J. Partial Diff. Eqs. **15**(2002), 89–96
©International Academic Publishers Vol.15 No.1

ASYMPTOTIC BEHAVIOR OF GLOBAL SMOOTH SOLUTIONS TO THE EULER-POISSON SYSTEM IN SEMICONDUCTORS

Ju Qiangchang
(Academy of Mathematics and Systems Sciences, CAS,
Beijing 100080, China)
(E-mail: juqc@math08.math.ac.cn)

(Received Sep. 27, 2001)

Abstract In this paper, we establish the global existence and the asymptotic behavior of smooth solution to the initial-boundary value problem of Euler-Poisson system which is used as the bipolar hydrodynamic model for semiconductors with the nonnegative constant doping profile.

Key Words Bipolar hydrodynamic model, semiconductors, asymptotic, smooth solution.

2000 MR Subject Classification 35L65, 76X05, 35M10, 35L70, 35Q60

Chinese Library Classification O175.29.

1. Introduction

We are concerned with the large time behavior of smooth solutions to the one-dimensional Euler-Poisson system which is used as the bipolar hydrodynamic model for semiconductors in the case of two carriers, i.e. electron and hole. Namely

$$n_t + (nu)_x = 0, \tag{1.1}$$

$$h_t + (hv)_x = 0, \tag{1.2}$$

$$(nu)_t + (nu^2 + p(n))_x = n\phi_x - \frac{nu}{\tau_n}, \tag{1.3}$$

$$(hv)_t + (hv^2 + q(h))_x = -h\phi_x - \frac{hv}{\tau_h}, \tag{1.4}$$

$$\phi_{xx} = n - h - d(x), \tag{1.5}$$

$(t, x) \in (0, \infty) \times (0, 1)$, where (n, h) and (u, v) are densities and velocities for electrons and holes, respectively, $j = nu$ and $k = hv$ stand for the electron and hole current densities, ϕ denotes the electrostatic potential and we denote $E = \phi_x$ as the electric field, and $d(x)$ describes fixed charged background ions. The pressure functions $p(n)$ and $q(h)$ are taken as

$$p(n) = \frac{n^{\gamma_n}}{\gamma_n},\ \gamma_n > 1, \quad q(h) = \frac{h^{\gamma_h}}{\gamma_h},\ \gamma_h > 1. \tag{1.6}$$

τ_n and τ_h are the momentum relaxation times for electrons and holes, respectively. τ_n and τ_h are constants in the present paper. Furthermore, $\tau_n = \tau_h = 1$ for convenience.

Recently, the hydrodynamic model of semiconductors has attracted a lot of attention, due to its function to describe hot electron effects which are not accounted for in the classical drift-diffusion model. Rigorous results have been obtained in various papers. Most of them are concerned with the unipolar case — the case of one carrier type, i.e. electron. Also, there are a few results on the bipolar case which is of more importance and physical meaning. Fang and Ito [1] showed the existence of weak solutions to the system (1.1)-(1.5) using the viscosity argument. Natalini [6], Hsiao and Zhang [4], considered the relaxation limit problem from the bipolar hydrodynamic model to the drift-diffusion equations. Zhu and Hattori [7] showed the existence of smooth solutions to Cauchy problem of (1.1)-(1.5) and discussed the asymptotic stability of the steady state solution, when the doping profile is close to zero.

In present paper, we consider the initial boundary value problems for (1.1)-(1.5) with the following initial data

$$(n, h, j, k)(x, 0) = (n_0, h_0, j_0, k_0)(x), \quad x \in (0, 1), \tag{1.7}$$

and the insulating boundary conditions

$$j(0, t) = 0 = j(1, t), \tag{1.8}$$

$$k(0, t) = 0 = k(1, t), \tag{1.9}$$

$$E(0, t) = 0. \tag{1.10}$$

To provide some insights into the above evolutionary problem, we take the doping profile $d(x)$ as a nonnegative constant d. Our main purpose in this paper is to investigate the global existence and the asymptotic behavior of the smooth solution to (1.1)-(1.5) and (1.7)-(1.10). More precisely, we prove that when the initial data are small perturbations of a stationary solution to the system, the global smooth solution to (1.1)-(1.5) and (1.7)-(1.10) exists and tends to the stationary solution, as $t \to \infty$, exponentially. The steady state solution concerned satisfies the following system:

$$p(\bar{n})_x = \bar{n}\bar{E}, \tag{1.11}$$

$$q(\bar{h})_x = -\bar{h}\bar{E}, \tag{1.12}$$

$$\bar{E}_x = \bar{n} - \bar{h} - d, \tag{1.13}$$

with the boundary condition

$$\bar{E}(0) = 0 = \bar{E}(1). \tag{1.14}$$

We only consider the case with $\bar{n} > 0$ and $\bar{h} > 0$. For the problem (1.11)-(1.14), we have the following Lemma which is proved in the next section.

Proposition 1.1 *The problem* (1.11)-(1.14) *admits only the constant solution with* $\bar{E} = 0$ *and* $\bar{n} - \bar{h} = d$.

Now, Let $(\bar{n}, \bar{h}, 0)$ with $\bar{n} - \bar{h} = d$ be a fixed solution to (1.11)-(1.14) with $\bar{n} > 0, \bar{h} > 0$. The initial datum (n_0, h_0) is chosen so that

$$\int (n_0(x) - \bar{n}) = 0, \quad \int (h_0(x) - \bar{h}) = 0. \tag{1.15}$$

It follows from (1.1), (1.2), (1.8), (1.9) and (1.15) that

$$\int (n(x,t) - h(x,t) - d) = 0,$$

which with (1.5) and (1.10) implies

$$E(1,t) = 0, \quad \text{for all } t > 0. \tag{1.16}$$

Set

$$\psi(x,t) = \int_0^x (n - \bar{n}) ds, \quad \eta(x,t) = \int_0^x (h - \bar{h}) ds. \tag{1.17}$$

It is easy to verify that

$$\psi_t = -j, \quad \eta_t = -k, \quad E(x) = \psi - \eta. \tag{1.18}$$

The main result is given as the follows.

Theorem 1.2 *Assume* $\psi(\cdot, 0) \in H^3$, $\eta(\cdot, 0) \in H^3$, $j_0, k_0 \in H^2$. *Then there exists a positive number* ϵ, *such that if*

$$\|(\psi(\cdot,0), \eta(\cdot,0))\|_{H^3} + \|(j_0, k_0)\|_{H^2} \leq \epsilon, \tag{1.19}$$

then the problem (1.1)-(1.5) *and* (1.7)-(1.10) *has a unique smooth solution* (n, h, j, k, E) *satisfying*

$$\|(n - \bar{n}, h - \bar{h})\|_{H^2} + \|E(\cdot,t)\|_{H^3} + \|(j,k)\|_{H^2} \leq c \exp(-\alpha t), \tag{1.20}$$

for some positive constants c *and* α.

Remark 1.3 The frictional terms j and k, and the electric field E play essential roles in the existence and the exponential decay of the global smooth solution.

The proof of Theorem 1.2 given in Section 2 is based on the energy method. ψ and η satisfy two nonlinear wave equations with friction. We reformulate the two wave equations in the appropriate form by the stationary solution. We focus on the estimates of the damping terms and the coupled terms of electron and hole from the Poisson equation (1.5).

2. The Proof of Proposition 1.1 and Theorem 1.2

The proof of Proposition 1.1

(1.14) implies

$$\bar{n}_x(0) = 0 = \bar{n}_x(1), \tag{2.1}$$

and

$$\bar{h}_x(0) = 0 = \bar{h}_x(1). \tag{2.2}$$

In fact, from (1.11) and (1.13), we have

$$-(\frac{p(\bar{n})_x}{\bar{n}})_x = -\bar{n} + \bar{h} + d. \tag{2.3}$$

Multiplying (2.3) by n and integrating it over (0,1), one has

$$\int \frac{p'(\bar{n})}{\bar{n}}\bar{n}_x^2 = \int(-\bar{n}^2 + \bar{n}\bar{h} + \bar{n}d), \tag{2.4}$$

with the help of the integration by parts and (2.1). Similarly, it holds from (1.12) and (2.2) that

$$\int \frac{q'(\bar{h})}{\bar{h}}\bar{h}_x^2 = \int(-\bar{h}^2 + \bar{n}\bar{h} - \bar{h}d). \tag{2.5}$$

Combining (2.4) with (2.5) yields

$$\begin{aligned}\int(\frac{p'(\bar{n})}{\bar{n}}\bar{n}_x^2 + \frac{q'(\bar{h})}{\bar{h}}\bar{h}_x^2) &= -\int[(\bar{n}-\bar{h})^2 - 2d(\bar{n}-\bar{h}) + d(\bar{n}-\bar{h})] \\ &= -\int[(\bar{n}-\bar{h}) - d]^2 \le 0,\end{aligned} \tag{2.6}$$

with the help of $\int(\bar{n}-\bar{h}) = d$ due to (1.14) and (1.13). Noting d is a nonnegative constant, (2.6) with (1.6) implies $\bar{n}_x = \bar{h}_x = 0$, namely, $\bar{n}$ and $\bar{h}$ are positive constants. Thus, $\bar{n} - \bar{h} = d$ and $\bar{E} = 0$. The proof is completed.

To prove Theorem (2.1), we first establish the following a priori estimates.

Lemma 2.1 *There exist some positive constants $\delta > 0$ and $\beta > 0$, such that for any $T > 0$, if*

$$\sup_{0\le t\le T}(\|(\psi(\cdot,t),\eta(\cdot,t))\|_{H^3} + \|(\psi_t(\cdot,t),\eta_t(\cdot,t))\|_{H^2}) \le \delta, \tag{2.7}$$

then

$$\begin{aligned}&\|(\psi(\cdot,t),\eta(\cdot,t))\|_{H^3}^2 + \|(\psi_t(\cdot,t),\eta_t(\cdot,t))\|_{H^2}^2 \\ &\quad\le c(\|(\psi(\cdot,0),\eta(\cdot,0)\|_{H^3}^2 + \|(\psi_t(\cdot,0),\eta_t(\cdot,0))\|_{H^2}^2)\exp(-\beta t),\end{aligned} \tag{2.8}$$

for any $t \in [0,T]$.

Proof Due to (1.15), (1.3) and (1.4), we have

$$\psi(0,t) = 0 = \psi(1,t), \quad \eta(0,t) = 0 = \eta(1,t), \tag{2.9}$$

$$\psi_{xx}(0,t) = 0 = \psi_{xx}(1,t), \quad \eta_{xx}(0,t) = 0 = \eta_{xx}(1,t). \tag{2.10}$$

With the help of (1.17) and (1.18), $\psi(x,t)$ and $\eta(x,t)$ satisfy the following wave equations

$$\psi_{tt} + \psi_t - [p(\psi_x + \bar{n}) - p(\bar{n})]_x - (\frac{\psi_t^2}{\psi_x + \bar{n}})_x + \psi_x(\psi - \eta) + \bar{n}(\psi - \eta) = 0, \tag{2.11}$$

$$\eta_{tt} + \eta_t - [q(\eta_x + \bar{h}) - q(\bar{h})]_x - (\frac{\eta_t^2}{\eta_x + \bar{h}})_x + \eta_x(\eta - \psi) + \bar{h}(\eta - \psi) = 0. \tag{2.12}$$

We rewrite (2.11), (2.12) in another forms by $(\bar{n}, \bar{h})$ as follows.

$$\psi_{tt} + \psi_t - p'(\bar{n})\psi_{xx} + \bar{n}(\psi - \eta) = F_1, \tag{2.13}$$

$$\eta_{tt} + \eta_t - p'(\bar{h})\eta_{xx} + \bar{h}(\eta - \psi) = F_2, \tag{2.14}$$

where

$$F_1 = [p(\psi_x + \bar{n}) - p(\bar{n}) - p'(\bar{n})\psi_x]_x + (\frac{\psi_t^2}{\psi_x + \bar{n}})_x - \psi_x(\psi - \eta), \tag{2.15}$$

$$F_2 = [q(\eta_x + \bar{h}) - q(\bar{h}) - q'(\bar{h})\eta_x]_x + (\frac{\eta_t^2}{\eta_x + \bar{h}})_x - \eta_x(\eta - \psi). \tag{2.16}$$

We now devote to the a priori estimates of the solution (ψ, η) to the equations (2.13) and (2.14). Since the boundary conditions (2.9) and (2.10) for (ψ, η), the estimates over F_1, F_2 are similar to those in Hsiao and Yang [3]. We omit the detail here. We focus on the left terms of (2.13) and (2.14) which embody the differences between unipolar and bipolar cases.

Performing $\partial_x^i (i = 0, 1, 2)$ to (2.13) and (2.14), we have, respectively,

$$\partial_x^i \psi_{tt} + \partial_x^i \psi_t - p'(\bar{n})\partial_x^i \psi_{xx} + \bar{n}(\partial_x^i \psi - \partial_x^i \eta) = \partial_x^i F_1, \tag{2.17}$$

$$\partial_x^i \eta_{tt} + \partial_x^i \eta_t - q'(\bar{h})\partial_x^i \eta_{xx} + \bar{h}(\partial_x^i \eta - \partial_x^i \psi) = \partial_x^i F_2. \tag{2.18}$$

Here and in the sequel, ∂_x^i denotes differentiating i times with respect to x. Multiplying (2.17) and (2.18) by $\bar{h}\partial_x^i \psi$ and $\bar{n}\partial_x^i \eta$, respectively, and integrating them over $(0,1)$, we have

$$\begin{aligned}&\frac{d}{dt}\int \bar{h}(\partial_x^i \psi \partial_x^i \psi_t + \frac{(\partial_x^i \psi)^2}{2}) + \bar{h}\int p'(\bar{n})(\partial_x^{i+1}\psi)^2 - \bar{h}\int (\partial_x^i \psi_t)^2 + \int \bar{h}\bar{n}(\partial_x^i \psi - \partial_x^i \eta)\partial_x^i \psi \\ =&\bar{h}\int \partial_x^i F_1 \partial_x^i \psi,\end{aligned} \tag{2.19}$$

$$\begin{aligned}&\frac{d}{dt}\int \bar{n}(\partial_x^i \eta \partial_x^i \eta_t + \frac{(\partial_x^i \eta)^2}{2}) + \bar{n}\int q'(\bar{h})(\partial_x^{i+1}\eta)^2 - \bar{n}\int (\partial_x^i \eta_t)^2 + \int \bar{h}\bar{n}(\partial_x^i \eta - \partial_x^i \psi)\partial_x^i \eta \\ =&\bar{n}\int \partial_x^i F_2 \partial_x^i \eta.\end{aligned} \tag{2.20}$$

In view of (2.7), (2.9) and (2.10), we have

$$\sum_{i=0}^{2}\int \partial_x^i F_1 \partial_x^i \psi \leq O(\delta)\int (\sum_{k=0}^{2}(\partial_x^k \psi_t)^2+\sum_{k=0}^{3}(\partial_x^k\psi)^2), \tag{2.21}$$

$$\sum_{i=0}^{2}\int \partial_x^i F_2 \partial_x^i \eta \leq O(\delta)\int (\sum_{k=0}^{2}(\partial_x^k \eta_t)^2+\sum_{k=0}^{3}(\partial_x^k\eta)^2), \tag{2.22}$$

with the help of the Taylor expansion and the integration by parts. By (2.19)-(2.22), we have

$$\begin{aligned}&\frac{d}{dt}\int\sum_{i=0}^{2}[\bar h(\partial_x^i\psi\partial_x^i\psi_t+\frac{(\partial_x^i\psi)^2}{2})+\bar n(\partial_x^i\eta\partial_x^i\eta_t+\frac{(\partial_x^i\eta)^2}{2})]\\ &\quad+a_1\int\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\ -\bar h\sum_{i=0}^{2}\int(\partial_x^i\psi_t)^2-\bar n\sum_{i=0}^{2}\int(\partial_x^i\eta_t)^2\\ \leq& O(\delta)\int\Big[\sum_{k=0}^{2}((\partial_x^k\psi_t)^2+(\partial_x^k\eta_t)^2)+\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\Big],\end{aligned} \tag{2.23}$$

where we have used the poincáre inequality due to (2.9) and a_1 is a positive constant.

Now multiplying (2.17) and (2.18) by $\bar h\partial_x^i\psi_t$ and $\bar n\partial_x^i\eta_t$, respectively, and integrating them over $(0,1)$, we get

$$\begin{aligned}&\frac{d}{dt}\int\bar h\Big[\frac{(\partial_x^i\psi_t)^2}{2}+\frac{\bar n}{2}(\partial_x^i\psi)^2+\frac{1}{2}p'(\bar n)(\partial_x^i\psi_x)^2\Big]+\bar h\int(\partial_x^i\psi_t)^2-\bar n\bar h\int\partial_x^i\psi_t\partial_x^i\eta\\ =&\bar h\int\partial_x^iF_1\partial_x^i\psi_t,\end{aligned} \tag{2.24}$$

and

$$\begin{aligned}&\frac{d}{dt}\int\bar n\Big[\frac{(\partial_x^i\eta_t)^2}{2}+\frac{\bar h}{2}(\partial_x^i\eta)^2+\frac{1}{2}q'(\bar h)(\partial_x^i\eta_x)^2\Big]+\bar n\int(\partial_x^i\eta_t)^2-\bar n\bar h\int\partial_x^i\eta_t\partial_x^i\psi\\ =&\bar n\int\partial_x^iF_2\partial_x^i\eta_t.\end{aligned} \tag{2.25}$$

From (2.7), (2.9) (2.10) and integration by parts, we get

$$\begin{aligned}\sum_{i=0}^{2}\int\partial_x^iF_1\partial_x^i\psi_t\leq&-\frac{d}{dt}\int\Big[\sum_{i=0}^{2}(\frac{p'(\psi_x+\bar n)-p'(\bar n)}{2})(\partial_x^i\psi_x)^2+\frac{i\psi_t^2\psi_{xx}^2}{2^i(\psi_x+\bar n)^2}\Big]\\ &+O(\delta)\int(\sum_{k=0}^{2}(\partial_x^k\psi_t)^2+\sum_{k=0}^{3}(\partial_x^k\psi)^2),\end{aligned} \tag{2.26}$$

and

$$\begin{aligned}\sum_{i=0}^{2}\int\partial_x^iF_2\partial_x^i\eta_t\leq&-\frac{d}{dt}\int\Big[\sum_{i=0}^{2}(\frac{q'(\eta_x+\bar h)-q'(\bar h)}{2})(\partial_x^i\eta_x)^2+\frac{i\eta_t^2\eta_{xx}^2}{2^i(\eta_x+\bar h)^2}\Big]\\ &+O(\delta)\int(\sum_{k=0}^{2}(\partial_x^k\eta_t)^2+\sum_{k=0}^{3}(\partial_x^k\eta)^2).\end{aligned} \tag{2.27}$$

(2.24)-(2.27) imply

$$\begin{aligned}
&\frac{d}{dt}\int\Big\{\sum_{i=0}^{2}\bar{h}\Big[\frac{(\partial_x^i\psi_t)^2}{2}+\frac{\bar{n}}{2}(\partial_x^i\psi)^2+\frac{1}{2}p'(\bar{n})(\partial_x^i\psi_x)^2\\
&\qquad\qquad+(\frac{p'(\psi_x+\bar{n})-p'(\bar{n})}{2})(\partial_x^i\psi_x)^2-\frac{i\psi_t^2\psi_{xx}^2}{2^i(\psi_x+\bar{n})^2}\Big]\\
&\qquad+\sum_{i=0}^{2}\bar{n}\Big[\frac{(\partial_x^i\eta_t)^2}{2}+\frac{\bar{h}}{2}(\partial_x^i\eta)^2+\frac{1}{2}q'(\bar{h})(\partial_x^i\eta_x)^2\\
&\qquad\qquad+(\frac{q'(\eta_x+\bar{h})-q'(\bar{h})}{2})(\partial_x^i\eta_x)^2-\frac{i\eta_t^2\eta_{xx}^2}{2^i(\eta_x+\bar{h})^2}\Big]\\
&\qquad-\bar{n}\bar{h}\sum_{i=0}^{2}\partial_x^i\psi\partial_x^i\eta\Big\}+\bar{h}\sum_{i=0}^{2}\int(\partial_x^i\psi_t)^2+\bar{n}\sum_{i=0}^{2}\int(\partial_x^i\eta_t)^2\\
&\quad\le O(\delta)\int\Big[\sum_{k=0}^{2}((\partial_x^k\psi_t)^2+(\partial_x^k\eta_t)^2)+\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\Big].
\end{aligned}\tag{2.28}$$

(2.23)+2×(2.28) gives

$$\begin{aligned}
&\frac{d}{dt}\int G+a_2\int\Big[\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)+\sum_{i=0}^{2}((\partial_x^i\psi_t)^2+(\partial_x^i\eta_t)^2)\Big]\\
&\quad\le O(\delta)\int\Big[\sum_{k=0}^{2}((\partial_x^k\psi_t)^2+(\partial_x^k\eta_t)^2)+\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\Big],
\end{aligned}\tag{2.29}$$

where $a_2>0$ and

$$\begin{aligned}
G=&\sum_{i=0}^{2}[\bar{h}(\partial_x^i\psi\partial_x^i\psi_t+\frac{(\partial_x^i\psi)^2}{2})+\bar{n}(\partial_x^i\eta\partial_x^i\eta_t+\frac{(\partial_x^i\eta)^2)}{2})]\\
&+\sum_{i=0}^{2}\bar{h}\Big[(\partial_x^i\psi_t)^2+\bar{n}(\partial_x^i\psi)^2+p'(\bar{n})(\partial_x^{i+1}\psi)^2\\
&\qquad+(p'(\psi_x+\bar{n})-p'(\bar{n}))(\partial_x^{i+1}\psi_x)^2-\frac{i\psi_t^2\psi_{xx}^2}{2^{i-1}(\psi_x+\bar{n})^2}\Big]\\
&+\sum_{i=0}^{2}\bar{n}\Big[(\partial_x^i\eta_t)^2+\bar{h}(\partial_x^i\eta)^2+q'(\bar{h})(\partial_x^{i+1}\eta)^2\\
&\qquad+(q'(\eta_x+\bar{h})-q'(\bar{h}))(\partial_x^{i+1}\eta)^2-\frac{i\eta_t^2\eta_{xx}^2}{2^{i-1}(\eta_x+\bar{h})^2}\Big]\\
&-2\bar{n}\bar{h}\sum_{i=0}^{2}\partial_x^i\psi\partial_x^i\eta.
\end{aligned}\tag{2.30}$$

It is clear that

$$\begin{aligned}
&a_3\int\Big[\sum_{k=0}^{2}((\partial_x^k\psi_t)^2+(\partial_x^k\eta_t)^2)+\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\Big]\\
&\quad\le\int Gdx\le a_4\int\Big[\sum_{k=0}^{2}((\partial_x^k\psi_t)^2+(\partial_x^k\eta_t)^2)+\sum_{k=0}^{3}((\partial_x^k\psi)^2+(\partial_x^k\eta)^2)\Big], \qquad (2.31)
\end{aligned}$$

for some positive constants a_3 and a_4, where we have used the Poincáre inequality due to (2.9). Thus, (2.29)-(2.31) imply (2.8). Lemma 2.1 is proved.

Proof of Theorem 1.2

Based on Lemma 2.1, the proof of Theorem 1.2 is standard. In fact, combining the standard theory of existence and uniqueness of local (in time) solutions (see, for instance Majda [5],) with the estimates (2.8), we can extend the local solution by the usual continuation arguments and show that the estimates (2.8) hold globally(see, for instance Hsiao and Luo [2]) if the perturbation $\|(\psi(\cdot,0),\eta(\cdot,0))\|_{H^3}+\|(\psi_t(\cdot,0),\eta(\cdot,0))\|_{H^2}$ is sufficiently small.

References

[1] Fang W. and Ito K., Weak solution to a one dimensional hydrodynamic model of two carrier types for semiconductors, *Nonlinear Anal.*, **28**(1997), 947-963.

[2] Hsiao L. & Luo T., Nonlinear diffusive phenomena of solutions for the system of compressible adiabatic flow through porous media, *J. Diff. Eqns.*,**125**(1996), 329-365.

[3] Hsiao L. & Yang T., Asymptotics of initial boundary value problems for hydrodynamic and drift diffusion models for semiconductors, *J. Diff. Eqns.*,**170**(2001), 472-493.

[4] Hsiao L. & Zhang K., The Global weak solution and relaxation limits of the initial-boundary problem to the bipolar hydrodynamic model for semiconductors, *Math. Models and Methods in Appl. Sci.*, **10** (2000), 1333-1361.

[5] Majda A., Compressible Fluid Flow and Systems of Conservation laws in Several Space Variables, Springer-Verlag, Berlin/New York, 1984.

[6] Natalini R., The bipolar hydrodynamic model for semiconductors and the drift-diffusion equation, *J.Math.Anal.Appl.*, **198** (1996), 262-281.

[7] Zhu C. & Hattori H., Stability of steady solutions for an isentropic hydrodynamic model of semiconductors of two species, *J. Diff. Eqns.*, **166** (2000), 1-32.