(x,t)$$

where the scalar function $h(x,t)\in C^1(Q_T)$. It can be easily see that the function ψ_0 belongs to the following space

$$G=L^\infty(0,T;H_0^1(\Omega))\cap L^\infty(Q_T)$$

Substituting the function $\psi_0(x,t)$ for $\psi(x,t)$ in the integral relation of Definition, we have

$$\iint_{Q_T} \frac{Z^{(0)}}{1+|Z^{(0)}|} \cdot Z_t^{(0)} h(x,t)dxdt = \iint_{Q_T} F(x,t,Z^{(0)}) \cdot \frac{Z^{(0)}}{1+|Z^{(0)}|} h(x,t)dxdt$$

From which, it follows that

$$Z^{(0)} \cdot Z_t^{(0)} = F(x,t,Z^{(0)}) \cdot Z^{(0)}, \text{ a.e. in } Q_T$$

With the condition (I), we can easily obtain that

$$\frac{d}{dt}|Z^{(0)}(x,t)| \leqslant |b||Z^{(0)}(x,t)| + |f(x,t,0)|$$

Which gives

$$|Z^{(0)}(x,t)| \leqslant (\| \varphi(x) \|_{L^\infty(\Omega)} + \int_0^T \| f(\cdot,t,0) \|_{L^\infty(\Omega)} dt)\exp(|b|T)$$

The proof of the Lemma is now completed.

Consequently, by the maximum-norm estimate (17), we know that the weak solution $Z^{(\varepsilon)}(x,t)$ of problem (P_ε^*) is also a solution of the problem $(P_\varepsilon)(\varepsilon \geqslant 0)$.

Theorem 2 *Under the conditions of Theorem 1, then the homogeneous boundary value problem (P_ε) has, at least, one global weak solution $Z^{(\varepsilon)}(x,t)(\varepsilon \geqslant 0)$, and*

$$Z^{(\varepsilon)}(x,t) \in L^\infty(0,T;H_0^1(\Omega)) \cap W^{1,3/2}(0,T;L^{3/2}(\Omega)) \cap L^\infty(Q_T)$$

4. The Convergence as $\varepsilon \to 0$ and $\alpha \to 0$

Theorem 3 *Let $Z^{(\varepsilon)}(x,t)(\varepsilon \geqslant 0)$ be a weak solution of the problem (P_ε) obtained in Theorem 2. Then there exists a function $Z \in B$ and a sequence of number $\varepsilon_k > 0 (\varepsilon_k \to 0$ as $k \to \infty)$, such that the solution $Z^{(\varepsilon_k)}(x,t)$ of the problem (P_{ε_k}) converges weakly to $Z(x,t)$ in B, and the limiting function $Z(x,t)$ is a weak solution of the problem (P_0).*

Proof Taking notice of the estimates (10), (11) and (14), we can easily prove the result of the Theorem. Which is omitted.

In the rest of the section we denote by $Z^{(a)}(x,t)$ the weak solution of the problem (P_0). We shall verify that the solution $Z^{(a)}(a>0)$ converges to a function $Z(x,t)\in G$, as $a\to 0$, and $Z(x,t)$ is a weak solution of the following problem

$$\begin{cases} Z_t = \beta Z\times \triangle Z + f(x,t,Z) \\ (5),(6) \end{cases} \tag{3'}$$

where β is an arbitrary constant.

Theorem 4 *Let $Z^{(a)}(x,t)(a>0)$ be a solution obtained in Theorem 2. Then there exists a function $Z(x,t)\in G$ and a subsequence $Z^{(a^k)}$ of $Z^{(a)}(x,t)$ such that $Z^{(a^k)}$ converges weakly star to a function $Z(x,t)$ in G as $a^k\to 0$, and the limiting function $Z(x,t)$ is a weak solution of the problem (3′)(5)(6).*

Proof For any function

$$\psi(x,t)\in C^1(Q_T) \text{ with } \psi(x,T)=0$$

we need to prove that there exists a limiting function $Z(x,t)$, and which satisfies the following integral relation

$$\iint_{Q_T}[\psi_t\cdot Z-\beta\nabla\psi * (Z\times\nabla Z)+\psi\cdot f]dxdt+\int_\Omega \psi(x,0)\cdot\varphi(x)dx=0 \tag{19}$$

where $*$ denotes the scalar product in R^3 with respect to x.

Indeed, the weak solution $Z^{(a)}(x,t)$ of the problem (P_0) satisfies the equality

$$\iint_{Q_T}[\psi_t\cdot Z^{(a)}-\beta\nabla\psi * (Z^{(a)}\times\nabla Z^{(a)})+\psi\cdot f]dxdt$$

$$+\int_\Omega \psi(x,0)\cdot\varphi(x)dx$$

$$=a\iint_{Q_T}\sum_{i=1}^{3}(Z^{(a)}\times Z^{(a)}_{x_i})_{x_i}\cdot(Z^{(a)}\times\psi)dxdt \qquad (a>0) \tag{19'}$$

By the conclusion of Lemma 4 and Lemma 8, we see that there exists a constant

M' independent of α, such that

$$\sup_{0\leqslant t\leqslant T}\| Z^{(\alpha)}(\cdot,t)\|_{H^1(\Omega)}+\| Z^{(\alpha)}(x,t)\|_{L^\infty(Q_4)}\leqslant M'$$

$$\|\sum_{i=1}^{3}(Z^{(\alpha)}\times Z^{(\alpha)}_{x_i})_{x_i}\|_{L^2(Q_T)}\leqslant\frac{M'}{\sqrt{\alpha}} \tag{20}$$

For the purpose of taking the limiting process $\alpha\to 0$ in the equality (19′), we need the following estimates

Lemma 9 *For the weak solution $Z^{(\alpha)}(x,t)$, there exists a constant M'_0 independent of α, such that*

$$\sup_{0\leqslant t\leqslant T}\| Z^{(\alpha)}_t(\cdot,t)\|_{H^{-2}(\Omega)}\leqslant M'_0 \tag{21}$$

Proof By using the estimates (20) and the definition of weak solution, we can obtain easily that

$$\int_\Omega Z^{(\alpha)}_t\cdot\psi dx\leqslant M'_0\|\psi\|_{H^2(\Omega)}$$

for any function $\psi(x)\in C^2(\overline{\Omega})$.

Which implies the estimate (21).

Lemma 10 *We have*

$$\| Z^{(\alpha)}(\cdot,t_2)-Z^{(\alpha)}(\cdot,t_1)\|_{L^2(\Omega)}\leqslant C|t_2-t_1|^{1/3} \tag{22}$$

Proof With the estimates (20) and (21), repeating the procedure exactly as in [6], we can immediately get the result of the Lemma.

Therefore, by using the estimates (20)(22), we have, (as $\alpha\to 0$)

$$Z^{(\alpha)}(x,t)\to Z(x,t) \text{ in } L^\infty(0,T;H^1_0(\Omega))\cap L^\infty(Q_T) \text{ weakly star}$$

$$Z^{(\alpha)}\times\nabla Z^{(\alpha)}\to Z\times\nabla Z \text{ in } L^\infty(0,T;L^{3/2}(\Omega)) \text{ weakly star}$$

$$Z^{(\alpha)}(x,t)\to Z(x,t) \text{ in } L^2(\Omega) \text{ strongly for any } t\in[0,T]$$

We can now pass to the limit $(\alpha\to 0)$ in the equality (19′), and which yields (19)

The proof of Theorem 4 is now completed.

Part Ⅱ. Blow-up Property for the Problem $(\overline{P}_\varepsilon)$

In this part we restrict our investigation on the following special problem

$$(\overline{P}_\varepsilon)\quad \begin{cases} Z_t = \varepsilon\triangle Z - \alpha Z\times(Z\times\triangle Z) + \beta Z\times\triangle Z + |Z|^p Z & (23)\\ Z|_{\partial\Omega} = 0 & (24)\\ Z(x,0) = Z_0(x) & (25)\end{cases}$$

where the constant $p>0$.

1. Local Existence

We apply the Galerkin procedure as above. For each N we define an approximate solution $Z_N(t)$ of (23)—(25) as follows

$$Z_N(t) = \sum_{i=1}^{N} a_{iN}(t)W_i(x) \tag{26}$$

and

$$\begin{aligned}(Z_N'(t),W_s) = {} & \varepsilon(\triangle Z_N,W_s) - \alpha(Z_N\times(Z_N\times\triangle Z_N),W_s)\\ & + \beta(Z_N\times\triangle Z_N,W_s) + (f(Z_N),W_s) \qquad (27)\\ (Z_N(0),W_s) = {} & (Z_0,W_s), \qquad s=1,\cdots,N \qquad (28)\end{aligned}$$

where $f(Z)=|Z|^p Z$.

The local existence of solution of the problem (23)—(25) is ensured by the following theorem:

Theorem 5 *Suppose that $0<p\leqslant 2$ and $Z_0(x)\in H^2(\Omega)\cap H_0^1(\Omega)$. Then there exists a positive constant T_0 such that in the interval $[0,T_0]$ the problem (23)—(25) ($\varepsilon>0$) has a generalized solution $Z(x,t)$, and*

$$Z(x,t)\in B = L^\infty(0,T;H_0^1)\cap L^2(0,T;H^2)\cap W^{1,3/2}(0,T;L^{3/2})$$

Proof We multiply (27) by $-\lambda_s a_{sN}(t)$ and add the resulting equations for $s=1,2,\cdots,N$. We have

$$\frac{d}{dt}\|\nabla Z_N(\cdot,t)\|^2_{L^2(\Omega)} + \varepsilon\|\triangle Z_N(\cdot,t)\|^2_{L^2(\Omega)}$$

$$+ 2a \| Z_N \times \triangle Z_N \|^2_{L^2(\Omega)} \leqslant C \int_\Omega |Z_N|^{2p+2} dx \tag{29}$$

Since the constant $p \in (0,2]$, by using Sobolev's embedding theorem and Poincare's inequality, we get

$$\| Z_N \|_{L^{2p+2}(\Omega)} \leqslant C_1 \| Z_N \|_{H^1_0(\Omega)} \leqslant C_2 \| \nabla Z_N \|_{L^2(\Omega)}$$

Therefore, from (29) it follows that

$$\frac{d}{dt} \| \nabla Z_N(\cdot,t) \|^2_{L^2(\Omega)} + \varepsilon \| \triangle Z_N(\cdot,t) \|^2_{L^2(\Omega)}$$
$$+ 2a \| Z_N \times \triangle Z_N \|^2_{L^2(\Omega)} \leqslant C_0 \| \nabla Z_N(\cdot,t) \|^{2p+2}_{L^2(\Omega)} \tag{30}$$

where the constant C_0 is independent of N.

From (30), we see that if $0 < T_0 < 1/(2C_0 p \| Z_0 \|^{2p}_{H^1_0})$,then there exists a constant $C(T_0)$ independent of N, such that

$$\| \triangle Z_N(x,t) \|_{L^2(Q_{T_0})} + a \| Z_N \times \triangle Z_N \|_{L^2(Q_{T_0})} \leqslant C(T_0)$$
$$\sup_{0 \leqslant t \leqslant T_0} \| Z_N(\cdot,t) \|_{H^1(\Omega)} \leqslant C(T_0)$$

With the above a priori bounds, we repeat the same procedure, which is omitted, exactly as in Part I, the existence of generalized solution $Z(x,t)$ of the problem (23)—(25) can be immediately obtained and $Z(x,t) \in B$, such that

$$Z_t = \varepsilon \triangle Z - aZ \times (Z \times \triangle Z) + \beta Z \times \triangle Z + f(Z)$$

pointwise almost everywhere in Q_{T_0}.

The proof of the Theorem is now completed.

2. Blow-up Property

Without loss of generality, we assume that $\varepsilon = 1$, and denote

$$AZ = -\triangle Z$$
$$BZ = -aZ \times (Z \times \triangle Z) + \beta Z \times \triangle Z$$
$$g(Z) = \int_0^1 (f(sZ), Z) ds$$

and $\overline{Z}_t = -AZ + f(Z)$. Then obviously we have the following results:

$$(Z_t, Z) = (\overline{Z}_t, Z), \quad 2(a_0 + 1)g(Z) \leqslant (Z, f(Z))$$

where $0 < a_0 \leqslant p/2$.

Theorem 6 *Let $Z(x,t)$ be a generalized solution to the boundary value problem (23)—(25). Suppose that*

$$Z_0(x) \in H^2(\Omega) \cap H_0^1(\Omega)$$
$$g(Z_0) > \frac{1}{2}(Z_0, AZ_0)$$

Then there exists a finite time T, estimable from above, such that

$$\lim_{t \to T^-} \int_0^t \| Z(\cdot, t) \|^2_{L^2(\Omega)} dt = +\infty$$
$$\lim_{t \to T^-} \sup_{0 \leqslant \tau \leqslant t} \| Z(\cdot, \tau) \|_{L^2(\Omega)} = +\infty$$

Proof We employ the so-called concavity argument. Setting $\overset{*}{Z}(x,t) = Z(t)$. Let T_0, β_0, τ be positive constants to be determined later. We construct a function as follows ($t \in [0, T_0]$)

$$F(t) = \int_0^t (Z, Z) d\eta + (T_0 - t)(Z_0, Z_0) + \beta_0 (t + \tau)^2 \tag{31}$$

From which we infer that

$$F'(t) = 2\int_0^t (Z, Z_\eta) d\eta + 2\beta_0 (t + \tau) \tag{32}$$

Obviously, we see that $F'(0) = 2\beta_0 \tau > 0$ and that $F(t) > 0$ for $t \in [0, T_0]$. Therefore, $F^{-a_0}(t)$ is defined on $[0, T_0]$ with $a_0 > 0$.

Supposing we could show that

$$FF'' - (a_0 + 1)(F')^2 \geqslant 0$$

then since

$$(F^{-a_0})'' = -a_0 F^{-a_0 - 2}(FF'' - (a_0 + 1)(F')^2)$$

it follows that $F^{-a_0}(t)$ is concave, and

$$F(t) \geqslant F^{(1+1/a_0)}(0)(F(0) - a_0 t F'(0))^{-1/a_0}$$

Therefore, as $t \to T(\leqslant F(0)/a_0F'(0))$, from below, we see that $F(t) \to +\infty$. Which means the assertion of the theorem.

In fact, returning to (32), we have

$$F'(t) = 2(Z,Z_t) + 2\beta_0 = \int_0^t (Z,Z_\eta)_\eta d\eta + 2(Z_t,Z)_0 + 2\beta_0$$

or

$$\begin{aligned} F'(t) = & 4(a_0+1)\left(\int_0^t (\overline{Z}_\eta,\overline{Z}_\eta)d\eta + \beta_0\right) \\ & + 2\int_0^t ((Z_\eta,Z)_\eta - 2(a_0+1)(\overline{Z}_\eta,\overline{Z}_\eta))d\eta \\ & + 2((Z_t,Z)_0 - (2a_0+1)\beta_0) \end{aligned} \tag{33}$$

Combining the identities (31)—(33), we get

$$\begin{aligned} FF'' - (a_0+1)(F')^2 = & F[4(a_0+1)\left(\int_0^t (\overline{Z}_\eta,\overline{Z}_\eta)d\eta + \beta_0\right) \\ & + 2\int_0^t ((Z_\eta,Z)_\eta - 2(a_0+1)(\overline{Z}_\eta,\overline{Z}_\eta))d\eta \\ & + 2((Z_t,Z)_0 - (2a_0+1)\beta_0)] \\ & - (a_0+1)\left(2\int_0^t (Z,Z_\eta)d\eta + 2\beta_0(t+\tau)\right)^2 \\ = & 4(a_0+1)S^2 + \\ & + 4(a_0+1)(T_0-t)(Z_0,Z_0)\left(\int_0^t (\overline{Z}_\eta,\overline{Z}_\eta)d\eta + \beta_0\right) \\ & + 2F\int_0^t ((Z_\eta,Z)_\eta - 2(a_0+1)(\overline{Z}_\eta,\overline{Z}_\eta))d\eta \\ & + 2F((Z_t,Z)_0 - (2a_0+1)\beta_0) \end{aligned} \tag{34}$$

where

$$\begin{aligned} S^2 = & \left(\int_0^t (Z,Z)d\eta + \beta_0(t+\tau)^2\right)\cdot\left(\int_0^t (\overline{Z}_\eta,\overline{Z}_\eta)d\eta + \beta_0\right) \\ & - \left(\int_0^t (Z,\overline{Z}_\eta)d\eta + \beta_0(t+\tau)\right)^2 \geqslant 0 \end{aligned}$$

where we have used the Schwarz′ inequality. Thus from (34), it follows that

$$\begin{aligned} & FF'' - (a_0+1)(F')^2 \\ & \geqslant 2F\int_0^t ((Z_\eta,Z)_\eta - 2(a_0+1)(\overline{Z}_\eta,\overline{Z}_\eta))d\eta \\ & \quad + 2F((Z_t,Z)_0 - (2a_0+1)\beta_0) \\ & = -2F\int_0^t ((Z,AZ)_\eta - 2(a_0+1)(\overline{Z}_\eta,AZ))d\eta \end{aligned}$$

$$+ 2F\int_0^t((Z,f(Z))_\eta - 2(\alpha_0+1)(\overline{Z}_\eta, f(Z))d\eta$$

$$+ 2F((\overline{Z}_t, Z)_0 - (2\alpha_0+1)\beta_0)$$

$$= - 2F\int_0^t((Z,AZ)_\eta - 2(\alpha_0+1)(Z_\eta, AZ))d\eta$$

$$+ 2F\int_0^t((Z,f(Z))_\eta - 2(\alpha_0+1)(Z_\eta, f(Z))d\eta$$

$$- 4F(\alpha_0+1)\int_0^t(BZ,AZ)d\eta$$

$$+ 4F(\alpha_0+1)\int_0^t(BZ,f(Z))d\eta$$

$$+ 2F((\overline{Z}_t, Z)_0 - (2\alpha_0+1)\beta_0)$$

Applying the facts:

$$Z_t = \overline{Z}_t + BZ, (BZ,AZ) \leqslant 0$$

and

$$(BZ, f(Z)) = 0$$

which gives

$$FF'' - (\alpha_0+1)(F')^2 \geqslant - 2F\int_0^t((Z,AZ)_\eta - 2(\alpha_0+1)(Z_\eta, AZ))d\eta$$

$$+ 2F\int_0^t((Z,f(Z))_\eta - 2(\alpha_0+1)(Z_\eta, f(Z))d\eta$$

$$+ 2F((\overline{Z}_t, Z)_0 - (2\alpha_0+1)\beta_0)$$

On account of

$$(Z,AZ)_\eta = 2(Z_\eta, AZ)$$

and

$$g(Z) = \int_0^t(Z_\eta, f(Z))d\eta + g(Z_0)$$

we have

$$FF'' - (\alpha_0+1)(F')^2$$

$$\geqslant 4\alpha_0 F\int_0^t(Z_\eta, AZ)d\eta + 2F((Z,f(Z)) - 2(\alpha_0+1)g(Z))$$

$$+ 2F(2(\alpha_0+1)g(Z_0) - (Z_0, AZ_0) - (2\alpha_0+1)\beta_0)$$

$$\geqslant 4(\alpha_0+1)F\left(g(Z_0) - \frac{1}{2}(Z_0, AZ_0) - \frac{(2\alpha_0+1)\beta_0}{2(\alpha_0+1)}\right) \tag{35}$$

Therefore, for any $\beta_0 > 0$ satisfies

$$(2\alpha_0+1)\beta_0 \leqslant 2(\alpha_0+1)(g(Z_0) - \tfrac{1}{2}(Z_0, AZ_0))$$

from (35), we finally obtain

$$FF'' - (\alpha_0 + 1)(F')^2 \geqslant 0 \quad \text{or} \quad (F^{-\alpha_0}(t))'' \leqslant 0, \quad \forall\, t \in [0, T_0]$$

Which means that the generalized solution $Z(x,t)$ blow-up in a finite time T.

Furthermore, by a straightforward computation, we can check easily that the constant T such that

$$T < (T_0(Z_0, Z_0) + \beta_0 \tau^2)/(2\alpha_0 \beta_0 \tau)$$

where

$$\alpha_0 = \frac{p}{2}, \beta_0 = 2(\alpha_0 + 1)(g(Z_0) - \frac{1}{2}\frac{(Z_0, AZ_0))}{(2\alpha_0 + 1)}$$

and $\tau = \dfrac{(Z_0, Z_0)}{\alpha_0 \beta_0}$.

The assertion of the theorem is now proved.

Remark We state that there are many initial vectors Z_0 such that

$$g(Z_0) > \frac{1}{2}(Z_0, AZ_0)$$

In fact, for any function $V_0(x) \in H^2 \cap H_0^1$, which satisfies

$$(V_0, f(V_0)) > 0$$

we set $Z_0 = sV_0$, here s is a constant such that

$$s^2 g(V_0) = s^2 \int_0^1 (f(tV_0), V_0) dt$$

$$= \frac{1}{p+2} s^p (f(V_0), V_0) > \frac{1}{2}(V_0, AV_0)$$

Then we have

$$g(Z_0) = \int_0^1 (f(tsV_0), sV_0) dt = \frac{s^{p+2}}{p+2}(f(V_0), V_0)$$

$$> \frac{s^2(V_0, AV_0)}{2} = \frac{(Z_0, AZ_0)}{2}$$

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